# Exercise 8.3 (Solutions)

Textbook of Algebra and Trigonometry for Class XI
Available online @ http://www.mathcity.org, Version: 1.0.5

***Binomial Theorem when n is negative or fraction:***

When n is negative or fraction and $\left|x\right|<1$ then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots$$

Where the general term of binomial expansion is

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots\ldots\big(n-(r-1)\big)}{r!}x^r$$

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***Question # 1 (i)***

$$(1-x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(-x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-x)^3 + \ldots\ldots$$

$$= 1 - \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}x^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3\cdot 2}(-x^3) + \ldots\ldots$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots\ldots$$

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***Question # 1 (ii) and (iii)*** *Do yourself as above*

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***Question # 1 (iv)***

$$(4-3x)^{\frac{1}{2}} = \left[4\left(1-\frac{3x}{4}\right)\right]^{\frac{1}{2}} = (4)^{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{\frac{1}{2}} = 2\left(1-\frac{3x}{4}\right)^{\frac{1}{2}}$$

$$= 2\left[1 + \frac{1}{2}\left(-\frac{3x}{4}\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{3x}{4}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(-\frac{3x}{4}\right)^3 + \ldots\ldots\right]$$

$$= 2\left[1 - \frac{3x}{8} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{9x^2}{16}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3\cdot 2}\left(-\frac{27x^3}{64}\right) + \ldots\ldots\right]$$

$$= 2\left[1 - \frac{3x}{8} - \frac{1}{8}\left(\frac{9x^2}{16}\right) - \frac{1}{16}\left(\frac{27x^3}{64}\right) + \ldots\ldots\right]$$

$$= 2\left[1 - \frac{3x}{8} - \frac{9x^2}{128} - \frac{27x^3}{1024} + \ldots\ldots\right]$$

$$= 2 - \frac{3x}{4} - \frac{9x^2}{64} - \frac{27x^3}{512} + \ldots\ldots$$

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***Question # 1 (v)***

$$(8-2x)^{\frac{1}{2}} = (8)^{-1}\left(1-\frac{2x}{8}\right)^{-1} = \frac{1}{8}\left(1-\frac{x}{4}\right)^{-1}$$ *Now do yourself*

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***Question # 1 (vi)***

*Do yourself*

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**Question # 1 (vii)**

$$\frac{(1-x)^{-1}}{(1+x)^2}=(1-x)^{-1}(1+x)^{-2}$$
$$=\left(1+(-1)(-x)+\frac{(-1)(-1-1)}{2!}(-x)^2+\frac{(-1)(-1-1)(-1-2)}{3!}(-x)^3+..................\right)$$
$$\times\left(1+(-2)(x)+\frac{(-2)(-2-1)}{2!}(x)^2+\frac{(-2)(-2-1)(-2-2)}{3!}(x)^3+..................\right)$$
$$=\left(1+x+\frac{(-1)(-2)}{2}(x^2)+\frac{(-1)(-2)(-3)}{3\cdot 2}(-x^3)+..................\right)$$
$$\times\left(1-2x+\frac{(-2)(-3)}{2}(x)^2+\frac{(-2)(-3)(-4)}{3\cdot 2}(x)^3+..................\right)$$
$$=\left(1+x+x^2+x^3+..................\right)\times\left(1-2x+3x^2-4x^3+..................\right)$$
$$=1+(x-2x)+(x^2-2x^2+3x^2)+(x^3-2x^3+3x^3-4x^3)..................$$
$$=1-x+2x^2-2x^3+..................$$

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**Question # 1 (viii)**

*Do yourself as above*

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**Question # 1 (ix)**

$$\frac{(4+2x)^{\frac{1}{2}}}{2-x}=(4+2x)^{\frac{1}{2}}(2-x)^{-1}=(4)^{\frac{1}{2}}\left(1+\frac{2x}{4}\right)^{\frac{1}{2}}(2)^{-1}\left(1-\frac{x}{2}\right)^{-1}$$
$$=(4)^{\frac{1}{2}}\left(1+\frac{x}{2}\right)^{\frac{1}{2}}(2)^{-1}\left(1-\frac{x}{2}\right)^{-1}=2\left(1+\frac{x}{2}\right)^{\frac{1}{2}}\frac{1}{2}\left(1-\frac{x}{2}\right)^{-1}=\left(1+\frac{x}{2}\right)^{\frac{1}{2}}\left(1-\frac{x}{2}\right)^{-1}$$
$$=\left(1+\frac{x}{2}\right)^{\frac{1}{2}}\left(1-\frac{x}{2}\right)^{-1}$$
$$=\left(1+\frac{1}{2}\left(\frac{x}{2}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{x}{2}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{x}{2}\right)^3+......................\right)$$
$$\times\left(1+(-1)\left(-\frac{x}{2}\right)+\frac{(-1)(-1-1)}{2!}\left(-\frac{x}{2}\right)^2+\frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{x}{2}\right)^3+..................\right)$$
$$=\left(1+\frac{x}{4}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{x^2}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3\cdot 2}\left(\frac{x^3}{8}\right)+......................\right)$$
$$\times\left(1+\frac{x}{2}+\frac{(-1)(-2)}{2}\left(\frac{x^2}{4}\right)+\frac{(-1)(-2)(-3)}{3\cdot 2}\left(-\frac{x^3}{8}\right)+..................\right)$$
$$=\left(1+\frac{x}{4}-\frac{x^2}{32}+\frac{x^3}{128}+......................\right)\times\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+..................\right)$$
$$=1+\left(\frac{x}{4}+\frac{x}{2}\right)+\left(-\frac{x^2}{32}+\frac{x^2}{8}+\frac{x^2}{4}\right)+\left(\frac{x^3}{128}-\frac{x^3}{64}+\frac{x^3}{16}+\frac{x^3}{8}\right)+..................$$
$$=1+\frac{3x}{4}+\frac{11x^2}{32}+\frac{23x^3}{128}+..................$$

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**Question # 1 (x)**

$$(1+x-2x^2)^{\frac{1}{2}}=\left(1+(x-2x^2)\right)^{\frac{1}{2}}$$

$$=1+\frac{1}{2}(x-2x^2)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(x-2x^2)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(x-2x^2)^3+\ldots\ldots\ldots$$

$$=1+\frac{1}{2}(x-2x^2)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}(x^2-4x^3+4x^4)$$

$$+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3\cdot 2}\left(x^3+3(x)^2(-2x^2)+3(x)(-2x^2)^2-(2x^2)^3\right)+\ldots\ldots\ldots$$

$$=1+\frac{1}{2}(x-2x^2)-\frac{1}{8}(x^2-4x^3+4x^4)+\frac{1}{16}\left(x^3-6x^4+12x^5-8x^6\right)+\ldots\ldots\ldots$$

$$=1+\frac{1}{2}x-\frac{2}{2}x^2-\frac{1}{8}x^2-\frac{4}{8}x^3+\frac{4}{8}x^4+\frac{1}{16}x^3-\frac{6}{16}x^4+\frac{12}{16}x^5-\frac{8}{16}x^6+\ldots\ldots\ldots$$

$$=1+\frac{1}{2}x-x^2-\frac{1}{8}x^2-\frac{1}{2}x^3+\frac{1}{2}x^4+\frac{1}{16}x^3-\frac{3}{8}x^4+\frac{3}{4}x^5-\frac{1}{8}x^6+\ldots\ldots\ldots$$

$$=1+\frac{1}{2}x-\frac{9}{8}x^2-\frac{9}{16}x^3+\ldots\ldots\ldots$$

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**Question # 1 (xi)** *Do yourself as above*

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**Question # 2 (i)**

$$\sqrt{99}=(99)^{\frac{1}{2}}=(100-1)^{\frac{1}{2}}=(100)^{\frac{1}{2}}\left(1-\frac{1}{100}\right)^{\frac{1}{2}}$$

$$=10\left(1+\frac{1}{2}\left(-\frac{1}{100}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{1}{100}\right)^2+\ldots\ldots\ldots\right)$$

$$=10\left(1-\frac{1}{200}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{1}{10000}\right)+\ldots\ldots\ldots\right)$$

$$=10\left(1-0.005-\frac{1}{8}(0.0001)+\ldots\ldots\ldots\right)$$

$$=10(1-0.005-0.0000125+\ldots\ldots\ldots)$$

$$\approx 10(0.9949875)=9.949875$$

$$\approx 9.950$$

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**Question # 2 (ii)**

$(0.98)^{\frac{1}{2}}=(1-0.02)^{\frac{1}{2}}$ *Now do yourself*

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**Question # 2 (iii)**

$(1.03)^{\frac{1}{3}}=(1+0.03)^{\frac{1}{3}}$ *Now do yourself*

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**Question # 2 (iv)**

$\sqrt[3]{65}=(65)^{\frac{1}{3}}=(64-1)^{\frac{1}{3}}=(64)^{\frac{1}{3}}\left(1-\frac{1}{64}\right)^{\frac{1}{3}}$ *Now do yourself*

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***Question # 2 (v)***

$$\sqrt[4]{17}=(17)^{\frac{1}{4}}=(16-1)^{\frac{1}{4}}=(16)^{\frac{1}{4}}\left(1-\frac{1}{16}\right)^{\frac{1}{4}}$$

*Now do yourself*

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***Question # 2 (vi)***

$$\sqrt[5]{31}=(31)^{\frac{1}{5}}=(32-1)^{\frac{1}{5}}=(32)^{\frac{1}{5}}\left(1-\frac{1}{32}\right)^{\frac{1}{5}}$$

*Now do yourself*

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***Question # 2 (vii)***

$$\frac{1}{\sqrt[3]{998}}=\frac{1}{(998)^{\frac{1}{3}}}=(998)^{-\frac{1}{3}}=(1000-2)^{-\frac{1}{3}}=(1000)^{-\frac{1}{3}}\left(1-\frac{2}{1000}\right)^{-\frac{1}{3}}$$

$$=\left(10^3\right)^{-\frac{1}{3}}\left(1-\frac{1}{500}\right)^{-\frac{1}{3}}$$

$$=\left(\frac{1}{10}\right)\left(1+\left(-\frac{1}{3}\right)\left(-\frac{1}{500}\right)+\frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\left(-\frac{1}{500}\right)^2+\ldots\ldots\ldots\right)$$

$$=\left(\frac{1}{10}\right)\left(1+\left(\frac{1}{1500}\right)+\frac{-\frac{1}{3}\left(-\frac{4}{3}\right)}{2}\left(\frac{1}{250000}\right)+\ldots\ldots\ldots\right)$$

$$=\left(\frac{1}{10}\right)\left(1+(0.0006667)+\frac{2}{9}(0.000004)+\ldots\ldots\ldots\right)$$

$$=\left(\frac{1}{10}\right)(1+0.0006667+0.00000089+\ldots\ldots\ldots)$$

$$\approx\left(\frac{1}{10}\right)(1.00066759)=0.100066759 \quad \approx 0.100 \quad \text{Answer}$$

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***Question # 2 ( viii)***

$$\frac{1}{\sqrt[5]{252}}=\frac{1}{(252)^{\frac{1}{5}}}=(252)^{-\frac{1}{5}}=(243+9)^{-\frac{1}{5}}=(243)^{-\frac{1}{5}}\left(1+\frac{9}{243}\right)^{-\frac{1}{5}}$$

$$=\left(3^5\right)^{-\frac{1}{5}}\left(1+\frac{1}{27}\right)^{-\frac{1}{5}}$$

*Now do yourself as above*

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***Question # 2 (ix)***

$$\frac{\sqrt{7}}{\sqrt{8}}=\sqrt{\frac{7}{8}}=\left(\frac{7}{8}\right)^{\frac{1}{2}}=\left(1-\frac{1}{8}\right)^{\frac{1}{2}}$$

$$=1+\frac{1}{2}\left(-\frac{1}{8}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{1}{8}\right)^2+\ldots\ldots\ldots$$

$$=1-\frac{1}{16}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{1}{64}\right)+\ldots\ldots\ldots$$

$$=1-\frac{1}{16}-\frac{1}{8}\left(\frac{1}{64}\right)+\ldots\ldots\ldots$$

$$= 1 - \frac{1}{16} - \frac{1}{512} + \dots\dots\dots\dots$$
$$= 1 - 0.0625 - 0.00195 + \dots\dots\dots\dots$$
$$\approx 0.93555 \quad \approx 0.936 \quad \text{Answer}$$

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***Question # 2 (x)***

$$(0.998)^{-\frac{1}{3}} = (1 - 0.002)^{-\frac{1}{3}}$$ *Now do yourself as above*

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***Question # 2 (xi)***

$$\frac{1}{\sqrt[6]{486}} = \frac{1}{(486)^{\frac{1}{6}}} = (486)^{-\frac{1}{6}} = (729 - 243)^{-\frac{1}{6}} = (729)^{-\frac{1}{6}}\left(1 - \frac{243}{729}\right)^{-\frac{1}{6}}$$

$$= \left(3^6\right)^{-\frac{1}{6}}\left(1 - \frac{1}{3}\right)^{-\frac{1}{6}}$$ *Now do yourself*

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***Question # 2 (xii)***

$$(1280)^{\frac{1}{4}} = (1296 - 16)^{\frac{1}{4}} = (1296)^{\frac{1}{4}}\left(1 - \frac{16}{1296}\right)^{\frac{1}{4}} = \left(6^4\right)^{\frac{1}{4}}\left(1 - \frac{1}{81}\right)^{\frac{1}{4}}$$

*Now do yourself*

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***Question # 3 (i)***

$$\frac{\left(1 + x^2\right)}{(1 + x)^2} = \left(1 + x^2\right)(1 + x)^{-2}$$
$$= \left(1 + x^2\right)\left(1 + (-2)(x) + \frac{(-2)(-2-1)}{2!}(x)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}(x)^3 + \dots\dots\dots\dots\right)$$
$$= \left(1 + x^2\right)\left(1 - 2x + \frac{(-2)(-3)}{2}(x)^2 + \frac{(-2)(-3)(-4)}{3\cdot 2}(x)^3 + \dots\dots\dots\dots\right)$$
$$= \left(1 + x^2\right)\left(1 - 2x + 3x^2 - 4x^3 + \dots\dots\dots\dots\right)$$
$$= \left(1 + x^2\right)\left(1 + (-1)2x + (-1)^2 3x^2 + (-1)^3 4x^3 + \dots\dots\dots\dots\right)$$

Following in this way we can write

$$\frac{\left(1 + x^2\right)}{(1 + x)^2} = \left(1 + x^2\right)(1 + (-1)2x + (-1)^2 3x^2 + (-1)^3 4x^3 + \dots\dots\dots + (-1)^{n-2}(n-1)x^{n-2} +$$
$$(-1)^{n-1}(n)x^{n-1} + (-1)^n(n+1)x^n + \dots\dots\dots)$$

So taking only terms involving $x^n$ we get

$$(-1)^n(n+1)x^n + (-1)^{n-2}(n-1)x^n$$
$$= (-1)^n(n+1)x^n + (-1)^n(-1)^{-2}(n-1)x^n$$
$$= (-1)^n(n+1)x^n + (-1)^n(n-1)x^n \qquad \because (-1)^{-2} = 1$$
$$= (n + 1 + n - 1)(-1)^n x^n = (2n)(-1)^n x^n$$

Thus the coefficient of term involving $x^n$ is $(2n)(-1)^n$

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***Question # 3 (ii)***

Hint:

After solving you will get

$$\frac{\left(1 + x^2\right)}{(1 - x)^2} = \left(1 + x^2\right)(1 + 2x + 3x^2 + 4x^3 + \dots\dots\dots + (n-1)x^{n-2} + (n)x^{n-1} + (n+1)x^n + \dots\dots\dots)$$

*Do yourself as above*

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### Question # 3 (iii)

$$\frac{(1+x)^3}{(1-x)^2}=(1+x)^3(1-x)^{-2}$$

$$=(1+x)^3\left(1+(-2)(-x)+\frac{(-2)(-2-1)}{2!}(-x)^2+\frac{(-2)(-2-1)(-2-2)}{3!}(-x)^3+\dots\dots\dots\right)$$

$$=(1+x)^3\left(1+2x+\frac{(-2)(-3)}{2}(x)^2+\frac{(-2)(-3)(-4)}{3\cdot 2}(-x^3)+\dots\dots\dots\right)$$

$$=\left(1+3x+3x^2+x^3\right)\left(1+2x+3x^2+4x^3+\dots\dots\dots\right)$$

Following in this way we can write

$$\frac{(1+x)^3}{(1-x)^2}=\left(1+3x+3x^2+x^3\right)(1+2x+3x^2+4x^3+\dots\dots+(n-2)x^{n-3}+(n-1)x^{n-2}$$
$$+(n)x^{n-1}+(n+1)x^n+\dots\dots)$$

So taking only terms involving $x^n$ we have term

$$(n+1)x^n+3(n)x^n+3(n-1)x^n+(n-2)x^n$$
$$=\left((n+1)+3(n)+3(n-1)+(n-2)\right)x^n$$
$$=\left(n+1+3n+3n-3+n-2\right)x^n$$
$$=\left(8n-4\right)x^n$$

Thus the coefficient of term involving $x^n$ is $\left(8n-4\right)$.

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### Question # 3 (iv)

$$\frac{(1+x)^2}{(1-x)^3}=(1+x)^2(1-x)^{-3}$$

$$=(1+x)^2\left(1+(-3)(-x)+\frac{(-3)(-3-1)}{2!}(-x)^2+\frac{(-3)(-3-1)(-3-2)}{3!}(-x)^3+\dots\dots\dots\right)$$

$$=(1+x)^2\left(1+(-3)(-x)+\frac{(-3)(-4)}{2}(-x)^2+\frac{(-3)(-4)(-5)}{3\cdot 2}(-x)^3+\dots\dots\dots\right)$$

$$=\left(1+2x+x^2\right)\left(1+3x+\frac{(3)(4)}{2}(x^2)+\frac{(4)(5)}{2}(x^3)+\dots\dots\dots\right)$$

$$=\left(1+2x+x^2\right)\left(1+\frac{(2)(3)}{2}x+\frac{(3)(4)}{2}x^2+\frac{(4)(5)}{2}x^3+\dots\dots\dots\right)$$

Following in this way we can write

$$\frac{(1+x)^2}{(1-x)^3}=\left(1+2x+x^2\right)\left(1+\frac{(2)(3)}{2}x+\frac{(3)(4)}{2}x^2+\frac{(4)(5)}{2}x^3+\dots\dots\dots\right.$$
$$\left.+\frac{(n-1)(n)}{2}x^{n-2}+\frac{(n)(n+1)}{2}x^{n-1}+\frac{(n+1)(n+2)}{2}x^n+\dots\dots\right)$$

So taking only terms involving $x^n$ we have term

$$\frac{(n+1)(n+2)}{2}x^n+2\frac{(n)(n+1)}{2}x^n+\frac{(n-1)(n)}{2}x^n$$
$$=\left((n+1)(n+2)+2(n)(n+1)+(n-1)(n)\right)\frac{x^n}{2}$$
$$=\left(n^2+n+2n+2+2n^2+2n+n^2-n\right)\frac{x^n}{2}$$
$$=\left(4n^2+4n+2\right)\frac{x^n}{2}\quad=2\left(2n^2+2n+1\right)\frac{x^n}{2}$$
$$=\left(2n^2+2n+1\right)x^n$$

Thus the coefficient of term involving $x^n$ is $\left(2n^2+2n+1\right)$.

**Question # 3 (v)**

Since we know that

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + ..............$$

Therefore

$$\left(1 - x + x^2 - x^3 + ............\right)^2 = \left((1+x)^{-1}\right)^2 = (1+x)^{-2}$$

$$= 1 + (-2)(x) + \frac{(-2)(-2-1)}{2!}(x)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}(x)^3 + ..................$$

$$= 1 - 2x + \frac{(-2)(-3)}{2}(x)^2 + \frac{(-2)(-3)(-4)}{3\cdot 2}(x)^3 + ..................$$

$$= 1 - 2x + 3x^2 - 4x^3 + ..................$$

$$= 1 + (-1)2x + (-1)^2 3x^2 (-1)^3 4x^3 + ..................$$

Following in this way we can write

$$= 1 + (-1)2x + (-1)^2 3x^2 (-1)^3 4x^3 + .................. + (-1)^n (n+1)x^n + .........$$

So the term involving $x^n = (-1)^n (n+1)x^n$

And hence coefficient of term involving $x^n$ is $(-1)^n (n+1)$

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**Question # 4 (i)**

$$\text{L.H.S} = \frac{1-x}{\sqrt{1-x}} = \frac{1-x}{(1-x)^{\frac{1}{2}}} = (1-x)^{1-\frac{1}{2}} = (1-x)^{\frac{1}{2}}$$

$$= 1 + \left(\frac{1}{2}\right)(-x) + \text{ squares and higher power of } x.$$

$$= 1 - \frac{1}{2}x = \text{R.H.S} \quad \text{Proved}$$

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**Question # 4 (ii)**

Since $\dfrac{\sqrt{1+2x}}{\sqrt{1-x}} = (1+2x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$

Now $(1+2x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(2x) +$ squares and higher power of $x$.

$$\approx 1 + x$$

Now $(1-x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-x) +$ squares and higher power of $x$.

$$\approx 1 + \frac{1}{2}x$$

$$\frac{\sqrt{1+2x}}{\sqrt{1-x}} \approx (1+x)\left(1 + \frac{1}{2}x\right)$$

$$= 1 + x + \frac{1}{2}x \qquad \text{ignoring term involving } x^2.$$

$$= 1 + \frac{3}{2}x \quad \text{Proved.}$$

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**Question # 4 (iii)**

$$\frac{(9+7x)^{\frac{1}{2}} - (16+3x)^{\frac{1}{4}}}{4+5x} = \left((9+7x)^{\frac{1}{2}} - (16+3x)^{\frac{1}{4}}\right)(4+5x)^{-1}$$

Now $(9+7x)^{\frac{1}{2}} = 9^{\frac{1}{2}}\left(1 + \dfrac{7x}{9}\right)^{\frac{1}{2}}$

$$= (3^2)^{\frac{1}{2}}\left(1+\left(\frac{1}{2}\right)\left(\frac{7x}{9}\right)+ \textit{squres and higher of } x.\right)$$

$$\approx 3\left(1+\frac{7x}{18}\right) = 3+3\left(\frac{7x}{18}\right) = 3+\frac{7x}{6}$$

$$(16+3x)^{\frac{1}{4}} = (16)^{\frac{1}{4}}\left(1+\frac{3x}{16}\right)^{\frac{1}{4}}$$

$$= (2^4)^{\frac{1}{4}}\left(1+\left(\frac{1}{4}\right)\left(\frac{3x}{16}\right)+ \textit{square and higher power of } x\right)$$

$$\approx (2)\left(1+\frac{3x}{64}\right) = 2+2\left(\frac{3x}{64}\right) = 2+\frac{3x}{32}$$

$$(4+5x)^{-1} = 4^{-1}\left(1+\frac{5}{4}x\right)^{-1}$$

$$= \frac{1}{4}\left(1+(-1)\left(\frac{5}{4}x\right)+ \textit{squares and higher power of } x\right)$$

$$\approx \frac{1}{4}\left(1-\frac{5}{4}x\right) = \frac{1}{4}-\frac{5}{16}x$$

So $$\frac{(9+7x)^{\frac{1}{2}}-(16+3x)^{\frac{1}{4}}}{4+5x} \approx \left[\left(3+\frac{7x}{6}\right)-\left(2+\frac{3x}{32}\right)\right]\left(\frac{1}{4}-\frac{5}{16}x\right)$$

$$= \left[3+\frac{7x}{6}-2-\frac{3x}{32}\right]\left(\frac{1}{4}-\frac{5}{16}x\right) = \left(1+\frac{103}{96}x\right)\left(\frac{1}{4}-\frac{5}{16}x\right)$$

$$= \frac{1}{4}+\frac{103}{384}x-\frac{5}{16}x \quad = \frac{1}{4}-\frac{17}{384}x$$ Proved

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***Question # 4 (iv)*** *Do yourself*

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***Question # 4 (v)***

$$\frac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(8+5x)^{\frac{1}{3}}} = (1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}(8+5x)^{-\frac{1}{3}}$$

Now $$(1+x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(x)+ \textit{square and higher power of } x$$

$$\approx 1+\frac{1}{2}x$$

$$(4-3x)^{\frac{3}{2}} = 4^{\frac{3}{2}}\left(1-\frac{3}{4}x\right)^{\frac{3}{2}}$$

$$= \left(2^2\right)^{\frac{3}{2}}\left(1+\left(\frac{3}{2}\right)\left(-\frac{3}{4}x\right)+ \textit{square and higher power of } x\right)$$

$$\approx (2)^3\left(1-\frac{9}{8}x\right) = 8\left(1-\frac{9}{8}x\right)$$

$$(8+5x)^{-\frac{1}{3}} = (8)^{-\frac{1}{3}}\left(1+\frac{5}{8}x\right)^{-\frac{1}{3}}$$

$$= \left(2^3\right)^{-\frac{1}{3}}\left(1+\left(-\frac{1}{3}\right)\left(\frac{5}{8}x\right)+ \textit{square and higher power of } x\right)$$

$$\approx (2)^{-1}\left(1-\frac{5}{24}x\right) = \frac{1}{2}\left(1-\frac{5}{24}x\right)$$

So $\dfrac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(8+5x)^{\frac{1}{3}}} \approx \left(1+\dfrac{1}{2}x\right)8\left(1-\dfrac{9}{8}x\right)\dfrac{1}{2}\left(1-\dfrac{5}{24}x\right)$

$$= \frac{8}{2}\left(1+\frac{1}{2}x\right)\left(1-\frac{9}{8}x-\frac{5}{24}x\right)$$

$$= 4\left(1+\frac{1}{2}x\right)\left(1-\frac{4}{3}x\right) = 4\left(1+\frac{1}{2}x-\frac{4}{3}x\right) = 4\left(1-\frac{5}{6}x\right) \quad \text{Proved}$$

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***Question # 4 (vi)***

*Do yourself as above*

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***Question # 4 (vii)***

*Same as Question #4 (iii)*

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***Question # 5 (i)***

$\sqrt{1-x-2x^2} = \left(1-(x+2x^2)\right)^{\frac{1}{2}}$

$$= 1+\left(\frac{1}{2}\right)\left(-(x+2x^2)\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-(x+2x^2)\right)^2 + \textit{cube \& higher power of } x.$$

$$\approx 1-\left(\frac{1}{2}\right)(x+2x^2)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}(x+2x^2)^2$$

$$\approx 1-\frac{1}{2}x-\frac{1}{2}(2x^2)-\frac{1}{8}x^2 \quad = 1-\frac{1}{2}x-x^2-\frac{1}{8}x^2$$

$$= 1-\frac{1}{2}x-\frac{9}{8}x^2 \quad \text{Proved}$$

---

***Question # 5 (ii)***

$$\sqrt{\frac{1+x}{1-x}} = \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$$

Now

$$(1+x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)x+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}x^2 + \textit{cube \& higher power of } x.$$

$$\approx 1+\frac{1}{2}x+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}x^2 \quad = 1+\frac{1}{2}x-\frac{1}{8}x^2$$

$$(1-x)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\right)(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(-x)^2 + \textit{cube \& higher power of } x.$$

$$\approx 1+\frac{1}{2}x+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}x^2 \quad = 1+\frac{1}{2}x+\frac{3}{8}x^2$$

So

$$\sqrt{\frac{1+x}{1-x}} = \left(1+\frac{1}{2}x-\frac{1}{8}x^2\right)\left(1+\frac{1}{2}x+\frac{3}{8}x^2\right)$$

$$= 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{2}x+\frac{1}{4}x^2+\frac{3}{8}x^2 \quad = 1+x+\frac{1}{2}x^2 \quad \text{Proved}$$

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**Question # 6**

Since $x$ is nearly equal to 1 so suppose $x=1+h$,
where $h$ is so small that its square and higher powers be neglected

L.H.S = $px^p - qx^q$

$$= p(1+h)^p - q(1+h)^q$$

$$= p(1+ph+square\ \&\ higher\ \ power\ of\ \ x)$$
$$-q(1+qh+square\ \&\ higher\ \ power\ of\ \ h)$$

$$= p(1+ph) - q(1+qh)$$

$$= p+p^2h-q-q^2h \ \ldots\ldots\ldots\ldots\ldots\ (i)$$

Now R.H.S = $(p-q)x^{p+q}$

$$=(p-q)(1+h)^{p+q}$$

$$=(p-q)\big(1+(p+q)h+square\ \&\ higher\ \ power\ of\ \ h\big)$$

$$=(p-q)\big(1+(p+q)h\big) \quad =(p-q)\big(1+ph+qh\big)$$

$$= p+p^2h+pqh-q-pqh-q^2h$$

$$= p+p^2h-q-q^2h \ \ldots\ldots\ldots\ldots\ldots\ (ii)$$

From $(i)$ and $(ii)$

L.H.S = R.H.S Proved

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**Question # 7**

Since $p-q$ is small when compare

Therefore let $p-q=h \ \Rightarrow\ p=q+h$

$$\text{L.H.S} = \frac{(2n+1)p+(2n-1)q}{(2n-1)p+(2n+1)q} \quad = \frac{(2n+1)(q+h)+(2n-1)q}{(2n-1)(q+h)+(2n+1)q}$$

$$= \frac{2nq+q+2nh+h+2nq-q}{2nq-q+2nh-h+2nq+q} \quad = \frac{4nq+2nh+h}{4nq+2nh-h}$$

$$= \frac{4nq+2nh+h}{4nq\left(1+\frac{2nh-h}{4nq}\right)} \quad = \frac{4nq+2nh+h}{4nq}\left(1+\frac{2nh-h}{4nq}\right)^{-1}$$

$$= \frac{4nq+2nh+h}{4nq}\left(1+(-1)\left(\frac{2nh-h}{4nq}\right)+square\ \&\ higher\ \ power\ of\ \ x^2\right)$$

$$= \frac{4nq+2nh+h}{4nq}\left(1-\frac{2nh-h}{4nq}\right) \quad = \frac{4nq+2nh+h}{4nq}\left(\frac{4nq-2nh+h}{4nq}\right)$$

$$= \frac{16n^2q^2+8n^2hq+4nhq-8n^2hq+4nhq}{16n^2q^2} \qquad \text{ignoring squares of } h$$

$$= \frac{16n^2q^2+8nhq}{16n^2q^2} \quad = \frac{16n^2q^2}{16n^2q^2}+\frac{8nhq}{16n^2q^2}$$

$$=1+\frac{h}{2nq} \ \ldots\ldots\ldots\ldots\ldots\ (ii)$$

$$\text{Now R.H.S} = \left(\frac{p+q}{2q}\right)^{\frac{1}{n}} = \left(\frac{q+h+q}{2q}\right)^{\frac{1}{n}}$$

$$= \left(\frac{2q+h}{2q}\right)^{\frac{1}{n}} = \left(\frac{2q}{2q}+\frac{h}{2q}\right)^{\frac{1}{n}} = \left(1+\frac{h}{2q}\right)^{\frac{1}{n}}$$

$$=1+\left(\frac{1}{n}\right)\left(\frac{h}{2q}\right)+square\ \&\ higher\ \ power\ of\ \ h\,.$$

$$=1+\frac{h}{2nq} \quad \ldots\ldots\ldots\ldots\ldots.. (ii)$$

Form ($i$) and ($ii$)

L.H.S = R.H.S Proved

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***Question # 8***

Since $n$ and $N$ are nearly equal therefore consider $N=n+h$,
where $h$ is so small that its squares and higher power be neglected.

$$\text{L.H.S} = \left(\frac{n}{2(n+N)}\right)^{\frac{1}{2}} = \left(\frac{n}{2(n+n+h)}\right)^{\frac{1}{2}}$$

$$=\left(\frac{n}{2(2n+h)}\right)^{\frac{1}{2}} = \left(\frac{2(2n+h)}{n}\right)^{-\frac{1}{2}} = \left(\frac{4n+2h}{n}\right)^{-\frac{1}{2}} = \left(4+\frac{2h}{n}\right)^{-\frac{1}{2}}$$

$$=(4)^{-\frac{1}{2}}\left(1+\frac{2h}{4n}\right)^{-\frac{1}{2}} = (2^2)^{-\frac{1}{2}}\left(1+\frac{h}{2n}\right)^{-\frac{1}{2}}$$

$$=(2)^{-1}\left(1+\left(-\frac{1}{2}\right)\frac{h}{2n}+square\ \&\ higher\ \ power\ \ of\ \ h\right)$$

$$=\frac{1}{2}\left(1-\frac{h}{4n}\right) = \frac{1}{2}-\frac{h}{8n} \quad \ldots\ldots\ldots\ldots\ldots.. (i)$$

Now $\text{R.H.S} = \dfrac{8n}{9n-N}-\dfrac{n+N}{4n}$

$$=\frac{8n}{9n-(n+h)}-\frac{n+(n+h)}{4n} = \frac{8n}{9n-n-h}-\frac{n+n+h}{4n}$$

$$=\frac{8n}{8n-h}-\frac{2n+h}{4n} = \frac{8n}{8n\left(1-\frac{h}{8n}\right)}-\frac{2n+h}{4n} = \left(1-\frac{h}{8n}\right)^{-1}-\frac{2n+h}{4n}$$

$$=\left(1+(-1)\left(-\frac{h}{8n}\right)+square\ \&\ higher\ \ power\ \ of\ \ h\right)-\left(\frac{2n}{4n}+\frac{h}{4n}\right)$$

$$=\left(1+\frac{h}{8n}\right)-\left(\frac{1}{2}+\frac{h}{4n}\right) = 1+\frac{h}{8n}-\frac{1}{2}-\frac{h}{4n}$$

$$=\frac{1}{2}-\frac{h}{8n} \quad \ldots\ldots\ldots\ldots\ldots.. \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S Proved

---

***Question # 9 (i)***

$$1-\frac{1}{2}\left(\frac{1}{4}\right)+\frac{1\cdot 3}{2!\cdot 4}\left(\frac{1}{4}\right)^2-\frac{1\cdot 3\cdot 5}{3!\cdot 8}\left(\frac{1}{4}\right)^3+\ldots\ldots\ldots\ldots\ldots\ldots.$$

Suppose the given series be identical with

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

This implies $\quad nx = -\dfrac{1}{2}\left(\dfrac{1}{4}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

$$\frac{n(n-1)}{2!}x^2 = \frac{1\cdot 3}{2!\cdot 4}\left(\frac{1}{4}\right)^2 \quad \ldots\ldots\ldots\ldots (ii)$$

From ($i$) $\quad nx = -\dfrac{1}{8} \;\Rightarrow\; x = -\dfrac{1}{8n} \quad \ldots\ldots\ldots\ldots\ldots (iii)$

Putting value of $x$ in $(ii)$

$$\frac{n(n-1)}{2!}\left(-\frac{1}{8n}\right)^2=\frac{1\cdot 3}{2!\cdot 4}\left(\frac{1}{4}\right)^2$$

$$\Rightarrow \frac{n(n-1)}{2}\left(\frac{1}{64n^2}\right)=\frac{3}{2\cdot 4}\left(\frac{1}{16}\right)$$

$$\Rightarrow \frac{(n-1)}{128n}=\frac{3}{128} \quad \Rightarrow (n-1)=\frac{3}{128}\cdot 128n \quad \Rightarrow n-1=3n$$

$$\Rightarrow n-3n=1 \quad \Rightarrow -2n=1 \quad \Rightarrow \boxed{n=-\frac{1}{2}}$$

Putting value of $n$ in equation $(iii)$

$$x=-\frac{1}{8\left(-\frac{1}{2}\right)} \quad \Rightarrow \boxed{x=\frac{1}{4}}$$

So

$$(1+x)^n=\left(1+\frac{1}{4}\right)^{-\frac{1}{2}}=\left(\frac{5}{4}\right)^{-\frac{1}{2}}=\left(\frac{4}{5}\right)^{\frac{1}{2}}=\sqrt{\frac{4}{5}}$$

---

***Question # 9 (ii)*** *Do yourself as above*

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***Question # 9 (iii)***

$$1+\frac{3}{4}+\frac{3\cdot 5}{4\cdot 8}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12}+\ldots\ldots$$

Suppose the given series be identical with

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\ldots\ldots$$

This implies $\quad nx=\frac{3}{4}$ ……………………. $(i)$

$$\frac{n(n-1)}{2!}x^2=\frac{3\cdot 5}{4\cdot 8} \ldots\ldots\ldots\ldots (ii)$$

From $(i)$ $\quad nx=\frac{3}{4} \Rightarrow x=\frac{3}{4n}$ …………… $(iii)$

Putting value of $x$ in $(ii)$

$$\frac{n(n-1)}{2!}\left(\frac{3}{4n}\right)^2=\frac{3\cdot 5}{4\cdot 8}$$

$$\Rightarrow \frac{n(n-1)}{2}\left(\frac{9}{16n^2}\right)=\frac{15}{32}$$

$$\Rightarrow \frac{9(n-1)}{32n}=\frac{15}{32} \quad \Rightarrow 9(n-1)=\frac{15}{32}\cdot 32n \quad \Rightarrow 9n-9=15n$$

$$\Rightarrow 9n-15n=9 \quad \Rightarrow -6n=9 \quad \Rightarrow n=-\frac{9}{6} \quad \Rightarrow \boxed{n=-\frac{3}{2}}$$

Putting value of $n$ in equation $(iii)$

$$x=-\frac{3}{4\left(-\frac{3}{2}\right)} \Rightarrow \boxed{x=-\frac{1}{2}}$$

So $\quad (1+x)^n=\left(1-\frac{1}{2}\right)^{-\frac{3}{2}}=\left(\frac{1}{2}\right)^{-\frac{3}{2}}=(2)^{\frac{3}{2}}=\left(\sqrt{2}\right)^3=2\sqrt{2}$ $\quad$ Answer

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***Question # 9 (iv)*** *Do yourself as above*

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***Question # 10***

$$1+\frac{1}{4}+\frac{1\cdot 3}{4\cdot 8}+\frac{1\cdot 3\cdot 5}{4\cdot 8\cdot 12}+......................$$

Suppose the given series be identical with

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+..............................$$

This implies

$$nx=\frac{1}{4}\ ......................\ (i)$$

$$\frac{n(n-1)}{2!}x^2=\frac{1\cdot 3}{4\cdot 8}\ ............\ (ii)$$

From $(i)$ $\quad nx=\frac{1}{4}\ \Rightarrow\ x=\frac{1}{4n}\ ..............\ (iii)$

Putting value of $x$ in $(ii)$

$$\frac{n(n-1)}{2!}\left(\frac{1}{4n}\right)^2=\frac{1\cdot 3}{4\cdot 8}$$

$$\Rightarrow\ \frac{n(n-1)}{2}\left(\frac{1}{16n^2}\right)=\frac{3}{32}$$

$$\Rightarrow\ \frac{(n-1)}{32n}=\frac{3}{32}\ \Rightarrow\ (n-1)=\frac{3}{32}\cdot 32n\ \Rightarrow\ n-1=3n$$

$$\Rightarrow\ n-3n=1\ \Rightarrow\ -2n=1\ \Rightarrow\ \boxed{n=-\frac{1}{2}}$$

Putting value of $n$ in equation $(iii)$

$$x=\frac{1}{4\left(-\frac{1}{2}\right)}\ \Rightarrow\ \boxed{x=-\frac{1}{2}}$$

So $\quad (1+x)^n=\left(1-\frac{1}{2}\right)^{-\frac{1}{2}}=\left(\frac{1}{2}\right)^{-\frac{1}{2}}=(2)^{\frac{1}{2}}=\sqrt{2}$

Hence $\quad 1+\frac{1}{4}+\frac{1\cdot 3}{4\cdot 8}+\frac{1\cdot 3\cdot 5}{4\cdot 8\cdot 12}+......................=\sqrt{2}$ $\quad$ Proved

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***Question # 11***

$$y=\frac{1}{3}+\frac{1\cdot 3}{2!}\left(\frac{1}{3}\right)^2+\frac{1\cdot 3\cdot 5}{3!}\left(\frac{1}{3}\right)^3+......................$$

Adding 1 on both sides

$$1+y=1+\frac{1}{3}+\frac{1\cdot 3}{2!}\left(\frac{1}{3}\right)^2+\frac{1\cdot 3\cdot 5}{3!}\left(\frac{1}{3}\right)^3+......................$$

Suppose the given series be identical with

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+..............................$$

This implies

$$nx=\frac{1}{3}\ ......................\ (i)$$

$$\frac{n(n-1)}{2!}x^2=\frac{1\cdot 3}{2!}\left(\frac{1}{3}\right)^2\ ............\ (ii)$$

From $(i)$ $\quad nx=\frac{1}{3}\ \Rightarrow\ x=\frac{1}{3n}\ ..............\ (iii)$

Putting value of $x$ in ($ii$)

$$\frac{n(n-1)}{2!}\left(\frac{1}{3n}\right)^2=\frac{1\cdot 3}{2!}\left(\frac{1}{3}\right)^2$$

$$\Rightarrow \frac{n(n-1)}{2}\left(\frac{1}{9n^2}\right)=\frac{3}{2}\cdot\frac{1}{9}$$

$$\Rightarrow \frac{(n-1)}{18n}=\frac{1}{6} \quad \Rightarrow \quad (n-1)=\frac{1}{6}\cdot 18\,n \quad \Rightarrow \quad n-1=3n$$

$$\Rightarrow n-3n=1 \quad \Rightarrow \quad -2n=1 \quad \Rightarrow \quad \boxed{n=-\frac{1}{2}}$$

Putting value of $n$ in equation ($iii$)

$$x=\frac{1}{3\left(-\frac{1}{2}\right)} \quad \Rightarrow \quad \boxed{x=-\frac{2}{3}}$$

So $\quad (1+x)^n=\left(1-\frac{2}{3}\right)^{-\frac{1}{2}}=\left(\frac{1}{3}\right)^{-\frac{1}{2}}=(3)^{\frac{1}{2}}=\sqrt{3}$

This implies

$$1+y=\sqrt{3}$$

On squaring both sides

$$(1+y)^2=\left(\sqrt{3}\right)^2$$

$$\Rightarrow 1+2y+y^2=3 \quad \Rightarrow \quad 1+2y+y^2-3=0$$

$$\Rightarrow y^2+2y-2=0$$ Proved

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***Question # 12***

$$2y=\frac{1}{2^2}+\frac{1\cdot 3}{2!}\cdot\frac{1}{2^4}+\frac{1\cdot 3\cdot 5}{3!}\cdot\frac{1}{2^6}+\ldots\ldots\ldots\ldots$$

Adding 1 on both sides

$$1+2y=1+\frac{1}{2^2}+\frac{1\cdot 3}{2!}\cdot\frac{1}{2^4}+\frac{1\cdot 3\cdot 5}{3!}\cdot\frac{1}{2^6}+\ldots\ldots\ldots\ldots$$

Comparing above series with

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\ldots\ldots\ldots\ldots$$

After solving as above you will get $n=-\frac{1}{2}$ and $x=-\frac{1}{2}$, so

$$(1+x)^n=\left(1-\frac{1}{2}\right)^{-\frac{1}{2}}=\left(\frac{1}{2}\right)^{-\frac{1}{2}}=(2)^{\frac{1}{2}}=\sqrt{2}$$

This implies

$$1+2y=\sqrt{2}$$

On squaring both sides

$$(1+2y)^2=\left(\sqrt{2}\right)^2$$

$$\Rightarrow 1+4y+4y^2=4 \quad \Rightarrow \quad 1+4y+4y^2-2=0$$

$$\Rightarrow 4y^2+4y-1=0$$ Proved

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