Name: ..... **Reg. No.: CIIT/.....–BSE–...../ATK**

Assignment 3: Stochastic Process: BSE-VIII (Fall 2025)

Instructions:

- Please choose **all correct options (there might be more than one)** by filling or ticking or crossing the box.
- Spoiled or overwritten selection has no credit.
- Please don't use whitener or any other eraser.

Q. 1. Which of the following statements about the Markov property are correct?

- ☐ For a Markov chain, $P(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots) = P(X_{n+1} = j | X_n = i)$
- ☐ Simple random walk satisfies the Markov property
- ☐ The Markov property requires knowledge of at least two previous states
- ☐ The future state depends only on the present state, not on past states

Q. 2. Which of the following define a simple random walk?

- ☐ A Markov chain with state space $\mathbb{Z}$ and transition probabilities $P_{i,i+1} = p$, $P_{i,i-1} = 1 - p$
- ☐ A process where each step size follows a normal distribution
- ☐ $X_{n+1} = X_n + 1$ with probability p, $X_{n+1} = X_n - 1$ with probability q
- ☐ $X_0 = 0$, $X_n = X_{n-1} + Y_n$ where $Y_n = \pm 1$ with equal probability

Q. 3. For a simple symmetric random walk:

- ☐ The probability of moving left equals the probability of moving right
- ☐ The process always returns to the origin after an even number of steps
- ☐ It represents an unbiased random movement
- ☐ $p = q = \frac{1}{2}$

Q. 4. The Chapman-Kolmogorov equation states that:

- ☐ $P^{(m+n)} = P^{(m)} P^{(n)}$
- ☐ The n-step transition matrix equals P^n
- ☐ $P_{ij}^{(m+n)} = \sum_k P_{ik}^{(m)} P_{kj}^{(n)}$
- ☐ $P_{ij}^{(n)} = P_{ij}^n$ for all i, j

Q. 5. A stochastic process is defined as:

- ☐ A mathematical model for a system that evolves randomly over time
- ☐ A collection of random variables indexed by time
- ☐ A deterministic process with random initial conditions
- ☐ A sequence describing the evolution of a physical process through time

Q. 6. A discrete-time Markov chain is characterized by:

- ☐ Transition probabilities P_{ij} satisfying $\sum_j P_{ij} = 1$
- ☐ The Markov property: future depends only on the present
- ☐ A countable state space
- ☐ Independent increments

Q. 7. For a transition probability matrix P:

- ☐ Each row sums to 1
- ☐ Entries represent $P_{ij} = P(X_{n+1} = j | X_n = i)$
- ☐ Each column sums to 1
- ☐ The (i, j)-th entry gives the probability of moving from state i to state j

Q. 8. A second-order Markov chain:

- ☐ Satisfies the standard Markov property
- ☐ Can be converted to a first-order chain using augmented states
- ☐ Depends on the current and previous state
- ☐ Has state space $S \times S$ when converted to first-order

Q. 9. The state space of a stochastic process:

- ☐ Must always be numerical
- ☐ For a simple random walk is $\mathbb{Z}$
- ☐ Is the set of all possible values the random variables can assume
- ☐ Can be finite, countable, or continuous

Q. 10. For n-step transition probabilities:

- ☐ They can be computed using the Chapman-Kolmogorov equation
- ☐ $P_{ij}^{(n)} = P(X_{m+n} = j | X_m = i)$
- ☐ $P^{(n)} = P^n$ for a time-homogeneous Markov chain
- ☐ $P_{ij}^{(n)} = (P_{ij})^n$

Q. 11. The conditional law of total probability states that:

- ☐ It extends the regular law of total probability to conditional probabilities
- ☐ $P(A|C) = \sum_k P(A|B_k, C)P(B_k|C)$
- ☐ $P(A|C) = \sum_k P(A, B_k|C)$
- ☐ $P(A|C) = P(A, C)/P(C)$ only when events are independent

Q. 12. A university tracks where students prefer to study: Library (L), Cafeteria (C), and Ground (G). Each week, students may change their preferred study location according to the following probabilities:
• Students in the Library stay there with probability 0.2, move to the Cafeteria with 0.5.
• Students in the Cafeteria stay with 0.6, move to the Library with 0.25.
• Students in the Garden stay with 0.55, move to the Cafeteria with 0.35.
The current distribution of students in Library is 15% and Cafeteria is 35%.
Using this weekly transition pattern, what percentage of students will prefer each location after a month?.