

# Lecture 03: Discrete Mathematics

Course Title: Discrete Mathematics

Course Code: MTH211

Class: BSM-II

## Objectives

The main aim of the lecture is to

- *define the notion of tautologies and contradiction.*
- *define and discuss logical equivalence.*
- *discuss algebra of propositions.*
- *define conditional and biconditional statements.*

## References:

- S. Lipschutz and M. Lipson, Schaum's Outlines Discrete Mathematics, Third Edition, McGraw-Hill, 2007.
- K.H. Rosen, Discrete Mathematics and its Application, McGraw-Hill, 6th edition. 2007.
- K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall. New Jersey, 2003.
- <https://www.freepik.com/> (for background image)

## Tautologies and Contradictions

Some propositions  $P(p, q, \dots)$  contain only  $T$  in the last column of their truth tables or, in other words, they are true for any truth values of their variables. Such propositions are called *tautologies*.

Analogously, a proposition  $P(p, q, \dots)$  is called a *contradiction* if it contains only  $F$  in the last column of its truth table or, in other words, if it is false for any truth values of its variables.

**For example:** From the truth table,  $p \vee \neg p$  is tautology and  $p \wedge \neg p$  is contradiction.

$p$	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

$p$	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

## Logical Equivalence

Two propositions  $P(p, q, \dots)$  and  $Q(p, q, \dots)$  are said to be *logically equivalent*, or simply *equivalent* or *equal*, denoted by

$$P(p, q, \dots) \equiv Q(p, q, \dots)$$

if they have identical truth tables.

**For example**, Consider the truth tables of  $\neg(p \wedge q)$  and  $\neg p \vee \neg q$ :

Observe that both truth tables are the same, that is, both propositions are false in the first case and true in the other three cases.

$p$	$q$	$p \wedge q$	$\neg(p \wedge q)$	$p$	$q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	T	T	F	F	F
T	F	F	T	T	F	F	T	T
F	T	F	T	F	T	T	F	T
F	F	F	T	F	F	T	T	T
$(a) \neg(p \wedge q)$				$(b) \neg p \vee \neg q$				

Accordingly, we can write:  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

**Remark:** Let  $p$  be “Roses are red” and  $q$  be “Violets are blue.” Let  $S$  be the statement:

“It is not true that roses are red and violets are blue.”

Then  $S$  can be written in the form  $\neg(p \wedge q)$ . However, as noted above,  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .

Accordingly,  $S$  has the same meaning as the statement:

“Roses are not red, or violets are not blue.”

## Algebra of Propositions

Propositions satisfy various laws which are listed in the table below. (In this table,  $T$  and  $F$  are restricted to the truth values “True” and “False,” respectively.)

### Laws of the algebra of propositions

<b>Idempotent laws:</b>	(1a) $p \vee p \equiv p$	(1b) $p \wedge p \equiv p$
<b>Associative laws:</b>	(2a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$	(2b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
<b>Commutative laws:</b>	(3a) $p \vee q \equiv q \vee p$	(3b) $p \wedge q \equiv q \wedge p$
<b>Distributive laws:</b>	(4a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	(4b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
<b>Identity laws:</b>	(5a) $p \vee F \equiv p$ (6a) $p \vee T \equiv T$	(5b) $p \wedge T \equiv p$ (6b) $p \wedge F \equiv F$
<b>Involution law:</b>	(7) $\neg\neg p \equiv p$	
<b>Complement laws:</b>	(8a) $p \vee \neg p \equiv T$ (9a) $\neg T \equiv F$	(8b) $p \wedge \neg p \equiv F$ (9b) $\neg F \equiv T$
<b>DeMorgan's laws:</b>	(10a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$	(10b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

## Conditional and Biconditional Statements

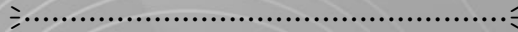
A statement of the form “If  $p$  then  $q$ ” is called *conditional* statement and is denoted by  $p \rightarrow q$ . The conditional  $p \rightarrow q$  is frequently read “ $p$  implies  $q$ ” or “ $p$  only if  $q$ .”

A statement of the form “ $p$  if and only if  $q$ ” is called *biconditional* statement and is denoted by  $p \leftrightarrow q$ .

The truth values of  $p \rightarrow q$  and  $p \leftrightarrow q$  are defined by the the following tables:

$p$	$q$	$p \rightarrow q$	$p$	$q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	F	F	T

(a)  $p \rightarrow q$                       (b)  $p \leftrightarrow q$



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