Question # 1 (i)

$$^{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20\cdot19\cdot18\cdot17!}{17!} = 20\cdot19\cdot18 = 6840$$

(ii)

$$^{16}P_4 = \frac{16!}{(16-4)!} = \frac{16!}{12!} = \frac{16\cdot15\cdot14\cdot13\cdot12!}{12!} = 16\cdot15\cdot14\cdot13 = 43680$$

Others do yourself

Question # 2 (i)

$$^{n}P_2 = 30 \quad \Rightarrow \quad \frac{n!}{(n-2)!} = 30 \quad \Rightarrow \quad \frac{n(n-1)(n-2)!}{(n-2)!} = 30$$

$$\Rightarrow \ n(n-1) = 30 \quad \Rightarrow \ n(n-1) = 6\cdot5 \quad \Rightarrow \ \boxed{n=6}$$

(ii)

$$^{11}P_n = 11\cdot10\cdot9 \quad \Rightarrow \quad \frac{11\cdot10\cdot9\cdot8!}{(11-n)!} = 11\cdot10\cdot9$$

$$\Rightarrow \ \frac{8!}{(11-n)!} = 1 \quad \Rightarrow \ 8! = (11-n)! \quad \Rightarrow \ 8 = 11-n \quad \Rightarrow \ n = 11-8 \quad \Rightarrow \ \boxed{n=3}$$

(iii)

$$^{n}P_4 : {}^{n-1}P_3 = 9:1 \quad \Rightarrow \quad \frac{^{n}P_4}{^{n-1}P_3} = \frac{9}{1} \quad \Rightarrow \quad {}^{n}P_4 = 9\ {}^{n-1}P_3$$

$$\Rightarrow \ \frac{n!}{(n-4)!} = 9\,\frac{(n-1)!}{(n-1-3)!} \qquad \Rightarrow \ \frac{n(n-1)!}{(n-4)!} = 9\,\frac{(n-1)!}{(n-4)!} \qquad \Rightarrow \ \boxed{n=9}$$

Question # 3 (i)

$$\text{R.H.S} = n\cdot {}^{n-1}P_{r-1} = n\cdot\frac{(n-1)!}{\big(n-1-(r-1)\big)} = \frac{n(n-1)!}{\big(n-1-r+1\big)} = \frac{n!}{(n-r)} = {}^{n}P_r = \text{L.H.S}$$

(ii)

$$\text{R.H.S} = {}^{n-1}P_r + r\cdot{}^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r\cdot\frac{(n-1)!}{(n-1-r+1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\cdot\frac{(n-1)!}{(n-r)!} \qquad = \frac{(n-1)!}{(n-r-1)!} + r\cdot\frac{(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left(1 + r\cdot\frac{1}{(n-r)}\right) \qquad = \frac{(n-1)!}{(n-r-1)!}\left(\frac{n-r+r}{(n-r)}\right)$$

$$= \frac{(n-1)!}{(n-r-1)!}\left(\frac{n}{(n-r)}\right) \qquad = \frac{n(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{n!}{(n-r)!} = {}^{n}P_r = \text{L.H.S}$$

Question # 4

Total number of flags = $n = 6$

Number of signal using one flag = $^{6}P_1 = 6$

Number of signal using two flag = $^{6}P_2 = 30$

Number of signal using three flag = $^{6}P_3 = 120$

Number of signal using four flag = ${}^6P_4 = 360$

Number of signal using five flag = ${}^6P_5 = 720$

Number of signal using six flag = ${}^6P_6 = 720$

Total number of signals = $6+30+120+360+720+720=1956$

Question # 6 (i)

Since number of letters in PLANE = n = 5

Therefore total words form $= {}^5P_5 = 120$

(ii)

Since number of letters in OBJECT = n = 6

Therefore total words forms $= {}^6P_6 = 720$

(iii)

Since number of letters in FASTING = n = 7

Therefore total words forms $= {}^7P_7 = 5040$

Question # 7

Number of digits = n = 5

So numbers forms taken 3 digits at a time = ${}^5P_3 = 60$

Question # 8

Number greater than 23000 can be formed as

Number of numbers of the form $23{*}{*}{*} = {}^3P_3 = 6$

Number of numbers of the form $25{*}{*}{*} = {}^3P_3 = 6$

Number of numbers of the form $26{*}{*}{*} = {}^3P_3 = 6$

Number of numbers of the form $3{*}{*}{*}{*} = {}^4P_4 = 24$

Number of numbers of the form $5{*}{*}{*}{*} = {}^4P_4 = 24$

Number of numbers of the form $6{*}{*}{*}{*} = {}^4P_4 = 24$

Thus the total number formed = 6 + 6 + 6 + 24 + 24 + 24 = 90

➢ ***Alternative*** (Submitted by **Waqas Ahmad** - FAZMIC Sargodha – 2004-06)

Permutation of 5 digits numbers = ${}^5P_5$ = 120

Numbers less than 23000 are of the form $1{*}{*}{*}{*}$

Then permutations $= {}^4P_4 = 24$

If number less than 23000 are of the form $21{*}{*}{*}$

Then permutations = ${}^3P_3$ = 6

Thus number greater than 23000 formed = $120-24-6$ = 90

Question # 9

Total number of digits = 5

(i) If we take 28 as a single digit then number of numbers = ${}^4P_4 = 24$

If we take 82 as a single digit then number of numbers = ${}^4P_4 = 24$

So the total numbers when 2 and 8 are next to each other = 24 + 24 = 48

(ii) Number of total permutation = ${}^5P_5 = 120$

thus number of numbers when 2 and 8 are not next to each other = 120 – 48 = 72

Question # 10

Since number of permutation of 6 digits = ${}^6P_6 = 720$

But 0 at extreme left is meaning less

so number of permutation when 0 is at extreme left = ${}^5P_5 = 120$

Thus the number formed by 6 digits = 720 – 120 = 600

Now if we fix 0 at ten place then number formed = ${}^5P_5 = 120$

Question # 11

Number of digits = 5

For multiple of 5 we must have 5 at extreme right so number formes = ${}^4P_4 = 24$

Question # 12

Total numbers of books = 8

Total number of permutation = ${}^8P_8 = 40320$

Let E_1 and E_2 denotes two English books then

Number of permutation when E_1E_2 place together = ${}^7P_7 = 5040$

Number of permutation when E_2E_1 place together = ${}^7P_7 = 5040$

So total permutation when E_1 and E_2 together = 5040 + 5040 = 10080

Required permutation when English books are not together = 40320 – 10080
= 30240

Question # 13

Let E_1, E_2, E_3 be the book on English and U_1, U_2, U_3, U_4, U_5 be the book on Urdu
Then the permutation when

books are arranged as $E_1, E_2, E_3, U_1, U_2, U_3, U_4, U_5$ = ${}^3P_3 \times {}^5P_5 = 6 \times 120 = 720$

books are arranged as $U_1, U_2, U_3, U_4, U_5, E_1, E_2, E_3$ = ${}^5P_5 \times {}^3P_3 = 120 \times 6 = 720$

so total permutation when books of same subject are together = 720 + 720
= 1440

Question # 14

Let the five boys be B_1, B_2, B_3, B_4, B_5 and the four girls are G_1, G_2, G_3, G_4 then there seats plane is $B_1, G_1, B_2, G_2, B_3, G_3, B_4, G_4, B_5$

Then the permutations = ${}^5P_1 \times {}^4P_1 \times {}^4P_1 \times {}^3P_1 \times {}^3P_1 \times {}^2P_1 \times {}^2P_1 \times {}^1P_1 \times {}^1P_1$
$= 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 2880$

Book: *Exercise 7.2*
Text Book of Algebra and Trigonometry Class XI
Punjab Textbook Board, Lahore.

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