Question # 1 (i)

PAKPATTAN

Number of letters = $n = 9$
Number of P's $= p = 2$
Number of A's $= q = 3$
Number of T's $= r = 2$

Thus the number of worlds formed

$$= \frac{n!}{p!\cdot q!\cdot r!} = \frac{9!}{2!\cdot 3!\cdot 2!}$$
$$= \frac{362880}{(2)(6)(2)} = 15120 \quad \textit{Answer}$$

(ii) & (iii)

Do yourself as above

(iv) ASSASSINATION

Number of letters $= n = 13$
Number of A's $= p = 3$
Number of S's $= q = 4$
Number of I's $= r = 2$
Number of N's $= s = 2$
So the number of words

$$= \frac{n!}{p!\cdot q!\cdot r!\cdot s!} = \frac{13!}{3!\cdot 4!\cdot 2!\cdot 2!}$$
$$= \frac{6227020800}{(6)(24)(2)(2)} = 10810800$$

Question # 2

If P is the first letter then words are of the form $\mathrm{P}*****$,
Where five $*$ can be replace with A,N,A,M,A.
So number of letters $= n = 5$
Number of A's $= p = 3$

So required permutations $= \frac{5!}{3!} = \frac{120}{6} = 20$

Question # 3

If C be the first letter and K is the last letter then words are of the form $\mathrm{C}******\mathrm{K}$ .Where each $*$ can be replaced with A,T,T,A,E,D.
So number of letters $= n = 6$
Number of A's $= p = 2$
Number of T's $= q = 2$

So required permutations $= \frac{n!}{p!\cdot q!} = \frac{6!}{2!\cdot 2!}$
$$= \frac{720}{(2)(2)} = 180$$

Question # 4

The number greater than 1000000 are of the following forms.

If numbers are of the form $2******$
Where each $*$ can be filled with $0,2,2,3,4,4$
Then number of digits $= n = 6$
Number of 2's $= p = 2$
Number of 4's $= q = 2$

So number formed $= \frac{n!}{p!\cdot q!} = \frac{6!}{2!\cdot 2!}$
$$= \frac{720}{(2)(2)} = 180$$

Now if numbers are of the form $3******$
Where each $*$ can be filled with $0,2,2,2,4,4$
Then number of digits $= n = 6$
Number of 2's $= p = 3$
Number of 4's $= q = 2$

So number formed $= \frac{n!}{p!\cdot q!} = \frac{6!}{3!\cdot 2!}$
$$= \frac{720}{(6)(2)} = 60$$

Now if numbers are of the form $4******$
Where each $*$ can be filled with $0,2,2,2,3,4$
Then number of digits $= n = 6$
Number of 2's $= p = 3$

So number formed $= \frac{n!}{p!} = \frac{6!}{3!}$
$$= \frac{720}{6} = 120$$

So required numbers greater than 1000000
$$= 180 + 60 + 120$$
$$= 360$$

Ø Alternative

(Submitted by Waqas Ahmad - FAZMIC Sargodha – 2004-06)

No. of digits = 7
No. of 2's = 3
No. of 4's = 2

Permutations of 7 digits number $= \frac{7!}{3!\cdot 2!}$
$$= \frac{5040}{(6)(2)} = 420$$

Number less than 1,000,000 are of the form $0******$,
Where each $*$ can be replaced with 2, 2, 3, 4, 4.
No. of digits = 6
No. of 2's = 3
No. of 4's = 2

So permutations $= \frac{6!}{3!\cdot 2!} = \frac{720}{(6)(2)} = 60$

Hence number greater than 1000000 $= 420 - 60$
$$= 360$$

Question # 5

Total number of digits $= n = 6$
Number of 2's $= p = 2$
Number of 3's $= q = 2$
Number of 4's $= r = 2$
So number formed by these 6 digits

$$= \frac{n!}{p!\,q!\,r!} = \frac{6!}{(2!)(2!)(2!)}$$
$$= \frac{720}{(2)(2)(2)} = 90$$

Question # 6

Total members $= n = 11$
Members in first committee $= p = 3$
Members in second committee $= q = 4$
Members in third committee $= r = 2$
Members in fourth committee $= s = 2$
So required number of committees

$$= \frac{n!}{p!\cdot q!\cdot r!\cdot s!} = \frac{11!}{3!\cdot 4!\cdot 2!\cdot 2!}$$

$$= \frac{39916800}{(6)(24)(2)(2)} = 69300$$

Question # 7

Number of D.C.O's $= 9$

Let D_1 and D_2 be the two D.C.O's insisting to sit together so consider them one.

If D_1D_2 sit together then permutations

$$= {}^9P_9 = 362880$$

If D_2D_1 sit together then permutations

$$= {}^9P_9 = 362880$$

So total permutations $= 362880 + 362880$
$= 725760$

Question # 8

Fixing one officer on a particular seat
We have permutations of remaining 11 officers

$$= {}^{11}P_{11} = 39916800$$

Question # 9

9 males can be seated on a round table

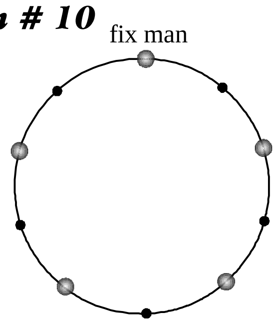
$$= {}^8P_8 = 40320$$

And 5 females can be seated on a round table

$$= {}^4P_4 = 24$$

So permutations of both $= 40320 + 24$
$= 967680$

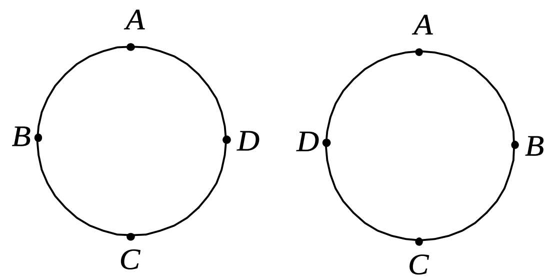
Question # 10



If we fix one man round a table
then their permutations $= {}^4P_4 = 24$
Now if women sit between the two men
then their permutations $= {}^5P_5 = 120$
So total permutations $= 24\times 120 = 2880$

Question # 11

Number of keys $= 4$
Fixing one key we have permutation $= {}^3P_3 = 6$

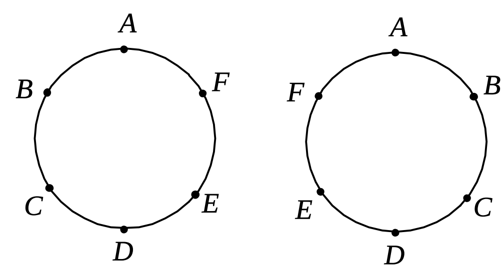


Since above figures of arrangement are reflections of each other

Therefore permutations $= \frac{1}{2}\times 6 = 3$

Question # 12

Number of beads $= 6$
Fixing one bead, we have permutation $= {}^5P_5$
$= 120$



Since above figures of arrangement are reflections of each other

Therefore permutations $= \frac{1}{2}\times 120 = 60$

Why $0! = 1$

We know that

$$10! = 10\cdot 9\cdot 8 .....3\cdot 2\cdot 1$$
$$= 10(10-1)(10-2).....3\cdot 2\cdot 1$$

And generally we write

$$n! = n(n-1)(n-2)....3\cdot 2\cdot 1$$

Similarly we write

$$(n-1)! = (n-1)(n-2)....3\cdot 2\cdot 1$$

So we can write

$$n! = n(n-1)!$$

$$\Rightarrow \frac{n!}{n} = (n-1)! \qquad \div\text{ing by } n$$

$$\text{i.e. } (n-1)! = \frac{n!}{n}$$

Putting $n=1$ in above

$$(1-1)! = \frac{1!}{1}$$

$$\Rightarrow 0! = 1 \qquad \textit{Proved}$$