



# COMSATS Institute of Information Technology Attock campus

## Department of Mathematics

### Assignment # 03

**Class:** MSc-IV  
**Subject:** Real Analysis II  
**Instructor:** Dr. Atiq ur Rehman

**Due Date:** 24-11-2016  
**Course Code:** MTH322  
**Marks:** 6

#### Question # 1:

Prove that a series of functions  $\sum f_n$  defined on  $[a, b]$  converges uniformly on  $[a, b]$  if and only if for every  $\varepsilon > 0$  and for all  $x \in [a, b]$ , there exist an integer  $N$  such that

$$|f_{n+1}(x) + f_{n+2}(x) + \dots + f_{n+p}(x)| < \varepsilon, \quad n \geq N, p \geq 1.$$

#### Question # 2:

Give the short answers of the following question not more than four lines:

- i. Define convergence of the sequence.
- ii. Define convergence of the series.
- iii. Define pointwise convergence of sequences of functions.
- iv. Give an example of the sequence of functions, which is uniformly convergent.
- v. Give an example of the sequence of functions, which is pointwise convergent but not uniformly convergent.
- vi. Is the sequence of functions  $\left\{ \frac{1}{x+n} \right\}$  is pointwise convergent on  $D = \{0, 1, 2, 3\}$ .
- vii. Define uniform convergence of series of functions.

#### Academic Honesty Requirements:

You are encouraged to work with others in the completion of assignments but it doesn't include copying. However, in the spirit of Academic Honesty, which includes crediting others for their contribution to your work, please include one of the following statements with every submitted assignment on title page:

1. I worked alone on this assignment.
2. I worked with the following: List their full names. Include their relationship to you if they are not also a member of this class.