Question # 1 (i)

$$^{12}C_3 = \frac{12!}{(12-3)!\,3!} = \frac{12!}{9!\,3!} = \frac{12\cdot 11\cdot 10\cdot 9!}{9!\,3!} = \frac{12\cdot 11\cdot 10}{3!} = \frac{1320}{6} = 220 \quad \text{Answer}$$

(ii) $$^{20}C_{17} = \frac{20!}{(20-17)!\,17!} = \frac{20!}{3!\,17!} = \frac{20\cdot 19\cdot 18\cdot 17!}{3!\,17!} = \frac{20\cdot 19\cdot 18}{3!} = \frac{6840}{6} = 1140 \quad \text{Answer}$$

(iii) $$^{n}C_4 = \frac{n!}{(n-4)!\,4!} = \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!\,4!} = \frac{n(n-1)(n-2)(n-3)}{4!} \quad \text{Answer}$$

Question # 2 (i)

Since $^{n}C_5 = {}^{n}C_4$

$\Rightarrow\ ^{n}C_{n-5} = {}^{n}C_4 \qquad \because\ ^{n}C_r = {}^{n}C_{n-r}$

$\Rightarrow\ n-5 = 4 \quad \Rightarrow\ n = 4+5 \quad \Rightarrow\ \boxed{n=9}$

(ii) $^{n}C_{10} = \dfrac{12\times 11}{2!} \quad \Rightarrow\ ^{n}C_{10} = \dfrac{12\cdot 11\cdot 10!}{2!\,10!} \quad \Rightarrow\ ^{n}C_{10} = \dfrac{12!}{(12-10)!\,10!}$

$\Rightarrow\ ^{n}C_{10} = {}^{12}C_{10} \quad \Rightarrow\ \boxed{n=12}$.

(iii)

Do yourself as Q # 2 (i)

Question # 3 (i)

$^{n}C_r = 35$ and $^{n}P_r = 210$

Since $^{n}C_r = 35 \quad \Rightarrow\ \dfrac{n!}{(n-r)!\,r!} = 35 \quad \Rightarrow\ \dfrac{n!}{(n-r)!} = 35\cdot r!$ .... (i)

Also $^{n}P_r = 210 \quad \Rightarrow\ \dfrac{n!}{(n-r)!} = 210$ .... (ii)

Comparing (i) and (ii)

$35\cdot r! = 210$

$\Rightarrow\ r! = \dfrac{210}{35} \quad \Rightarrow\ r! = 6 \quad \Rightarrow\ r! = 3! \quad \Rightarrow\ \boxed{r=3}$

Putting value of r in equation (ii)

$$\frac{n!}{(n-3)!} = 210$$

$$\Rightarrow\ \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 210$$

$$\Rightarrow\ n(n-1)(n-2) = 210 \quad \Rightarrow\ n(n-1)(n-2) = 7\cdot 6\cdot 5$$

$$\Rightarrow\ \boxed{n=7}$$

(ii) $^{n-1}C_{r-1} : {}^{n}C_r : {}^{n+1}C_{r+1} = 3:6:11$

First consider $^{n-1}C_{r-1} : {}^{n}C_r = 3:6 \quad \Rightarrow\ \dfrac{(n-1)!}{(n-1-r+1)!\,(r-1)!} : \dfrac{n!}{(n-r)!\,r!} = 3:6$

$$\Rightarrow\ \frac{(n-1)!}{(n-r)!\,(r-1)!} : \frac{n!}{(n-r)!\,r!} = 3:6 \quad \Rightarrow\ \frac{\dfrac{(n-1)!}{(n-r)!\,(r-1)!}}{\dfrac{n!}{(n-r)!\,r!}} = \frac{3}{6}$$

$$\Rightarrow \frac{(n-1)!}{(n-r)!\,(r-1)!}\times\frac{(n-r)!\,r!}{n!}=\frac{1}{2} \qquad \Rightarrow \frac{(n-1)!}{(r-1)!}\times\frac{r!}{n!}=\frac{1}{2}$$

$$\Rightarrow \frac{(n-1)!}{(r-1)!}\times\frac{r(r-1)!}{n(n-1)!}=\frac{1}{2} \qquad \Rightarrow \frac{r}{n}=\frac{1}{2} \qquad \Rightarrow n=2r \ \ldots\ldots\ldots (i)$$

Now consider

$$^{n}C_{r} : {}^{n+1}C_{r+1}=6:11 \qquad \Rightarrow \frac{n!}{(n-r)!\,r!}:\frac{(n+1)!}{(n+1-r-1)!\,(r+1)!}=6:11$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!}:\frac{(n+1)!}{(n-r)!\,(r+1)!}=6:11 \qquad \Rightarrow \frac{\dfrac{n!}{(n-r)!\,r!}}{\dfrac{(n+1)!}{(n-r)!\,(r+1)!}}=\frac{6}{11}$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!}\times\frac{(n-r)!\,(r+1)!}{(n+1)!}=\frac{6}{11} \qquad \Rightarrow \frac{n!}{r!}\times\frac{(r+1)!}{(n+1)!}=\frac{6}{11}$$

$$\Rightarrow \frac{n!}{r!}\times\frac{(r+1)r!}{(n+1)n!}=\frac{6}{11} \qquad \Rightarrow \frac{(r+1)}{(n+1)}=\frac{6}{11} \qquad \Rightarrow 11(r+1)=6(n+1)$$

$$\Rightarrow 11(r+1)=6(2r+1) \qquad \because\ n=2r$$

$$\Rightarrow 11r+11=12r+6 \qquad \Rightarrow 11r-12r=6-11$$

$$\Rightarrow -r=-5 \qquad \Rightarrow \boxed{r=5}$$

Putting value of r in equation (ii)

$$n=2(5) \qquad \Rightarrow \boxed{n=10}$$

Question # 4 (i)

(a) 5 sided polygon has 5 vertices,
so joining two vertices we have line segments = $^{5}C_{2}=10$
Number of sides = 5
So number of diagonals = 10 – 5 = 5

(b) 5 sided polygon has 5 vertices,
so joining any three vertices we have triangles = $^{5}C_{3}=10$

Question # 4 (ii)

(a) 8 sided polygon has 8 vertices
So joining any two vertices we have line segments = $^{8}C_{2}=28$
Number of sides = 8
So number of diagonals = 28 – 8 = 20

(b) 8 sided polygon has 8 vertices,
so joining any three vertices we have triangles = $^{8}C_{3}=56$.

Question # 4 (iii)

Do yourself as above.

Queston # 5

Number of boys = 12
So committees formed taking 3 boys $= {}^{12}C_{3}=220$
Number of girls = 8
So committees formed by taking 2 girls = $= {}^{8}C_{2}=28$
Now total committees formed including 3 boys and 2 girls = $220\times 28=6160$

Question # 6

Number of persons $=8$

Since two particular persons are included in every committee so we have to find combinations of 6 persons 3 at a time = ${}^{6}C_{3}=20$

Hence number of committees = 20

Question # 7

The number of player = 15

So combination, taking 11 player at a time = ${}^{15}C_{11}=1365$

Now if one particular player is in each collection

then number of combination = ${}^{14}C_{10}=1001$

Question # 8

L.H.S = ${}^{16}C_{11}+{}^{16}C_{10}$

$$=\frac{16!}{(16-11)!\ 11!}+\frac{16!}{(16-10)!\ 10!}=\frac{16!}{5!\ 11!}+\frac{16!}{6!\ 10!}$$

$$=\frac{16!}{5!\ 11\cdot 10!}+\frac{16!}{6\cdot 5!\ 10!}=\frac{16!}{10!\ 5!}\left(\frac{1}{11}+\frac{1}{6}\right)$$

$$=\frac{16!}{10!\ 5!}\left(\frac{6+11}{66}\right)=\frac{16!}{10!\ 5!}\left(\frac{17}{66}\right)=\frac{16!}{10!\ 5!}\left(\frac{17}{11\cdot 6}\right)$$

$$=\frac{17\cdot 16!}{11\cdot 10!\ 6\cdot 5!}=\frac{17!}{11!\ 6!}=\frac{17!}{11!\ (17-11)!}={}^{17}C_{11}=\text{R.H.S}$$

Alternative

L.H.S = ${}^{16}C_{11}+{}^{16}C_{10}$ = 4368 + 8008 = 12276 ……… (*i*)

R.H.S = ${}^{17}C_{11}=12376$ ……… (*ii*)

From (*i*) and (*ii*)

L.H.S = R.H.S

Question # 9

Number of men = 8

Number of women = 10

(*i*) We have to form combination of 4 women out of 10 and 3 men out of 8

$$={}^{10}C_{4}\times{}^{8}C_{3}=210\times 36=11760$$

(*ii*) At the most 4 women means that women are less than or equal to 4, which implies the following possibilities $(1W,6M),(2W,5M),(3W,4M),(4W,3M),(7M)$

$$={}^{10}C_{1}\times{}^{8}C_{6}+{}^{10}C_{2}\times{}^{8}C_{5}+{}^{10}C_{3}\times{}^{8}C_{4}+{}^{10}C_{4}\times{}^{8}C_{3}+{}^{8}C_{7}$$
$$=(10)(28)+(45)(56)+(120)(70)+(210)(56)+(8)$$
$$=280+2520+8400+11760+8$$
$$=22968$$

(*iii*) At least 4 women means that women are greater than or equal to 4, which implies the following possibilities $(4W,3M),(5W,2M),(6W,1M),(7W)$

$$={}^{10}C_{4}\times{}^{8}C_{3}+{}^{10}C_{5}\times{}^{8}C_{2}+{}^{10}C_{6}\times{}^{8}C_{1}+{}^{10}C_{7}$$
$$=(210)(56)+(252)(28)+(210)(8)+120$$
$$=11760+7056+1680+120$$
$$=20616$$

Question # 10

$$\text{L.H.S} = {}^nC_r + {}^nC_{r-1} = \frac{n!}{(n-r)!\; r!} + \frac{n!}{(n-(r-1))!\; (r-1)!}$$

$$= \frac{n!}{(n-r)!\; r!} + \frac{n!}{(n-r+1)!\; (r-1)!}$$

$$= \frac{n!}{(n-r)!\; r(r-1)!} + \frac{n!}{(n-r+1)(n-r)!\; (r-1)!}$$

$$= \frac{n!}{(n-r)!\; (r-1)!}\left(\frac{1}{r} + \frac{1}{(n-r+1)}\right)$$

$$= \frac{n!}{(n-r)!\; (r-1)!}\left(\frac{n-r+1+r}{r(n-r+1)}\right)$$

$$= \frac{n!}{(n-r)!\; (r-1)!}\left(\frac{n+1}{r(n-r+1)}\right)$$

$$= \frac{(n+1)n!}{(n-r+1)(n-r)!\;\; r(r-1)!}$$

$$= \frac{(n+1)!}{(n-r+1)!\;\; r!} = \frac{(n+1)!}{(n+1-r)!\;\; r!}$$

$$= {}^{n+1}C_r = \text{R.H.S}$$

Made by: Atiq ur Rehman (atiq@mathcity.org), URL: http://www.mathcity.org

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نوٹ
اگر آپ کو ان نوٹس میں کوئی غلطی یا خامی نظر آئے یا آپ کے پاس ان سوالات کےاس سے آسان حل موجود ہوں تو آپ اوپر دیئے گئے ای میل ایڈرس پر ہمیں بھیج سکتے ہیں۔ یا پھر آپ بذریہ خط یاپھر خود اس پتا پر اطلاع دے سکتے ہیں۔
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