M.Phil Admission Test Fall 2010

Quaid-e-Azam University, Islamabad.

Mathematics , Dated: 18-8-2010, http://www.mathcity.org

Time: 90 minutes

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Note: Section I is compulsory, Section II is for Applied Mathematics and Section III for Pure Mathematics candidates.

Use separate page for each question.

Section I

Q. 1. If $0 < a < b$, determine $\lim\limits_{n\to\infty}\left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$.

Q. 2. Compute the integral or prove its divergence $\int\limits_0^1 \frac{dx}{\sqrt{1-x^2}}$.

Q. 3. a) In the group $(\mathbb{C}-\{0\},\cdot)$ mention an element with order n.

b) In the group $(\mathbb{C}-\{0\},\cdot)$ mention an element with order ∞.

c) When a group is of finite order?

d) Give an example of a multiplicative group with order 8 containing a prime field with 3 elements.

e) Establish an isomorphism between the groups $(\mathbb{R},+)$ and $(\mathbb{R}^+,\cdot)$.

Q. 4. Let $X \neq \phi$ and $x_0 \in X$. Show that $\Im = \{X\} \cup \{A \subseteq X : x_0 \notin A\}$ forms a topology on X.

Q. 5. Solve $x^2y^3 + x(1+y^2)y' = 0$.

Q. 6. Let T be a linear operator on $\mathbb{R}^3$ defined by $T(x,y,z) = (2y+z, x-4y, 3x)$. Find the matrix of T in the basis $\{(1,1,1),(1,1,0),(1,0,0)\}$.

Q. 7. Define a conservative force. Prove that the work done by such a force around a closed loop is zero. Is the converse true?

Q. 8. Let C denote the circle $|z| = 1$, taken in the positive sense. Evaluate the integral $\int\limits_C \exp(z+\frac{1}{z})dz$.

Q. 9. Find the first fundamental form of the surface:

$x = \{3(\cos\phi\sin\theta), 3(\sin\phi\sin\theta), 3(\cos\theta)\} \qquad 0 \le \theta \le \pi, 0 \le \phi \le 2\pi$,

Q. 10. Find the contracted tensor components of F_b^a, where

$F_{db}^{ac} = A_d^a B_b^c$ and $A_d^a = \begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}$, $B_b^c = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 0 & 9 \end{bmatrix}$.

Q. 11. Show that $\left(\frac{\Delta^2}{E}\right)(e^x)\cdot\frac{E(e^x)}{\Delta^2(e^x)} = e^x$.

Q. 12. Solve $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t};\ o < x < \infty, t > 0$

$u(0,t) = A,\ t \ge 0$

$u(x,0) = 0,\ 0 < x < \infty$

u and $\frac{\partial u}{\partial x}$ both tend to zero when $x \to \infty$.

Q. 13. Let $T : N \to M$ be a surjective bounded and linear operator from a normed space N to a normed space M. If there exists a positive real number b such that $\|Tx\| \ge b\|x\|$ for all $x \in N$, then T^{-1} exists and is bounded.

Section II

Q. 14. Find the solution of $\operatorname{Sin} nx = \int_0^x e^{x-t}u(t)dt$.

Q. 15. Is the motion $u = \frac{kx}{x^2+y^2}, v = \frac{ky}{x^2+y^2}, w = 0$ kinematically possible for an incompressible fluid (k is constant).

Q. 16. Write the Lorentz transformations as a rotation about a fixed axis, hence prove that the Lorentz transformations form a rotation group (Lorentz -Poincare group).

Q. 17. Let $\widehat{A}$ be an operator such that for a state vector $|\Psi\rangle$:
$|\Psi\rangle = 2i\,|+\rangle - 3\,|0\rangle + |-\rangle$,
$\widehat{A}\,|\Psi\rangle = 3\,|+\rangle + |0\rangle - i\,|-\rangle$.
Find the matrix and outer product representation for $\widehat{A}$ in basis $\{|+\rangle, |0\rangle, |-\rangle\}$.

Q. 18. Derive equation of motion for isotropic elastic medium.

Q. 19. Derive wave equations for conducting medium in terms of electric and magnetic fields.

Q. 20. Use Picard's method to approximate $y(0.1)$ given that $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$. By performing four iterations.

Section III

Q. 21. Let H be a Hilbert space and $A \subseteq H$. Then $\left(\overline{A}\right)^{\perp} = A^{\perp}$.

Q. 22. In how many ways can 7 boys and 2 girls be lined up in a row such that the girls must be separated by exactly 3 boys?

Q. 23. a) Prove that $\frac{\mathbb{Z}}{n\mathbb{Z}} \simeq \mathbb{Z}_n$.
b) Verify that F^n is an algebra over the field F.
c) For field F verify that if $k < n$ be positive integers, then
$0 \to F^k \xrightarrow{\sigma} F^n \xrightarrow{\xi} F^{n-k} \to 0$ is short exact sequence of F-algebras.
(Hint show that $\operatorname{Im}\sigma = \ker\xi$) (3+3+4)

Q. 24. a) Define a linear code.
b) In short exact sequence of $\mathbb{Z}_2$-spaces
$0 \to \mathbb{Z}_2^k \to \mathbb{Z}_2^n \to \mathbb{Z}_2^{n-k} \to 0$ explain the code is linear subspace of $\mathbb{Z}_2^n$ having dimension k but each code vector has n digits.
c) What is a cyclic code?
d) Code described in (b) has a specific name, explain. (2+3+2+3)

Q. 25. Show that retract of a projective module is projective.

Q. 26. Show that outer measure of an interval is its length.

Q. 27. Prove that every action gives rise to a permutation representation and vice versa.

Good Luck

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