Roll No. _____ **Subject: Math: IV** **M.A/M.Sc: Part- I / Composite, 1st-A/2011**

**Sig of Dy. Superintendent.** _____

---------- **Cut Here** ----------

# University of Sargodha

**M.A/M.Sc Part-1 / Composite, 1st -A/2011**

**Math: IV** **Mechanics**

**Maximum Marks: 40** **Fictitious #:** _____

**Time Allowed: 45 Min.** ***Objective Part*** **Signature of CSO:** _____

**Note:** **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

| | | |
|---|---|---|
| **1(a)** | Tick the correct option in the following<br>(i) The direction cosines of $x_3 - axis$ are<br>a) $(0,0,1)$ b) $(1,0,0)$ c) $(0,1,0)$ d) $(0,1,1)$<br>(ii) If $\vec{A}$ is conservative field , then<br>a) $\nabla.\vec{A}=0$ b) $\nabla\times\vec{A}=0$ c) both "a" & "b" d) none<br>(iii) Time derivative of Euler's angles represents the<br>a) linear velocity b) angular velocity c) linear acceleration d) none<br>(iv) The most general rigid body motion is _____ motion.<br>a) rectilinear b) screw c) projectile d) simple harmonic<br>(v) Mechanics of the elastic solid deals with the behavior of<br>a) liquids b) solids c) gases d) all of these | 05 |
| **1(b)** | (i) Temperature with in a body is a vector point function. T / F<br>(ii) First partial derivatives are the directional derivatives in the direction of coordinate axes. T / F<br>(iii) In curvilinear coordinate system the coordinate curves are straight lines. T / F<br>(iv) An index which is repeated in a given expression is called free index. T / F<br>(v) The table of direction cosines is called transformation matrix. T / F<br>(vi) The operator $\Delta$ is called a scalar invariant operator if its form is unchanged Under the rotation of coordinate axes. T / F<br>(vii) Hydrodynamics deals with the gases. T / F<br>(viii) The defining condition of a rigid is called the constraint of rigidity. T / F<br>(ix) A rigid body can have the motion of translation and rotation. T / F<br>(x) In rotational motion moment of inertia plays the same role as mass in linear motion T / F | **10** |
| **1(c)** | (i) Tensors of _____ rank can be added.<br>(ii) _____ is a vector at a point "P" normal to the surface $u_1 = c_1$ | **05** |

(iii) An axis of symmetry is always a _____ axis.

(iv) The rotation of a rigid body about a point can be described by using _____ angular coordinate system

(v) For non trivial solution the angular velocity of the system is _ _____ _ _

**Q. No 2** Write short answers of the following (20)

i) If $r^2 = x_1^2 - x_2^2 + x_3^2$ , then show that $\frac{\partial r}{\partial x_k} = \frac{x_k}{r}$

ii) If $\varphi = r^2 e^{-r}$ . Find $\nabla \varphi$ .

iii) Determine the constant "a" so that $\vec{V} = (x+3y)\hat{i} + (y-2x)\hat{j} + (x+az)\hat{k}$ is solenoidal.

iv) Find the area of the region bounded by the ellipse.

v) Express the gradient of a scalar point function in cartesian coordinates by using curvilinear coordinate system.

vi) If $A_i$ & $B_i$ are two first order tensors , then show that their product $A_i B_i$ is a second order tensor.

vii) Differentiate between inertial and non inertial frame of references.

viii) Using operators show that the fixed and rotating coordinate systems can be written as $D_f = D_r + \bar{\omega} X$

ix) Show that $I = Md^2$

x) In torque free motion of a rigid body abut "O" has equal principal axis of inertia. Prove that it must rotate with constant angular velocity.

# University of Sargodha

**M.A/M.Sc Part-1 / Composite, 1st -A/2011**

**Math: IV** **Mechanics**

**Maximum Marks: 60** **Time Allowed: 2:15 Hours**

***Subjective Part***

**Note: Attempt any three questions. All questions carry equal marks.**

**Q.No.3**

a) Verify the Stokes 's theorem in the plane $\bar{A}=(2x-y)\hat{i}-yz\,\hat{j}-y^2z\,\hat{k}$, where "S" is the upper half surface of the sphere $x^2+y^2+z^2=1$ and "C" is its boundary.

b) Derive an expression for curl of a vector in curvilinear coordinate system.

**Q.No.4**

a) Prove that $\nabla\left[\frac{\bar{A}.\bar{r}}{r^3}\right]+\nabla\times\left[\frac{\bar{A}.\times\bar{r}}{r^3}\right]=\bar{0}$ , for any arbitrary constant vector $\bar{A}$ .

b) Verify that the transformation between b the coordinates $x_1, x_2, x_3$ and $x_1', x_2', x_3'$ given by $x_1'=\frac{1}{3}(2x_1+2x_2-x_3)\; x_2'=\frac{1}{3}(2x_1-x_2+2x_3)\; ,x_3'=\frac{1}{3}(-x_1+2x_2+2x_3)$ is orthogonal and left handed. A vector $\bar{A}$ referred in $ox_1x_2x_3$ has components $(2,1,-2)$. Find its components in the new system $ox_1'x_2'x_3'$.

**Q.No.5**

a) Show that the finite rotation of rigid body don't commute , but infinitesimal rotations commute. Also show that sum of angular velocities is also an angular velocity.

b) If $\hat{i}\;\hat{j}\;\hat{k}$ denotes the unit vectors associated with a rotating coordinate system, obtain expressions for the derivatives $\frac{d\hat{i}}{dt}$ etc by an alternative method.

**Q.No.6**

a) Find the moment of inertia of a uniform triangular lamina of a mass "M" about one of its sides.

b) Describe momental ellipsoid.

**Q.No 7**

a) Derive Euler's equation for rigid body motion in a force field. Use these to obtain a complete solution of problem of free rotation of a symmetrical rigid body.

b) Describe how Euler's equation can be used to discuss the motion of solid cylinder rolling down on an inclined plane.