# Lecture 03

**Course Title:** Calculus with Analytic Geometry **Course Code:** MTH104

## Objectives

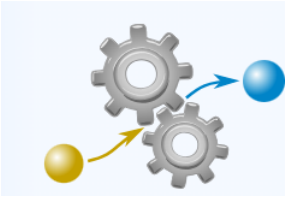
The main aim of the lecture is to discuss:

- *Functions*
- *Graph of the functions*

## References:


- Earl W. Swokowski, Calculus with Analytic Geometry, PWS Publisher, Boston, 1988.
- James Stewart, Calculus Early Transcendental, 6th Ed., Thomson Brooks/Cole, 2008.

## Function



It is like a machine that has input and an output.
And the output is related somehow to the input.

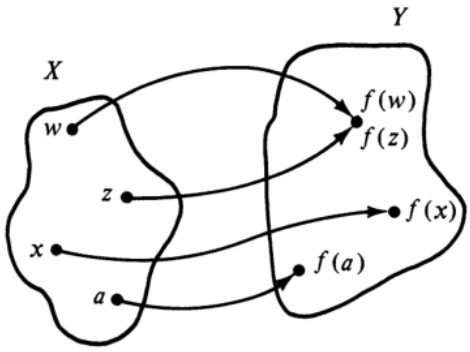
We will see many ways to think about functions, but there are always three main parts:

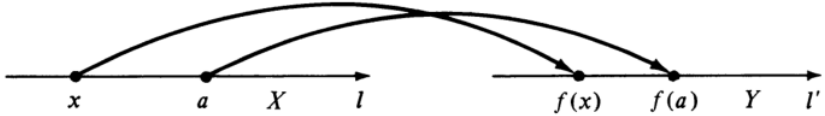
- The input (called domain)
- The relationship (pictorial or algebraic expressions or any other way)
- The output (called codomain or range)

**Definition (formal definition of the function)**

A function *f* from a set *X* to a set *Y* is a correspondence that assigns to each element *x* of *X,* a unique element *y* of *Y*. The element *y* is called the image of *x* under *f* and is denoted by *f*(*x*).

The set *X* is called the ***domain*** of the function. The ***range*** of the function consists of all images of elements of X.



Two functions *f* and *g* from *X* to *Y* are said to be **equal**, and we write

***f* = *g*, provided *f*(*x*) = *g*(*x*) for every *x* in *X*.**

For example,

if $g(x) = \frac{1}{2}(2x^2 - 6) + 3— 6)$ and $f(x) = x^2$ for all $x \in \mathbb{R}$, then $g = f$.


**Graph does not exist without**

- **Domain**
- **Expression to represent output (in calculus, we use algebraic expression to repression)**

**In calculus, if function is represented by only algebraic expression without giving domain, it means, we must consider all possible real values satisfying the given expression.**

**Example:**

(i) $f(x) = \sqrt{x-1}$

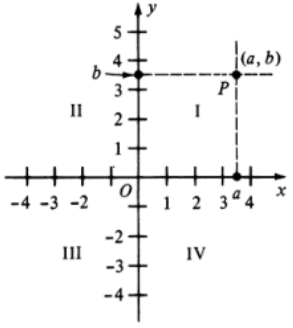
(ii) $g(x) = \frac{1}{x^2-1}$

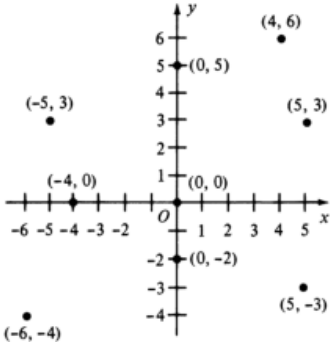
(iii) $h(x) = \sqrt{1-x}$

(iv) $k(x) = \sin^{-1} x$

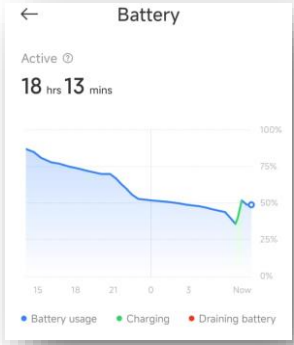
(v) $m(x) = \log(x+1)$

We have a very nice representation (graph) of the function, if we take $x$ horizontally on real line and $f(x)$ on vertical real line. In this case we use coordinates system in two dimensions (Section 1.2 of [1]), which was also part of the mathematics book of the FSc.

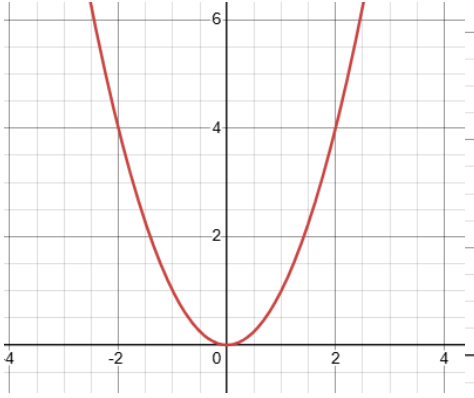


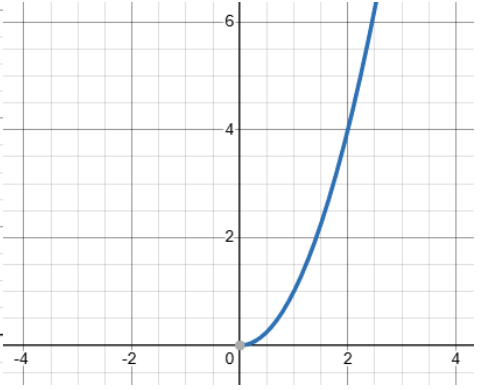



Let’s discuss a very common graph you’ll encounter in everyday life: the time versus battery percentage graph. This graph illustrates how the battery life of a device, such as your phone or laptop, depletes over time.



**What about these graphs? Are they different or the same?**



**Other notions, which you have already studies to consider:**

- Constant function
- Identity function
- Onto function
- One-to-one function
- Even function
- Odd function

**Point to ponder:** Use any software tool to draw the graph of the function instead of the classical way.

**Solve Exercise 1.4**