M.Phil Admission Test Spring 2009

QUAID-E-AZAM UNIVERSITY, ISLAMABAD.

Mathematics , Dated: 29-1-2009, http://www.mathcity.org

Time: 09:30 a.m. to 11.00 a.m. Dated: 29-01-2009

Note: Section 1 is compulsory, Section II-for Applied Mathematics and Section III is for Pure Mathematics candidates:

Section:I

Q.1 Find the local maximum, minimum or saddle point of the function

$$f(x,y) = (x^2+y)e^{y/2}$$

Q.2 (i). Let V be a vector space over the field F and W is its subspace. Prove that V/W is vector space over F.

(ii) $\mathbb{C}[x] = \{ f(x) = \sum_{i=0}^{n} z_i x^i \mid z_i \in \mathbb{C} \}$ is vector space over $\mathbb{R}$ and

$x\,\mathbb{C}[x] = \{ g(x) \in \mathbb{C}[x] \mid x \text{ divides } g(x) \}$ is subspace of $\mathbb{C}[x]$ over $\mathbb{R}$.

Q: Let $\beta(t)$ be a regular, non-unit speed curve in $\mathbb{R}^3$. Show that for torsion τ we have

$$\tau = [\dot{\beta}, \ddot{\beta}, \dddot{\beta}] / |\dot{\beta} \times \ddot{\beta}|^2$$

where the bracket denotes the scalar triple product.

Q: Let $d : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ be a metric on $\mathbb{R}$ defined by

$$d(x,y) = 0 \iff x = y$$

$$d(x,y) = |x| + |y| \iff x \neq y.$$

Find $S_2(2)$, $S_3(2)$ and $S_6(2)$. Prove or Disprove that $A = \{1, 2, 3, \ldots 20\}$ is an open set.

Q Show that with $W(x)=1$, the functions $\phi_i(x) = \cos(ix)$ $i = 0, 1, 2, \ldots$ are orthogonal over the interval $[-\pi, \pi]$. Construct the corresponding orthonormal set of functions.

Q. Show by an example that every bounded function need not be Riemann-integrable.

Q: Find the finite presentation of the symmetric group.

Q. Let $S_{\alpha\beta}$ be a symmetric tensor and $T^{\alpha\beta}$ be a skew symmetric tensor in an n-dimensional space.

(a) Work out the number of independent components of $S_{\alpha\beta}$.

(b) Work out the number of independent components of $T^{\alpha\beta}$.

(c) Show that $S_{\alpha\beta}T^{\alpha\beta} = 0$.

Q: Let C_N denote the boundary of a square formed by the lines

$$x = \pm(N+\tfrac{1}{2})\pi, \quad y = \pm(N+\tfrac{1}{2})\pi$$

where N is a positive integer, and let the orientation of C_N be counterclockwise.

Show that

$$\left| \int_{C_N} \frac{dz}{z^2 \sin z} \right| \leq \frac{16}{(2N+1)A}$$

where $|\sin z| \geq A$.

Q: A system consists of three particles, each of unit mass, with positions and velocities as follows

$\underline{r}_1 = \underline{i} + \underline{j}$ $\quad \underline{v}_1 = 2\underline{i}$

$\underline{r}_2 = \underline{j} + \underline{k}$ $\quad \underline{v}_2 = \underline{j}$

$\underline{r}_3 = \underline{k}$ $\quad \underline{v}_3 = \underline{i} + \underline{j} + \underline{k}$

(i) Find the position and velocity of the centre of mass.

(ii) Find the kinetic energy of the system.

Q: Find the inverse of $\begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ by LU decomposition method.

Q: Show that in an inner product space, $x \perp y$ if and only if $\|x + \alpha y\| \geq \|x\|$ for all scalars α.

Q: Solve the following first order p.d.e.

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = (1+x)e^{y}$$

Q. (a) Compute the x-component of velocity u for one-dimensional steady compressible flow when $\rho = 1 + x^2$. Assume for $x = 0$; $u = 4$.

(b) Verify whether the following functions are valid potential functions:

(i) $\phi = A(x^2 - y^2)$ $\quad$ (ii) $\phi = A\cos x$

Downloaded from: http://www.MathCity.org

Q: Give the Klein – Gordon and the Dirac wave equations. Why the Klein – Gordon equation is not a valid wave equation, prove.

Q. A train of rest length 30 metres is moving with 60 percent of the speed of light. Calculate its length as it appears to an observer at rest on the platform.

Q: Write the Lagrangian for a simple harmonic oscillator. Using Lagrange's equations of motion deduce the equations of motion for the harmonic oscillator.

Q Reduce the following integral equation to a differential equation

$$u(x) = \lambda \int_0^\infty e^{-|x-t|} u(t)\,dt$$

Q. NA Solve $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ in the range $0 \le x \le 0.1$ Using modified Euler's method when $h = 0.1$

Q One of the Maxwell's equations $\nabla \cdot \vec{D} = \rho$ can be derived from Gauss's law. Interpret this physically and derive Poisson's equation as well as Laplace's equation from it.

Q Let F be a field and $F^* = F \setminus \{0\}$. Show that $GL(3,F)/SL(3,F)$ is isomorphic to F^*.

Let R^2 be an inner product space and M be an independent subset of R^2. Find $M^\perp$.

Show that a commutative ring with identity is a field if and only if each ideal of it is prime.

Downloaded from: http://www.MathCity.org

Q. Indicate true or false.

a) $\mathbb{Z}$ is not a module over $\mathbb{Q}$.

b) Every module is a vector space.

c) Every vector space is a Free module.

d) $\mathbb{Q}/\mathbb{Z}$ is module over $\mathbb{Z}$.

e) $\mathbb{Q}$ is module over $\mathbb{Z}$.

Show that a $\mathbb{Z}$-module is injective if and only if it is divisible.

Show that a set of measure zero is measurable.

A number X is chosen at random from the series $4, 9, 14, 19, \ldots$ and another number Y is chosen from the series $1, 5, 9, 13, \ldots$ Each series has 100 terms. Find $P(X=Y)$.

****

For updates visit: ***http://www.MathCity.org***

****

Downloaded from: *http://www.MathCity.org*