Lecture 01: Discrete Mathematics

Course Title: Discrete Mathematics **Course Code:** MTH211 **Class:** BSM-II

Objectives

The main aim of the lecture is to define the notion of

- *proposition or statement*
- *compound proposition*
- *conjunction, disjunction and negation.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hil, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, MeGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.
- https://www.freepik.com/ (for background image)

Logic and Propositional Calculus:

Many algorithms and proofs use logical expressions such as:

“IF p THEN q” or “If p_1 AND p_2, THEN q_1 OR q_2”.

Therefore, it is necessary to know the cases in which these expressions are TRUE or FALSE, that is, to know the “truth value” of such expressions.

Proposition or Statements:

A *proposition* (or *statement*) is a declarative statement which is true or false, but not both.

Examples:

Consider, for example, the following six sentences:

(i) Ice floats in water.	(iii) $2 + 2 = 4$	(v) Where are you going?
(ii) China is in Europe.	(iv) $2 + 2 = 5$	(vi) Do your homework.

The first four are propositions, the last two are not. Also, (i) and (iii) are true, but (ii) and (iv) are false.

Compound Propositions:

Many propositions are *composite*, that is, composed of *subpropositions* and various connectives discussed subsequently. Such composite propositions are called *compound propositions*. A proposition is said to be *primitive* if it cannot be broken down into simpler propositions, that is, if it is not composite.

Examples:

For example, the above propositions (i) through (iv) are primitive propositions. On the other hand, the following two propositions are composite:

"Roses are red and violets are blue." and "John is smart or he studies every night."

Remarks:

The fundamental property of a compound proposition is that its truth value is completely determined by the truth values of its subpropositions together with the way in which they are connected to form the compound propositions

Basic Logical Operations:

Now we discuss the three basic logical operations of *conjunction*, *disjunction*, and *negation* which correspond, respectively, to the English words "*and*," "*or*," and "*not*."

For examples:

i. Roses are red and violets are blue.
ii. John is smart or he studies every night.
iii. China is not in Europe.

Conjunction, $p \wedge q$

Any two propositions can be combined by the word "and" to form a compound proposition called the *conjunction* of the original propositions. Symbolically, $p \wedge q$ read "p and q," denotes the conjunction of p and q. Since $p \wedge q$ is a proposition it has a truth value, and this truth value depends only on the truth values of p and q defined as follows:

Definition:

If p and q are true, then $p \wedge q$ is true; otherwise $p \wedge q$ is false.

Examples:

Consider the following four statements:

(i) Ice floats in water and 2 + 2 = 4.

(ii) Ice floats in water and 2 + 2 = 5.

(iii) China is in Europe and 2 + 2 = 4.

(iv) China is in Europe and 2 + 2 = 5.

Only the first statement is true. Each of the others is false since at least one of its substatements is false.

Disjunction, $p \vee q$

Any two propositions can be combined by the word "or" to form a compound proposition called the *disjunction* of the original propositions. Symbolically,

$$p \vee q \text{ read "}p \text{ or } q\text{,"}$$

denotes the disjunction of p and q. The truth value of $p \vee q$ depends only on the truth values of p and q as follows:

Definition:

If p and q are false, then $p \vee q$ is false; otherwise $p \vee q$ is true.

Examples:

Consider the following four statements:

(i) Ice floats in water or 2 + 2 = 4.

(ii) Ice floats in water or 2 + 2 = 5.

(iii) China is in Europe or 2 + 2 = 4.

(iv) China is in Europe or 2 + 2 = 5.

Only the last statement (iv) is false. Each of the others is true since at least one of its sub-statements is true.

Negation, $\neg p$

Given any proposition *p*, another proposition, called the *negation* of *p*, can be formed by writing “It is not true that . . .” or “It is false that . . .” before *p* or, if possible, by inserting in *p* the word “not.” Symbolically, the negation of *p*, read “not *p*,” is denoted by $\neg p$.

The truth value of $\neg p$ depends on the truth value of *p* as follows:

Definition: If *p* is true, then $\neg p$ is false; and if *p* is false, then $\neg p$ is true.

Example:

Consider the following six statements:

(a_1) Ice floats in water. (a_2) It is false that ice floats in water. (a_3) Ice does not float in water.

(b_1) 2 + 2 =5 (b_2) It is false that 2 + 2 = 5. (b_3) $2 + 2 \neq 5$.

Then (a_2) and (a_3) are each the negation of (a_1); and (b_2) and (b_3) are each the negation of (b_1). Since (a_1) is true, (a_2) and (a_3) are false; and since (b_1) is false, (b_2) and (b_3) are true.

Remark:

The logical notation for the connectives "and," "or," and "not" is not completely standardized. For example, some texts use:

$$p \,\&\, q,\ p \cdot q \text{ or } pq \text{ for } p \wedge q,$$

$$p + q \text{ for } p \vee q, \text{ and}$$

$$p',\ \overline{p} \text{ or } {\sim}p \text{ for } \neg p.$$

THANKS FOR YOUR ATTENTION