

Subject: Math: IV-VI(viii)/IX-XI(viii) M.A/M.Sc: Part- II / Composite, 1st -A/2011

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University of Sargodha

M.A/M.Sc Part-II / Composite, 1st -A/2011

Math: IV-VI(viii)/IX-XI(viii) Special Theory of Relativity & Analytical Dynamics

Maximum Marks: 40

Fictitious #: _____

Time Allowed: 45 Min.

Objective Part

Signature of CSO: _____

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q. 1. A: Choose the correct option:

(5)

- i. The SPECIAL RELATIVITY deals with _____
a). Dynamics b). Uniform motion c). Any motion d). None of these
- ii. The FOUR VELOCITY is defined by
a). $\frac{dx^\mu}{dt}$ b). $\frac{dx^\mu}{dx^\nu}$ c). $\frac{dx^\mu}{d\tau}$ d). Both a & c
- iii. The TIME DILATION is defined by _____
a). $\delta t = \delta t'$ b). $\delta t = \frac{\delta t'}{\gamma}$ c). $\frac{\delta t}{\gamma} = \delta t'$ d). None of these
- iv. In D' ALEMBERT,s PRINCIPLE the virtual displacements are
a). Independent b). Dependent c). Arbitrary d). both b & c
- v. The constraints given by $f = (x, y, z, \dot{x}, \dot{y}, \dot{z}, t)$ are known as _____ constraints
a). Non-holonomic b). Scleronomic non-holonomic
c). Holonomic d). Rheonomic non-holonomic

B: Write True or False against each statement:

(10)

- i. The FOUR VECTOR is an invariant quantity.
- ii. The FOUR MOMENTUM is a scalar quantity.
- iii. $E_0 = mc^2$ is called the rest-energy.
- iv. All FOUR VELOCITY vectors have not a constant magnitude.
- v. The TIME-LIKE vectors represents the actual path of a physical object in space over time.
- vi. The LAGRANGIAN of a free particle is $L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$.
- vii. If, in a given LAGRANGIAN, $df(q_1, q_2, \dots, q_n)/dt$ is added, the Lagrange's EOM do not change.

- viii. The non-holonomic constraint equation involve the quantities depending upon the position or possibly the time.
- ix. The constraints given by $f(x,y,z)=0$ are known as REONOMIC HOLONOMIC constraints.
- x. $F = -ma$ represents the reversed effective force.

C: Fill in the blank:

(5)

- i. The determinant of the LORENTZ transformation is _____.
- ii. For null vectors the magnitude of velocity v is equal to _____.
- iii. The mass of a stable nucleus is always _____ the sum of the masses of its constituent particles.
- iv. The binding energy is usually measured in _____.
- v. The HAMILTONIAN function is obtained from _____ transformation.

Q.2. Write short answer of the following :

(20)

- i. Write down the postulates of SPECIAL RELATIVITY.

- ii. Define LENGTH-CONTRACTION.

- iii. Write down one-dimensional LORENTZ transformations.

- iv. A car moves at a speed 160 km/h. If the length of the car is 2.4 m, calculate the decrease in the length as noted by a stationary observer.

- v. Define binding energy with example.

- vi. Define generalized coordinates

- vii. Define FOUR MOMENTUM.

- viii. Define LAGRANGE bracket.

- ix. Define D' ALEMBERT's PRINCIPLE.

- x. Considering one particle case, show that the LAGRANGIAN EOM simply yields the Newton's second law of motion.

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Math: IV-VI(viii)/IX-XI(viii) Special Theory of Relativity & Analytical Dynamics

Time Allowed: 2:15 Hours

Maximum Marks: 60

Subjective Part

Note: Attempt any three questions. All questions carry equal marks.

Q. 3. a. Derive General LORENTZ transformations (10)

b. Let $r_\nu = r_\nu(q_\alpha)$; $\alpha = 1, 2, 3, \dots, n$ then prove that kinetic energy satisfies the

relations; (i) $T = \sum_{\alpha=1}^n \sum_{\beta=1}^n a_{\alpha\beta} \dot{q}_\alpha \dot{q}_\beta$ where $a_{\alpha\beta}$ are the functions of q_α

$$(ii). \dot{q}_1 \frac{\partial T}{\partial \dot{q}_1} + \dot{q}_2 \frac{\partial T}{\partial \dot{q}_2} + \dots + \dot{q}_n \frac{\partial T}{\partial \dot{q}_n} = 2T \quad (10)$$

Q. 4. a. When an observer records an event at $x = 3.2 \times 10^8 \text{ m}$ and $t = 2.5 \text{ s}$ in a rest frame S . Find its respective coordinates in frame S' moving with velocity $0.38c$. (10)

b. Derive LAGRANGE equations of motion for non-conservative system, when constraints are non-holonomic. (10)

Q 5. a. Explain NULL CONE structure (10)

b. Show directly that the following transformations are CANONICAL: (10)

$$(i). P = \log\left(\frac{1}{q} \sin p\right), Q = q \cot p \quad (ii). P = \log(\sin p), Q = p \tan q$$

Q. 6. a. Prove that $f \cdot v = c^2 \frac{dm}{dt}$, then show that $f_\mu = \left(f, \frac{1}{c^2} (f \cdot v)\right)$, where f_μ is the force defined for a system of particle and f represents the 3- force. (10)

b. Derive LAGRANGE equation of motion for holonomic system. Also, show that, for a conservative system, it reduces to

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_\alpha} \right) = \frac{\partial L}{\partial q_\alpha} \quad \text{where } L = T - V \quad (10)$$

Q. 7. a. With usual notations, prove that $E^2 = P^2 c^2 + m_0 c^4$. (10)

b. State and prove NOETHER's theorem. (10)