

Lecture 10: Discrete Mathematics

Course Title: Discrete Mathematics

Course Code: MTH211

Class: BSM-II

Objectives

The main aim of the lecture is to discuss

- *Types of relations.*
- *Reflexive, symmetric and antisymmetric.*

References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hill, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, McGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.

Types of Relations:

Here we discuss a number of important types of relations defined on a set A .

Reflexive Relations

A relation R on a set A is reflexive if aRa for every $a \in A$, that is, if $(a, a) \in R$ for every $a \in A$.

Thus R is not reflexive if there exists $a \in A$ such that $(a, a) \notin R$.

Example 1

Consider the following five relations on the set $A = \{1, 2, 3, 4\}$:

$$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$$

$$R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$$

$$R_3 = \{(1, 3), (2, 1)\}$$

$$R_4 = \emptyset, \text{ the empty relation}$$

$$R_5 = A \times A, \text{ the universal relation}$$

Since A contains the four elements 1, 2, 3, and 4, a relation R on A is reflexive if it contains the four pairs $(1, 1)$, $(2, 2)$, $(3, 3)$, and $(4, 4)$. Thus only R_2 and the universal relation $R_5 = A \times A$ are reflexive. Note that R_1 , R_3 , and R_4 are not reflexive since, for example, $(2, 2)$ does not belong to any of them.

Example 2

Consider the following five relations:

- (1) Relation \leq (less than or equal) on the set \mathbb{Z} of integers.
- (2) Set inclusion \subseteq on a collection C of sets.
- (3) Relation \perp (perpendicular) on the set L of lines in the plane.
- (4) Relation \parallel (parallel) on the set L of lines in the plane.
- (5) Relation $|$ of divisibility on the set \mathbb{N} of positive integers.
(Recall $x | y$ if there exists z such that $xz = y$.)

The relation (3) is not reflexive since no line is perpendicular to itself. Also (4) is not reflexive since no line is parallel to itself. The other relations are reflexive; that is, $x \leq x$ for every $x \in \mathbb{Z}$, $A \subseteq A$ for any set $A \in C$, and $n | n$ for every positive integer $n \in \mathbb{N}$.

Symmetric Relations

A relation R on a set A is symmetric if whenever aRb then bRa , that is, if whenever $(a, b) \in R$ then $(b, a) \in R$.

A relation R is not symmetric if there exists $a, b \in A$ such that $(a, b) \in R$ but $(b, a) \notin R$.

Example 3

Consider the following five relations on the set $A = \{1, 2, 3, 4\}$:

$$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$$

$$R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$$

$$R_3 = \{(1, 3), (2, 1)\}$$

$$R_4 = \emptyset, \text{ the empty relation}$$

$$R_5 = A \times A, \text{ the universal relation}$$

A relation R_1 is not symmetric since $(1, 2) \in R_1$ but $(2, 1) \notin R_1$. R_3 is not symmetric since $(1, 3) \in R_3$ but $(3, 1) \notin R_3$.

The other relations are symmetric.

Example 4

Consider the following five relations:

- (1) Relation \leq (less than or equal) on the set \mathbb{Z} of integers.
- (2) Set inclusion \subseteq on a collection C of sets.
- (3) Relation \perp (perpendicular) on the set L of lines in the plane.
- (4) Relation \parallel (parallel) on the set L of lines in the plane.
- (5) Relation $|$ of divisibility on the set \mathbb{N} of positive integers.
(Recall $x | y$ if there exists z such that $xz = y$.)

The relation \perp is symmetric since if line a is perpendicular to line b then b is perpendicular to a .

Also, \parallel is symmetric since if line a is parallel to line b then b is parallel to line a .

The other relations are not symmetric.

For example:

$$3 \leq 4 \text{ but } 4 \not\leq 3; \{1, 2\} \subseteq \{1, 2, 3\} \text{ but } \{1, 2, 3\} \not\subseteq \{1, 2\}; \text{ and } 2 | 6 \text{ but } 6 \not| 2.$$

Antisymmetric Relations

A relation R on a set A is antisymmetric if whenever aRb and bRa then $a = b$, that is, if $a \neq b$ and aRb then $b \not R a$.

Thus, R is not antisymmetric if there exist distinct elements a and b in A such that aRb and bRa .

Example 5

Consider the following five relations on the set $A = \{1, 2, 3, 4\}$:

$$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$$

$$R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$$

$$R_3 = \{(1, 3), (2, 1)\}$$

$$R_4 = \emptyset, \text{ the empty relation}$$

$$R_5 = A \times A, \text{ the universal relation}$$

A relation R_2 is not antisymmetric since $(1, 2)$ and $(2, 1)$ belong to R_2 , but $1 \neq 2$. Similarly, the universal relation R_5 is not antisymmetric. All the other relations are antisymmetric.

Example 6

Consider the following five relations:

- (1) Relation \leq (less than or equal) on the set \mathbb{Z} of integers.
- (2) Set inclusion \subseteq on a collection C of sets.
- (3) Relation \perp (perpendicular) on the set L of lines in the plane.
- (4) Relation \parallel (parallel) on the set L of lines in the plane.
- (5) Relation $|$ of divisibility on the set \mathbb{N} of positive integers.
(Recall $x | y$ if there exists z such that $xz = y$.)

The relation \leq is antisymmetric since whenever $a \leq b$ and $b \leq a$ then $a = b$.

Set inclusion \subseteq is antisymmetric since whenever $A \subseteq B$ and $B \subseteq A$ then $A = B$.

Also, divisibility on \mathbb{N} is antisymmetric since whenever $m | n$ and $n | m$ then $m = n$. (Note that divisibility on \mathbb{Z} is not antisymmetric since $3 | -3$ and $-3 | 3$ but $3 \neq -3$.)

The relations \perp and \parallel are not antisymmetric.

Remark

The properties of being symmetric and being antisymmetric are not negatives of each other. For example, the relation $R = \{(1, 3), (3, 1), (2, 3)\}$ is neither symmetric nor antisymmetric. On the other hand, the relation $R' = \{(1, 1), (2, 2)\}$ is both symmetric and antisymmetric.

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