COMSATS University Islamabad

Attock Campus

Department of Mathematics

Assignment # 01

Class: BSM-VIII
Subject: Complex Analysis
Instructor: Dr. Atiq ur Rehman

Due Date: 30-09-2025
Course Code: MTH324
Marks: 36

Instructions:

1. Please submit the handwritten assignment and don't share with other classmates.
2. Read Chapter 01 of the book to answer questions following the terminologies of the book.

Question # 0: You are kindly requested to write the following statement exactly as provided and to sign if handwritten or type your initials if composed.

"I have read the instructions and understand what I am writing."

- **Complex numbers (as ordered pairs)**
- **Equality of complex numbers**
- **Addition and multiplication**
- **The number $\boldsymbol{i}$**

Question # 1: Define set of complex numbers. Prove that $(3,1)(3,-1)\left(\frac{1}{5},\frac{1}{10}\right) = (2,1)$.

Question # 2: If $(x, y) \in \mathbb{C}$ such that

$$(x, y)(u, v) = (1, 0).$$

Then fine the value of u and v.

Question # 3: Solve the equation $z^2 + z + 1 = 0$ for $z = (x, y)$ by writing

$$(x, y)(x, y) + (x, y) + (1, 0) = (0, 0).$$

- **Representation $\boldsymbol{x + iy}$**
- **Real part and imaginary part**

Question # 4: If $i = (1,0)$ then prove that $i^2 = -1$. Also show that $i^n = 1$ if n is **divisible** by 4.

Question # 5: Show that

(a) $\mathrm{Re}(iz) = -\mathrm{Im}\, z$ (b) $\mathrm{Im}(iz) = \mathrm{Re}\, z$.

Question # 6: Use calculator to solve

$$\frac{1+2i}{3-4i} + \frac{2-i}{5i}.$$

If z represents the above number, then write $\mathrm{Re}\, z$, $\mathrm{Im}\, z$.

Question # 7: Verify the following equality of complex numbers by using calculator:

$$(2+i) + (3-2i)\cdot(1+i) - \frac{4+i}{1-i} = (2+3i)\cdot(1-i) + (5-i) - \frac{3-2i}{2+i}.$$

- **Complex plane (Argand diagram)**
- **Modulus (absolute value)**
- **Complex conjugate**

Conjugate

Question # 8: Locate the numbers $z_1 + z_2$ and $z_1 - z_2$ vectorially when
(a) $z_1 = 2i$ and $z_2 = \frac{2}{3} - i$ (b) $z_1 = (-3, 1)$ and $z_2 = (1, 4)$

Also find the maximum and minimum value of $|z_1 + z_2|$ and $|z_1 - z_2|$.

Question # 9: ***Using the properties of moduli***, prove that if $|z_3| \neq |z_4|$, then

$$\frac{Re(z_1 + z_2)}{|z_3 + z_4|} \leq \frac{|z_1| + |z_2|}{\big||z_3| - |z_4|\big|}.$$

Question # 10: By factorizing $z^4 - 4z^2 + 3$ into two quadratic factors and using

$$\big||z_1| - |z_2|\big| \leq |z_1 \pm z_2|$$

and if z lies on the circle $|z| = 2$, then

$$\left|\frac{1}{z^4 - 4z^2 + 3}\right| \leq \frac{1}{3}.$$

Question # 11: Prove that $|z|^2 = |\bar{z}|^2 = z\bar{z}$

Question # 12: Use properties of conjugates and moduli to prove

$$|(2\bar{z} + 7)(\sqrt{2} + i)(1 + i\sqrt{2})| = 3|2z + 7|.$$

Question # 13: Sketch the set of points:
(a) $|z - 1 - i| = 1$ (b) $|z + i| \leq 3$.

- **Argument (angle)**
- **Polar (trigonometric) form**
- **Exponential form (Euler's formula)**
- **Multiplication and division in polar form**
- **Powers of complex numbers (De Moivre's theorem)**

Question # 14: Find the principal argument
(a) $1 + i$ (b) $5 - 5i$ (c) $\sqrt{3} + i$

Question # 15: By writing the individual on the left as exponential form, prove that
(a) $i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + \sqrt{3}i)$ (b) $(1 + \sqrt{3}i)^{-10} = 2^{-11}(-1 + \sqrt{3}i)$.

- **Roots of complex numbers (nth roots)**

Question # 16: Find the cube root of $4 + i4\sqrt{3}$ and draw them as the corner point of certain triangle.

Question # 17: Factorize $z^4 + 4$ up to it linear factors using idea of 4th roots of z.

Question # 18: Draw: $z = 3 - 2i + 2e^{i\theta}$ with $-\frac{\pi}{2} \leq \theta \leq \pi$.
