QUAID-I-AZAM UNIVERSITY
DEPARTMENT OF MATHEMATICS

Test for M.Phil. Admission Spring – 2006

Time: 9:30 am. To 11: 00 a.m. Dated: 15-02-2006

NOTE:- Attempt as many questions as you can.

Q.1 Solve
$$u_{tt} = c^2 u_{xx} \quad , \quad x > 0,\ t > 0$$
$$u(0,t) = A_0 \sin \omega t \ , \quad t > 0$$
$$u(x,0) = u_t(x,0) = 0 \ , \quad x \geq 0$$
A_0 is constant and u is bounded.

Q.2 Find explicitly the centres of alternating group A_n and general linear group $GL(n,F)$.

Q.3. Show that If C is the boundary of the triangle with vertices at the points 0, $3i$ and -4, oriented in counter clockwise direction, then $\left| \int_C (e^z - \bar{z})\,dz \right| \leq 60$

Q.4 Write down basic laws of electromagnetism in Mathematical language and find their solutions.

Q.5 The position of a particle is given by the position vector
$$\underline{r}(t) = a \sin\theta\ \underline{i} + b\cos\theta\ \underline{j} + a\theta\ \underline{k}$$
where θ is a function of the time t.
If the mass of the particle is unit, then find the momentum of the particle at any time t. Also find the law of force at $\theta = \frac{\pi}{4}$.

Q.6 The dynamical behavior of a mechanical system is ~~describbed~~ described by the Lagrangian function

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} \omega x^2$$

Find Hamilton's canonical equations for the motion of the system.

Q.7 Show that the retract of an injective module is injective.

Q.8 Let N be the set of natural numbers. Let J consists of N and all those subsets of N which does not contain 2. Show that J is a topology on N. Is this topology compact? Prove or disprove.

Q.9 Determine whether the space of all continuous real valued functions defined over the interval $[0,1]$ form a Hilbert space with respect to any inner product.

Q.10 A deck of playing cards is shuffled and two cards are drawn one by one without replacement. what is the probability that the first card is greater in denomination then the second?

Q.11 Solve

$$\dot{x} = \int_0^x 3^{x-t} \, u(t)\, dt$$

Q.12 a) Give an example of a binary operation on $\{A, N, Z\}$.

b) Let $H \trianglelefteq G$ s.t. $[G:H] = 3$. Show that $x^3 \in H \quad \forall x \in G$.

Q.13 a) Decide true or false.

(i) The minimal polynomial divides the characteristic polynomial of a linear transformation.

(ii) $Q + X\,R[x]$ is subspace of R-space $R[x]$

(iii) $\dim_{\mathbb{Z}_2}(GF(8)) = 3$

(IV) Dimension of 0 space is 0.

(V) A finite dimensional vector space has only one basis.

b) verify that $\operatorname{Ker} T$ where T is linear transformation of a vector space V, is invariant under T.

Q.14 Find the Eigen values and Eigen functions of the following problem.

$$(x y')' + \lambda x^{-1} y = 0$$

$$y(1) = 0$$

$$y'(e) = 0$$

Q.15 If in a ring R with unity, $(xy)^2 = x^2 y^2$ for all $x, y \in R$, Then R is commutative.

Q.16 Test the convergence of the integral

$$\int_1^{\infty} \frac{\cos x + \sin 3x}{x^{5/4}}\, dx.$$

Q.17 If $\vec{F} = xy\,\hat{i} + y^2 z\,\hat{j} + z^3\,\hat{k}$, evaluate $\iint (\vec{F}\cdot\vec{n})\, dS$, where S is the unit cube defined by

$$0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1.$$

Q.18 The stream function and velocity potential for a flow are given by:

$$\psi = 2xy, \qquad \phi = x^2 - y^2$$

show that the conditions of continuity and irrotational flow are satisfied.

Q.19 Let $\underline{\alpha}(s)$ be a unit speed curve whose image lies on a sphere of radius r and center $\underline{m}$. Show that $K \neq 0$. If $\tau \neq 0$ then show that

$$\underline{\alpha} - \underline{m} = -\rho \underline{N} - \rho' \sigma \underline{B}$$

where $\rho = \frac{1}{K}$, $\sigma = \frac{1}{\tau}$, $\underline{N}$ is the principal normal vector and $\underline{B}$ is the binormal vector to the curve. (prime denotes differentiation w.r.t to s).

Q.20 Write the Lorentz transformation equations for an arbitrary 4-vector. Derive the Lorentz transformation equations for the electric and magnetic field components.

Q.21 Find the value of n to make the following integral

$$\int_0^{2\pi} (\sin x + \cos x)\, dx$$

correct upto six decimal place when using Composite Simpson Rule

Q.22 Calculate $\nabla^2 f(1,1)$ numerically when

$$f(x,y) = x^4 - 2x^2y^2 + y^4$$

Q.23 Obtain the geodesic equation for the surface

$$x = (r \sin\phi,\ r\cos\phi,\ r)$$

Also calculate the second fundamental form.

Q.24 Show that outer measure of an interval is its length.

Good Luck.