Exercise 7.2 (Solutions)

Question # 1

Given $A(2,5)$, $B(-1,1)$ and $C(2,-6)$

(i) $\overrightarrow{AB} = (-1-2)\underline{\hat{i}} + (1-5)\underline{\hat{j}} = -3\underline{\hat{i}} - 4\underline{\hat{j}}$

(ii) From above $\overrightarrow{AB} = -3\underline{\hat{i}} - 4\underline{\hat{j}}$

Also $\overrightarrow{CB} = (2+1)\underline{\hat{i}} + (-6-1)\underline{\hat{j}} = 3\underline{\hat{i}} - 7\underline{\hat{j}}$

Now

$$2\overrightarrow{AB} - \overrightarrow{CB} = 2\left(-3\underline{\hat{i}} - 4\underline{\hat{j}}\right) - \left(3\underline{\hat{i}} - 7\underline{\hat{j}}\right)$$
$$= -6\underline{\hat{i}} - 8\underline{\hat{j}} - 3\underline{\hat{i}} + 7\underline{\hat{j}}$$
$$= -9\underline{\hat{i}} - \underline{\hat{j}}$$

(iii) *Do yourself as above*

Question # 2

(i) $\underline{u} = \underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}}$

$\underline{v} = 3\underline{\hat{i}} - 2\underline{\hat{j}} + 2\underline{\hat{k}}$

$\underline{w} = 5\underline{\hat{i}} - \underline{\hat{j}} + 3\underline{\hat{k}}$

$$\underline{u} + 2\underline{v} + \underline{w} = \underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}} + 2\left(3\underline{\hat{i}} - 2\underline{\hat{j}} + 2\underline{\hat{k}}\right) + \left(5\underline{\hat{i}} - \underline{\hat{j}} + 3\underline{\hat{k}}\right)$$
$$= \underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}} + 6\underline{\hat{i}} - 4\underline{\hat{j}} + 4\underline{\hat{k}} + 5\underline{\hat{i}} - \underline{\hat{j}} + 3\underline{\hat{k}}$$
$$= 12\underline{\hat{i}} - 3\underline{\hat{j}} - 6\underline{\hat{k}}$$

(ii) *Do yourself*

(iii)

$$3\underline{v} + \underline{w} = 3\left(3\underline{\hat{i}} - 2\underline{\hat{j}} + 2\underline{\hat{k}}\right) + 5\underline{\hat{i}} - \underline{\hat{j}} + 3\underline{\hat{k}}$$
$$= 9\underline{\hat{i}} - 6\underline{\hat{j}} + 6\underline{\hat{k}} + 5\underline{\hat{i}} - \underline{\hat{j}} + 3\underline{\hat{k}}$$
$$= 14\underline{\hat{i}} - 7\underline{\hat{j}} + 9\underline{\hat{k}}$$

Now $\left|3\underline{v} + \underline{w}\right| = \sqrt{(14)^2 + (-7)^2 + (9)^2}$
$$= \sqrt{196 + 49 + 81} = \sqrt{326}$$

Question # 3

(i) $\underline{v} = 2\underline{\hat{i}} + 3\underline{\hat{j}} + 4\underline{\hat{k}}$

$$\Rightarrow \left|\underline{v}\right| = \sqrt{(2)^2 + (3)^2 + (4)^2}$$
$$= \sqrt{4 + 9 + 16} = \sqrt{29}$$

Unit vector of $\underline{v} = \underline{\hat{v}} = \dfrac{\underline{v}}{\left|\underline{v}\right|} = \dfrac{2\underline{\hat{i}} + 3\underline{\hat{j}} + 4\underline{\hat{k}}}{\sqrt{29}}$

$$= \frac{2}{\sqrt{29}}\underline{\hat{i}} + \frac{3}{\sqrt{29}}\underline{\hat{j}} + \frac{4}{\sqrt{29}}\underline{\hat{k}}$$

Hence direction cosines of $\underline{v}$ are

$$\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}.$$

(ii) *Do yourself as above.*

(iii) *Do yourself as (i)*

Question # 4

Since $\left|a\underline{\hat{i}} + \left(a+1\right)\underline{\hat{j}} + 2\underline{\hat{k}}\right| = 3$

$$\Rightarrow \sqrt{a^2 + \left(a+1\right)^2 + \left(2\right)^2} = 3$$
$$\Rightarrow \sqrt{a^2 + a^2 + 2a + 1 + 4} = 3$$

On squaring both sides

$$2a^2 + 2a + 5 = 9$$
$$\Rightarrow 2a^2 + 2a + 5 - 9 = 0$$
$$\Rightarrow 2a^2 + 2a - 4 = 0$$
$$\Rightarrow a^2 + a - 2 = 0$$
$$\Rightarrow a^2 + 2a - a - 2 = 0$$
$$\Rightarrow a\left(a+2\right) - 1\left(a+2\right) = 0$$
$$\Rightarrow \left(a+2\right)\left(a-1\right) = 0$$
$$\Rightarrow a + 2 = 0 \quad \text{or} \quad a - 1 = 0$$
$$\Rightarrow a = -2 \quad \text{or} \quad a = 1$$

Question # 5

Given $\underline{v} = \underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}}$

$$\left|\underline{v}\right| = \sqrt{(1)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{1 + 4 + 1} = \sqrt{6}$$

Now

$$\underline{\hat{v}} = \frac{\underline{v}}{\left|\underline{v}\right|} = \frac{\underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}}}{\sqrt{6}}$$
$$= \frac{1}{\sqrt{6}}\underline{\hat{i}} + \frac{2}{\sqrt{6}}\underline{\hat{j}} - \frac{1}{\sqrt{6}}\underline{\hat{k}}$$

Question # 6

Given $\underline{a} = 3\underline{\hat{i}} - \underline{\hat{j}} - 4\underline{\hat{k}}$

$\underline{b} = -2\underline{\hat{i}} - 4\underline{\hat{j}} - 3\underline{\hat{k}}$

$\underline{c} = \underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}}$

Suppose that

$$\underline{d} = 3\underline{a} - 2\underline{b} + 4\underline{c}$$
$$\Rightarrow \underline{d} = 3\left(3\underline{\hat{i}} - \underline{\hat{j}} - 4\underline{\hat{k}}\right) - 2\left(-2\underline{\hat{i}} - 4\underline{\hat{j}} - 3\underline{\hat{k}}\right) + 4\left(\underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}}\right)$$
$$= 9\underline{\hat{i}} - 3\underline{\hat{j}} - 12\underline{\hat{k}} + 4\underline{\hat{i}} + 8\underline{\hat{j}} + 6\underline{\hat{k}} + 4\underline{\hat{i}} + 8\underline{\hat{j}} - 4\underline{\hat{k}}$$
$$= 17\underline{\hat{i}} - 13\underline{\hat{j}} - 10\underline{\hat{k}}$$

Now

$$\left|\underline{d}\right| = \sqrt{\left(17\right)^2 + \left(-13\right)^2 + \left(-10\right)^2}$$

$= \sqrt{289+169+100} \quad = \sqrt{558} \quad = 3\sqrt{62}$

Now

$$\hat{\underline{d}} = \frac{\underline{d}}{|\underline{d}|} = \frac{17\hat{\underline{i}} - 13\hat{\underline{j}} - 10\hat{\underline{k}}}{3\sqrt{62}}$$

$$= \frac{17}{3\sqrt{62}}\hat{\underline{i}} - \frac{13}{3\sqrt{62}}\hat{\underline{j}} - \frac{10}{3\sqrt{62}}\hat{\underline{k}}$$

Question # 7

Consider $\underline{a} = 2\hat{\underline{i}} - 3\hat{\underline{j}} + 6\hat{\underline{k}}$

$$|\underline{a}| = \sqrt{(2)^2 + (-3)^2 + (6)^2}$$
$$= \sqrt{4+9+36} \quad = \sqrt{49} = 7$$

Now

$$\hat{\underline{a}} = \frac{\underline{a}}{|\underline{a}|} = \frac{2\hat{\underline{i}} - 3\hat{\underline{j}} + 6\hat{\underline{k}}}{7}$$
$$= \frac{2}{7}\hat{\underline{i}} - \frac{3}{7}\hat{\underline{j}} + \frac{6}{7}\hat{\underline{k}}$$

Let $\underline{b}$ be a vector having magnitude 4

i.e. $|\underline{b}| = 4$

Since $\underline{b}$ is parallel to $\underline{a}$

therefore $\hat{\underline{b}} = \hat{\underline{a}} = \frac{2}{7}\hat{\underline{i}} - \frac{3}{7}\hat{\underline{j}} + \frac{6}{7}\hat{\underline{k}}$

Now $\underline{b} = |\underline{b}|\hat{\underline{b}} = 4\left(\frac{2}{7}\hat{\underline{i}} - \frac{3}{7}\hat{\underline{j}} + \frac{6}{7}\hat{\underline{k}}\right)$

$$= \frac{8}{7}\hat{\underline{i}} - \frac{12}{7}\hat{\underline{j}} + \frac{24}{7}\hat{\underline{k}}$$

(ii) *Do yourself.*

Question # 8

Given $\underline{u} = 2\hat{\underline{i}} + 3\hat{\underline{j}} + 4\hat{\underline{k}}$

$\underline{v} = -\hat{\underline{i}} + 3\hat{\underline{j}} - \hat{\underline{k}}$

$\underline{w} = \hat{\underline{i}} + 6\hat{\underline{j}} + z\hat{\underline{k}}$

Since $\underline{u}$, $\underline{v}$ and $\underline{w}$ are sides of triangle therefore

$\underline{u} + \underline{v} = \underline{w}$

$\Rightarrow\ 2\hat{\underline{i}} + 3\hat{\underline{j}} + 4\hat{\underline{k}} - \hat{\underline{i}} + 3\hat{\underline{j}} - \hat{\underline{k}} = \hat{\underline{i}} + 6\hat{\underline{j}} + z\hat{\underline{k}}$

$\Rightarrow\ \hat{\underline{i}} + 6\hat{\underline{j}} + 3\hat{\underline{k}} = \hat{\underline{i}} + 6\hat{\underline{j}} + z\hat{\underline{k}}$

Equating coefficient of $\hat{\underline{k}}$ only, we have

$3 = z$ i.e. $\boxed{z = 3}$

Question # 9

Position vector (p.v) of point $A = 2\hat{\underline{i}} - \hat{\underline{j}} + \hat{\underline{k}}$

p.v of point $B = 3\hat{\underline{i}} + \hat{\underline{j}}$

p.v. of point $C = 2\hat{\underline{i}} + 4\hat{\underline{j}} - 2\hat{\underline{k}}$

p.v. of point $D = -\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}}$

$\overrightarrow{AB} =$ p.v. of $B -$ p.v. of A

$= 3\hat{\underline{i}} + \hat{\underline{j}} - 2\hat{\underline{i}} + \hat{\underline{j}} - \hat{\underline{k}} = \hat{\underline{i}} + 2\hat{\underline{j}} - \hat{\underline{k}}$

$\overrightarrow{CD}$ = p.v. of $D -$ p.v. of C

$= -\hat{\underline{i}} - 2\hat{\underline{j}} + \hat{\underline{k}} - 2\hat{\underline{i}} - 4\hat{\underline{j}} + 2\hat{\underline{k}}$

$= -3\hat{\underline{i}} - 6\hat{\underline{j}} + 3\hat{\underline{k}}$

$= -3\left(\hat{\underline{i}} + 2\hat{\underline{j}} - \hat{\underline{k}}\right) = -3\overrightarrow{AB}$

i.e. $\overrightarrow{CD} = l\,\overrightarrow{AB}$ where $l = -3$

Hence $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

Question # 10 (i)

$\underline{v} = 2\hat{\underline{i}} - 4\hat{\underline{j}} + 4\hat{\underline{k}}$

$$|\underline{v}| = \sqrt{(2)^2 + (-4)^2 + (4)^2}$$
$$= \sqrt{4+16+16} \quad = \sqrt{36} \quad = 6$$

Now $\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{2\hat{\underline{i}} - 4\hat{\underline{j}} + 4\hat{\underline{k}}}{6}$

$$= \frac{1}{3}\hat{\underline{i}} - \frac{2}{3}\hat{\underline{j}} + \frac{2}{3}\hat{\underline{k}}$$

The two vectors of length 2 and parallel to $\underline{v}$ are $2\hat{\underline{v}}$ and $-2\hat{\underline{v}}$.

$$2\hat{\underline{v}} = 2\left(\frac{1}{3}\hat{\underline{i}} - \frac{2}{3}\hat{\underline{j}} + \frac{2}{3}\hat{\underline{k}}\right) = \frac{2}{3}\hat{\underline{i}} - \frac{4}{3}\hat{\underline{j}} + \frac{4}{3}\hat{\underline{k}}$$

$$-2\hat{\underline{v}} = -2\left(\frac{1}{3}\hat{\underline{i}} - \frac{2}{3}\hat{\underline{j}} + \frac{2}{3}\hat{\underline{k}}\right) = -\frac{2}{3}\hat{\underline{i}} + \frac{4}{3}\hat{\underline{j}} - \frac{4}{3}\hat{\underline{k}}$$

Question # 10 (ii)

Given $\underline{v} = \hat{\underline{i}} - 3\hat{\underline{j}} + 4\hat{\underline{k}}$, $\underline{w} = a\hat{\underline{i}} + 9\hat{\underline{j}} - 12\hat{\underline{k}}$

Since $\underline{v}$ and $\underline{w}$ are parallel therefore there exists $l \in \mathbb{R}$ such that

$\underline{v} = l\,\underline{w}$

$\Rightarrow\ \hat{\underline{i}} - 3\hat{\underline{j}} + 4\hat{\underline{k}} = l\left(a\hat{\underline{i}} + 9\hat{\underline{j}} - 12\hat{\underline{k}}\right)$

$\Rightarrow\ \hat{\underline{i}} - 3\hat{\underline{j}} + 4\hat{\underline{k}} = a\,l\,\hat{\underline{i}} + 9\,l\,\hat{\underline{j}} - 12\,l\,\hat{\underline{k}}$

Comparing coefficients of $\hat{\underline{i}}$, $\hat{\underline{j}}$ and $\hat{\underline{k}}$

$1 = a\,l$ ....(i)

$-3 = 9\,l$ ....(ii)

$4 = -12\,l$ ....(iii)

From (ii) $l = -\frac{3}{9} \Rightarrow l = -\frac{1}{3}$

Putting in equation (i)

$1 = a\left(-\frac{1}{3}\right) \Rightarrow -3 = a$ i.e. $\boxed{a = -3}$

Question # 10 (c)

Consider $\underline{v} = \hat{\underline{i}} - 2\hat{\underline{j}} + 3\hat{\underline{k}}$

$$|\underline{v}| = \sqrt{(1)^2 + (-2)^2 + (3)^2}$$
$$= \sqrt{1+4+9} \quad = \sqrt{14}$$

Now

$$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{\underline{\hat{i}} - 2\underline{\hat{j}} + 3\underline{\hat{k}}}{\sqrt{14}}$$
$$= \frac{1}{\sqrt{14}}\underline{\hat{i}} - \frac{2}{\sqrt{14}}\underline{\hat{j}} + \frac{3}{\sqrt{14}}\underline{\hat{k}}$$

Let $\underline{a}$ be a vector having magnitude 5 i.e. $|\underline{a}| = 5$

Since $\underline{a}$ is parallel to $\underline{v}$ but opposite in direction,
therefore

$$\hat{\underline{a}} = -\hat{\underline{v}} = -\frac{1}{\sqrt{14}}\underline{\hat{i}} + \frac{2}{\sqrt{14}}\underline{\hat{j}} - \frac{3}{\sqrt{14}}\underline{\hat{k}}$$

Now

$$\underline{a} = |\underline{a}|\hat{\underline{a}} = 5\left(-\frac{1}{\sqrt{14}}\underline{\hat{i}} + \frac{2}{\sqrt{14}}\underline{\hat{j}} - \frac{3}{\sqrt{14}}\underline{\hat{k}}\right)$$
$$= -\frac{5}{\sqrt{14}}\underline{\hat{i}} + \frac{5}{\sqrt{14}}\underline{\hat{j}} - \frac{5}{\sqrt{14}}\underline{\hat{k}}$$

Question # 10 (d)

Suppose that $\underline{v} = 3\underline{\hat{i}} - \underline{\hat{j}} + 4\underline{\hat{k}}$ and

$\underline{w} = a\underline{\hat{i}} + b\underline{\hat{j}} - 2\underline{\hat{k}}$

$\because$ $\underline{v}$ and $\underline{w}$ are parallel

$\therefore$ there exists $l \in \mathbb{R}$ such that

$$\underline{v} = l\,\underline{w}$$
$$\Rightarrow\ 3\underline{\hat{i}} - \underline{\hat{j}} + 4\underline{\hat{k}} = l\left(a\underline{\hat{i}} + b\underline{\hat{j}} - 2\underline{\hat{k}}\right)$$
$$\Rightarrow\ 3\underline{\hat{i}} - \underline{\hat{j}} + 4\underline{\hat{k}} = al\,\underline{\hat{i}} + bl\,\underline{\hat{j}} - 2l\,\underline{\hat{k}}$$

Comparing coefficients of $\underline{\hat{i}}$, $\underline{\hat{j}}$ and $\underline{\hat{k}}$

$3 = al$ ....(i)

$-1 = bl$ ....(ii)

$4 = -2l$ ....(iii)

From equation (iii)

$$-\frac{4}{2} = l \quad \Rightarrow \quad l = -2$$

Putting value of l in equation (i)

$$3 = a(-2) \quad \Rightarrow \quad \boxed{a = -\frac{3}{2}}$$

Putting value of l in equation (ii)

$$-1 = b(-2) \quad \Rightarrow \quad \boxed{b = \frac{1}{2}}$$

Question # 11 (i)

$$\underline{v} = 3\underline{\hat{i}} - \underline{\hat{j}} + 2\underline{\hat{k}}$$
$$|\underline{v}| = \sqrt{(3)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{9+1+4} = \sqrt{14}$$

Let $\hat{\underline{v}}$ be unit vector along $\underline{v}$. Then

$$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{3\underline{\hat{i}} - \underline{\hat{j}} + 2\underline{\hat{k}}}{\sqrt{14}}$$
$$= \frac{3}{\sqrt{14}}\underline{\hat{i}} - \frac{1}{\sqrt{14}}\underline{\hat{j}} + \frac{2}{\sqrt{14}}\underline{\hat{k}}$$
$$\hat{\underline{v}} = \left[\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right]$$

Hence the direction cosines of $\underline{v}$ are

$$\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}.$$

Question # 11 (ii)

$$\underline{v} = 6\underline{\hat{i}} - 2\underline{\hat{j}} + \underline{\hat{k}}$$
$$|\underline{v}| = \sqrt{(6)^2 + (-2)^2 + (1)^2}$$
$$= \sqrt{36+4+1} = \sqrt{41}$$

Let $\hat{\underline{v}}$ be unit vector along $\underline{v}$. Then

$$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|} = \frac{6\underline{\hat{i}} - 2\underline{\hat{j}} + \underline{\hat{k}}}{\sqrt{41}}$$
$$= \frac{6}{\sqrt{41}}\underline{\hat{i}} - \frac{2}{\sqrt{41}}\underline{\hat{j}} + \frac{1}{\sqrt{41}}\underline{\hat{k}}$$
$$\hat{\underline{v}} = \left[\frac{6}{\sqrt{41}}, \frac{-2}{\sqrt{41}}, \frac{1}{\sqrt{41}}\right]$$

Hence the direction cosines of $\underline{v}$ are

$$\frac{6}{\sqrt{41}}, \frac{-2}{\sqrt{41}}, \frac{1}{\sqrt{41}}.$$

Question # 11 (iii)

$$P = (2,1,5),\ Q = (1,3,1)$$
$$\overrightarrow{PQ} = (1-2)\underline{\hat{i}} + (3-1)\underline{\hat{j}} + (1-5)\underline{\hat{k}}$$
$$= -\underline{\hat{i}} + 2\underline{\hat{j}} - 4\underline{\hat{k}}$$
$$\left|\overrightarrow{PQ}\right| = \sqrt{(-1)^2 + (2)^2 + (-4)^2}$$
$$= \sqrt{1+4+16} = \sqrt{21}$$

Let $\hat{\underline{v}}$ be unit vector along $\overrightarrow{PQ}$. Then

$$\hat{\underline{v}} = \frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \frac{-\underline{\hat{i}} + 2\underline{\hat{j}} - 4\underline{\hat{k}}}{\sqrt{21}}$$
$$= \frac{-1}{\sqrt{21}}\underline{\hat{i}} + \frac{2}{\sqrt{21}}\underline{\hat{j}} - \frac{4}{\sqrt{21}}\underline{\hat{k}}$$
$$\hat{\underline{v}} = \left[\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}\right]$$

Hence the direction cosines of $\overrightarrow{PQ}$ are

$$\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}.$$

Question # 12(i)

$45^\circ, 45^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 45^\circ + \cos^2 45^\circ + \cos^2 60^\circ = 1$$

L.H.S $= \cos^2 45^\circ + \cos^2 45^\circ + \cos^2 60^\circ$

$$=\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2$$
$$=\frac{1}{2}+\frac{1}{2}+\frac{1}{4} \quad = \quad \frac{5}{4} \neq \text{R.H.S}$$
Therefore given angles are not direction angles.

Question # 12(ii)

$30^\circ, 45^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 30^\circ+\cos^2 45^\circ+\cos^2 60^\circ=1$$
$$\text{L.H.S } =\cos^2 30^\circ+\cos^2 45^\circ+\cos^2 60^\circ$$
$$=\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2$$
$$=\frac{3}{4}+\frac{1}{2}+\frac{1}{4} \quad = \quad \frac{3}{2} \neq \text{R.H.S}$$
Therefore given angles are not direction angles.

Question # 12 (iii)

$30^\circ, 60^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 45^\circ+\cos^2 60^\circ+\cos^2 60^\circ=1$$
$$\text{L.H.S } =\cos^2 45^\circ+\cos^2 60^\circ+\cos^2 60^\circ$$
$$=\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2$$
$$=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$= \ 1 \ = \ \text{R.H.S}$$
Therefore given angles are direction angles.

Error Analyst
Muhammad Tayyab Riaz (2009-10) **Pakistan International School Al-Khobar, Saudi Arabia.** **Awais (2009-10)** **Punjab College, Lahore.** **Salman Ali (2009-2010)** **Superior College Multan.**

Book: *Exercise 7.2*
Calculus and Analytic Geometry
Mathematic 12
Punjab Textbook Board, Lahore.
Edition: August 2003.

Made by: ***Atiq ur Rehman*** *(atiq@mathcity.org)*

Printed: January 25, 2009
Updated: October 25, 2010