Exercise 6.8 (Solutions)

Q.No 1 $1, \frac{1}{3}, \frac{1}{9}, \ldots$

$a_1 = 1$, $r = \frac{a_2}{a_1} = \frac{1/3}{1} = \frac{1}{3}$, $n = 15$

Now $S_n = \frac{a_1(1-r^n)}{1-r}$

$= \frac{1\left(1-\left(\frac{1}{3}\right)^{15}\right)}{1-\frac{1}{3}} = \frac{1-\frac{1}{3^{15}}}{2/3}$

$= \frac{3}{2}\left[\frac{3^{15}-1}{3^{15}}\right] = \frac{3}{2}\left(\frac{14348907-1}{14348907}\right)$

$= \frac{43046718}{28697814}$

$= \frac{7174453}{4782969}$ ÷ing by 6. Ans

Q.No 2 i)

$0.2 + 0.22 + 0.222 + \ldots$

$= 2\left[0.1 + 0.11 + 0.111 + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{2}{9}\left[0.9 + 0.99 + 0.999 + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{2}{9}\left[(1-0.1) + (1-0.01) + (1-0.001) + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{2}{9}\left[(1+1+1+\ldots \text{ to } n \text{ terms}) - (0.1 + 0.01 + 0.001 + \ldots \text{ to } n \text{ terms})\right]$

$= \frac{2}{9}\left[n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots \text{ to } n \text{ terms}\right)\right]$

$a_1 = \frac{1}{10}$, $r = \frac{1}{10}$, $n = n$

$= \frac{2}{9}\left(n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-\frac{1}{10}}\right)$

$= \frac{2}{9}\left(n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{9/10}\right)$

$= \frac{2}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$ Answer

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ii) $3 + 33 + 333 + \ldots$ to n terms

$= \frac{1}{3}\left[9 + 99 + 999 + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{1}{3}\left[(10-1) + (100-1) + (1000-1) + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{1}{3}\left[(10 + 100 + 1000 + \ldots \text{ to } n \text{ terms}) - (1+1+1+\ldots \text{ to } n \text{ terms})\right]$

$= \frac{1}{3}\left[(10 + 100 + 1000 + \ldots \text{ to } n \text{ terms}) - n\right]$

$a_1 = 10$, $r = 10$, $n = n$

$= \frac{1}{3}\left[\frac{10(10^n-1)}{10-1} - n\right]$

$= \frac{1}{3}\left[\frac{10(10^n-1)}{9} - n\right]$ Answer

OR

$= \frac{1}{3}\left[\frac{10}{9}(10^n-1) - n\right]$ Answer

Q.No 3 i)

$1 + (a+b) + (a^2+ab+b^2) + \ldots$ to n terms

÷ing and ×ing by $a-b$.

$= \frac{a-b}{a-b}\left[1 + (a+b) + (a^2+ab+b^2) + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{1}{a-b}\left[(a-b) + (a-b)(a+b) + (a-b)(a^2+ab+b^2) + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{1}{a-b}\left[(a-b) + (a^2-b^2) + (a^3-b^3) + \ldots \text{ to } n \text{ terms}\right]$

$= \frac{1}{a-b}\left[(a + a^2 + a^3 + \ldots \text{ to } n \text{ terms}) - (b + b^2 + b^3 + \ldots \text{ to } n \text{ terms})\right]$

$= \frac{1}{a-b}\left[\frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1}\right]$

$= \frac{1}{a-b}\left[\frac{a(a^n-1)(b-1) - b(b^n-1)(a-1)}{(a-1)(b-1)}\right]$

$= \frac{a(b-1)(a^n-1) - b(a-1)(b^n-1)}{(a-b)(a-1)(b-1)}$ Answer

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ii)

$r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots$ to n terms

÷ing and ×ing by $1-k$.

$= \frac{1-k}{1-k}\left(r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots \text{ to } n \text{ terms}\right)$

$= \frac{1}{1-k}\left(r(1-k) + (1-k)(1+k)r^2 + (1-k)(1+k+k^2)r^3 + \ldots \text{ to } n \text{ terms}\right)$

$= \frac{1}{1-k}\left(r(1-k) + (1-k^2)r^2 + (1-k^3)r^3 + \ldots \text{ to } n \text{ terms}\right)$

$= \frac{1}{1-k}\left(r - rk + r^2 - r^2k^2 + r^3 - r^3k^3 + \ldots\ldots \text{ to } n \text{ terms}\right)$

$= \frac{1}{1-k}\left\{(r + r^2 + r^3 + \ldots \text{ to } n \text{ terms}) - (rk + r^2k^2 + r^3k^3 + \ldots \text{ to } n \text{ terms})\right\}$

$= \frac{1}{1-k}\left(\frac{r(r^n-1)}{r-1} - \frac{rk((rk)^n-1)}{rk-1}\right)$

$= \frac{r}{1-k}\left(\frac{r^n-1}{r-1} - \frac{k(r^nk^n-1)}{rk-1}\right)$ Answer

Q No 4 $2 + (1-i) + \frac{1}{i} + \ldots$ to 8 terms

$a_1 = 2$, $r = \frac{1-2i}{2}$; $n = 8$

Now $S_n = \frac{a_1(r^n-1)}{r-1}$

$= \frac{2\left(\left(\frac{1-2i}{2}\right)^8 - 1\right)}{\frac{1-2i}{2} - 1}$

$= \frac{2\left(\frac{(1-2i)^8}{2^8} - 1\right)}{\frac{1-2i-2}{2}}$

$= \frac{4\left(\frac{((1-2i)^2)^4 - 1}{256}\right)}{-1-2i}$

$= \frac{4\left(\frac{(1-2i+i^2)^4 - 256}{256}\right)}{-1-2i}$

$= \frac{4\{(1-2i-1)^4 - 256\}}{256(-1-2i)}$

$= \frac{\{(-2i)^4 - 256\}}{64(-1-2i)}$

$= \frac{(-2)^4(i)^4 - 256}{64(-1-2i)}$ $\quad\Big|\; i^4 = (i^2)^2 = (-1)^2 = 1$

$= \frac{16(1) - 256}{64(-1-2i)} = \frac{-240}{64(-1-2i)}$

$= \frac{-15}{-4(1+2i)} = \frac{15}{4(1+2i)} \cdot \frac{1-2i}{1-2i}$

$= \frac{15(1-2i)}{4((1)^2-(2i)^2)} = \frac{15(1-2i)}{4(2)} = \frac{15(1-2i)}{8}$ Answer

Q No 5 i) $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \ldots\ldots$

$a_1 = \frac{1}{5}$, $r = \frac{1}{5}$

Now $S = \frac{a_1}{1-r} = \frac{1/5}{1-1/5}$

$= \frac{1/5}{4/5} = \frac{1}{4}$ Answer

ii) & iii) Do yourself

iv) $2 + 1 + 0.5 + \ldots\ldots$

$a_1 = 2$, $r = \frac{1}{2} = 0.5$

Now $S = \frac{a_1}{1-r} = \frac{2}{1-0.5}$

$= \frac{2}{0.5} = 4$ Answer

v) $4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + \ldots\ldots$

$a_1 = 4$, $r = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Now $S = \frac{a_1}{1-r} = \frac{4}{1-1/\sqrt{2}}$

$= \frac{4}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1}$ Answer

OR

$= \frac{4\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{4((\sqrt{2})^2 + \sqrt{2})}{(\sqrt{2})^2 - (1)^2} = \frac{4(2+\sqrt{2})}{2-1}$

$= 4(2+\sqrt{2})$ Answer

vi) Do yourself

Q No 6 i) $1.\dot{3}\dot{4}$

$= 1.343434\ldots\ldots$

$= 1 + 0.343434\ldots\ldots$

$= 1 + (0.34 + 0.0034 + 0.000034 + \ldots\ldots)$

$a_1 = 0.34$, $r = \frac{0.0034}{0.34} = 0.01$

$= 1 + \frac{0.34}{1-0.01} = 1 + \frac{0.34}{0.99} = 1 + \frac{34}{99}$

$= \frac{99+34}{99} = \frac{133}{99}$ Answer

ii) $0.\dot{2}5\dot{9}$

$= 0.259259259\ldots\ldots$

$= 0.259 + 0.000259 + 0.000000259 + \ldots\ldots$

$a_1 = 0.259$, $r = \frac{0.000259}{0.259} = 0.001$

$= \frac{0.259}{1-0.001} = \frac{0.259}{0.999}$

$= \frac{259}{999}$ Answer

i) $1.1\dot{4}\dot{7}$

$= 1.1474747\ldots\ldots$

$= 1.1 + 0.0474747\ldots\ldots$

$= 1.1 + (0.047 + 0.00047 + 0.0000047 + \ldots\ldots)$

$a_1 = 0.047$, $r = \frac{0.00047}{0.047} = 0.01$

$= 1.1 + \frac{0.047}{1-0.01} = 1.1 + \frac{0.047}{0.99}$

$= \frac{11}{10} + \frac{47/1000}{99/100} = \frac{11}{10} + \frac{47}{1000}\cdot\frac{100}{99}$

$= \frac{11}{10} + \frac{47}{990} = \frac{1089+47}{990}$

$= \frac{1136}{990}$ Answer

Q No 7 $r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots$

÷ing & xing by $1-k$

$= \frac{1-k}{1-k}\left(r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots\right)$

$= \frac{1}{1-k}\left[(1-k)r + (1-k)(1+k)r^2 + (1-k)(1+k+k^2)r^3 + \ldots\ldots\right]$

$= \frac{1}{1-k}\left((1-k)r + (1-k^2)r^2 + (1-k^3)r^3 + \ldots\ldots\right)$

$= \frac{1}{1-k}\left(r - rk + r^2 - r^2k^2 + r^3 - r^3k^3 + \ldots\ldots\right)$

$= \frac{1}{1-k}\left\{(r + r^2 + r^3 + \ldots\ldots) - (rk + r^2k^2 + r^3k^3 + \ldots\ldots)\right\}$

$= \frac{1}{1-k}\left(\frac{r}{1-r} - \frac{rk}{1-rk}\right)$ $\because S = \frac{a_1}{1-r}$

$= \frac{1}{1-k}\left\{\frac{r(1-rk) - rk(1-r)}{(1-r)(1-rk)}\right\}$

$= \frac{r - \cancel{r^2k} - rk + \cancel{r^2k}}{(1-k)(1-r)(1-rk)}$

$= \frac{r(1-k)}{(1-k)(1-r)(1-rk)}$

$= \frac{r}{(1-r)(1-rk)}$ Answer

Q No 8 $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots\ldots$

$a_1 = \frac{x}{2}$, $r = \frac{\frac{1}{4}x^2}{x/2} = \frac{\frac{1}{8}x^3}{\frac{1}{4}x^2} = \frac{x}{2}$

So This is an infinite Geometric series also $0 < x < 2 \Rightarrow 0 < \frac{x}{2} < 1$

i.e $r = \frac{x}{2} < 1$ so solution exists.

$\Rightarrow y = \frac{a_1}{1-r} = \frac{x/2}{1-x/2}$

$\Rightarrow y = \frac{x/2}{\frac{2-x}{2}} \Rightarrow y = \frac{x}{2-x}$

$\Rightarrow y(2-x) = x \Rightarrow 2y - xy = x$

$\Rightarrow 2y = x + xy \Rightarrow 2y = x(1+y)$

$\Rightarrow \frac{2y}{1+y} = x$ i.e $x = \frac{2y}{1+y}$ Answer

Q No 9 Do yourself

Hint $0 < x < \frac{3}{2} \Rightarrow 0 < \frac{2}{3}x < 1$

Q No 10

Distance travel in 1st fall $= 27\,m$

" " " 2nd fall $= \frac{2}{3}\times 27 = 18\,m$

" " " 3rd fall $= \frac{2}{3}\times 18 = 12\,m$

So sequence of fall is

$27 + 18 + 12 + \ldots\ldots$

This is infinite geometric sequence if S_1 denotes distance travel by ball in fall then $a_1 = 27$, $r = \frac{18}{27} = \frac{2}{3} < 1$

$\Rightarrow S_1 = \frac{a_1}{1-r} = \frac{27}{1-2/3} = \frac{27}{1/3} = \cancel{54\,m}\ 81\,m$

Now

Distance travel in 1st rebound $= \frac{2}{3}\times 27 = 18$

" " " 2nd rebound $= \frac{2}{3}\times 18 = 12$

" " " 3rd rebound $= \frac{2}{3}\times 12 = 8$

So sequence of rebound is

$18+12+8+\cdots\cdots$

which is infinite Geometric series

If S_2 denotes distance travel by ball in rebound then $a_1 = 18$, $r = \frac{2}{3} < 1$

$$S_2 = \frac{a_1}{1-r} = \frac{18}{1-\frac{2}{3}} = \frac{18}{\frac{1}{3}} = 54$$

Now

total distance $= S_1 + S_2$

$= 81 + 54 = 135m$

Q No. 11 Same as Q No 10

Q No. 12 $y = 1 + 2x + 4x^2 + 8x^3 + \cdots\cdots$

i) $a_1 = 1$, $r = \frac{2x}{1} = \frac{4x^2}{2x} = 2x$

So $y = \frac{a_1}{1-r} = \frac{1}{1-2x}$

$\Rightarrow y = \frac{1}{1-2x} \Rightarrow y(1-2x) = 1$

$\Rightarrow y - 2xy = 1 \Rightarrow -2xy = 1-y$

$\Rightarrow 2xy = y-1 \Rightarrow x = \frac{1-y}{2x}$ proved

ii) Now series is convergent if

$|r| < 1$

$\Rightarrow |2x| < 1 \Rightarrow \pm 2x < 1$

$\Rightarrow \pm x < \frac{1}{2}$

$\Rightarrow x < \frac{1}{2}$ and $-x < \frac{1}{2}$

$\Rightarrow x < \frac{1}{2}$ and $x > -\frac{1}{2}$

$\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

hence series convergent if

$x \in (-\frac{1}{2}, \frac{1}{2})$ or $-\frac{1}{2} < x < \frac{1}{2}$

Q No 13 Do yourself as Q No 12

Q No 14

let the infinite geometric series

$a_1 + a_1 r + a_1 r^2 + \cdots\cdots$

then $S = \frac{a_1}{1-r}$

$\Rightarrow 9 = \frac{a_1}{1-r}$ $\because S = 9$ (given)

$\Rightarrow 9(1-r) = a_1$ ——— (i)

Now square of its terms.

$a_1^2 + a_1^2 r^2 + a_1^2 r^4 + \cdots\cdots$

then $S = \frac{a_1^2}{1-r^2}$ $\because a_1 = a_1^2,\ r = r^2$

$\Rightarrow \frac{81}{5} = \frac{a_1^2}{1-r^2}$ $\because S = \frac{81}{5}$ (given)

$\Rightarrow \frac{81}{5}(1-r^2) = a_1^2$

$\Rightarrow \frac{81}{5}(1-r)(1+r) = [9(1-r)]^2$ from (i)

$\Rightarrow \frac{\cancel{81}}{5}\cancel{(1-r)}(1+r) = \cancel{81}(1-r)^{\cancel{2}}$

$\Rightarrow \frac{1}{5}(1+r) = (1-r)$

$\Rightarrow (1+r) = 5 - 5r$

$\Rightarrow r + 5r = 5 - 1$

$\Rightarrow 6r = 4 \Rightarrow r = \frac{4}{6} = \frac{2}{3}$

putting in (i)

$9(1-\frac{2}{3}) = a_1 \Rightarrow 9(\frac{1}{3}) = a_1$

$\Rightarrow a_1 = 3$

Now

$a_1 r = (3)(\frac{2}{3}) = 2$

$a_1 r^2 = (3)(\frac{2}{3})^2 = (3)(\frac{4}{9}) = \frac{4}{3}$

$a_1 r^3 = (3)(\frac{2}{3})^3 = (3)(\frac{8}{27}) = \frac{8}{9}$

Thus

$3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots\cdots$

is the required series

——END——