Q.No.1 i) -2 and 8

$a=-2$ and $b=8$

Since $G.M = \pm\sqrt{ab}$

$= \pm\sqrt{(-2)(8)}$

$= \pm\sqrt{-16} = \pm 4i$

ii) $-2i$ and $8i$

$a=-2i$, $b=8i$

Since $G.M = \pm\sqrt{ab}$

$= \pm\sqrt{(-2i)(8i)}$

$= \pm\sqrt{-16i^2}$

$= \pm\sqrt{16}$ $\because i^2=-1$

$= \pm 4$

Q.No.2 i) Let G_1, G_2 are two G.Ms between 1 and 8.

then $1, G_1, G_2, 8$ are in G.P.

here $a_1 = 1$, $a_4 = 8$

$\Rightarrow a_1 r^3 = 8$

$\Rightarrow (1) r^3 = 8$

$\Rightarrow r^3 = (2)^3$

$\Rightarrow r = 2$

Now

$G_1 = a_2 = a_1 r = (1)(2) = 2$

$G_2 = a_3 = a_1 r^2 = (1)(2)^2 = (1)(4) = 4$

hence $2, 4$ are two G.Ms between 1 and 8

ii) Do yourself as (i)

Q.No.3 i) Let G_1, G_2 and G_3 are three G.Ms between 1 and 16.

then $1, G_1, G_2, G_3, 16$ are in G.P.

here $a_1 = 1$, $a_5 = 16$

$\Rightarrow a_1 r^4 = 16$

$\Rightarrow 1 \cdot r^4 = 16$

$\Rightarrow r^4 = (2)^4$

$\Rightarrow r = 2$

Now $G_1 = a_2 = a_1 r = (1)(2) = 2$

$G_2 = a_3 = a_1 r^2 = (1)(2)^2 = 4$

$G_3 = a_4 = a_1 r^3 = (1)(2)^3 = 8$

hence $2, 4, 8$ are three G.Ms between 1 and 16.

Q.No.3 (ii) Do yourself

Q.No.4 Let G_1, G_2, G_3 and G_4 are four G.Ms between 3 and 96. then $3, G_1, G_2, G_3, G_4, 96$ are in G.P.

here $a_1 = 3$, $a_6 = 96$

$\Rightarrow a_1 r^5 = 96$

$\Rightarrow 3 r^5 = 96$

$\Rightarrow r^5 = 96/3 = 32$

$\Rightarrow r^5 = (2)^5 \Rightarrow \boxed{r=2}$

Now

$G_1 = a_2 = a_1 r = (3)(2) = 6$

$G_2 = a_3 = a_1 r^2 = (3)(2)^2 = 12$

$G_3 = a_4 = a_1 r^3 = (3)(2)^3 = 24$

$G_4 = a_5 = a_1 r^4 = (3)(2)^4 = 48$

hence $6, 12, 24, 48$ are four G.Ms between 3 and 96.

Q.No.5

Suppose $A > G$

then $\frac{x+y}{2} > \pm\sqrt{xy}$

$\Rightarrow x+y > \pm 2\sqrt{xy}$

$\Rightarrow x+y \mp 2\sqrt{xy} > 0$

$\Rightarrow (\sqrt{x})^2 + (\sqrt{y})^2 \mp \sqrt{xy} > 0$

$\Rightarrow (\sqrt{x} \mp \sqrt{y})^2 > 0$

which is true as square is always +ive. hence $A > G$.

Q No 6 We know that

$$G\cdot M = \sqrt{ab} \text{ ------ (i)}$$

but we have given

$$G\cdot M = \frac{a^n+b^n}{a^{n-1}+b^{n-1}} \text{ ------ (ii)}$$

Comparing (i) and (ii)

$$\frac{a^n+b^n}{a^{n-1}+b^{n-1}} = \sqrt{ab}$$

$$\Rightarrow a^n+b^n = a^{1/2}b^{1/2}(a^{n-1}+b^{n-1})$$

$$\Rightarrow a^n+b^n = a^{n-1/2}b^{1/2}+a^{1/2}b^{n-\frac{1}{2}}$$

$$\Rightarrow a^n - a^{n-\frac{1}{2}}b^{1/2} = a^{1/2}b^{n-1/2} - b^n$$

$$\Rightarrow a^{n-\frac{1}{2}}(\cancel{a^{1/2}-b^{1/2}}) = b^{n-\frac{1}{2}}(\cancel{a^{1/2}-b^{1/2}})$$

$$\Rightarrow a^{n-1/2} = b^{n-1/2}$$

$$\Rightarrow \frac{a^{n-1/2}}{b^{n-1/2}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-1/2} = \left(\frac{a}{b}\right)^0 \quad \because \left(\frac{a}{b}\right)^0 = 1$$

$$\Rightarrow n-\frac{1}{2} = 0 \Rightarrow \boxed{n=1/2}$$

Q No 7 Let a and b be two +ive integers then by given condition.

$$A\cdot M = G\cdot M + 2$$

$$\Rightarrow \frac{a+b}{2} = \sqrt{ab}+2$$

$$\Rightarrow a+b = 2(\sqrt{ab}+2) \text{ ------ (i)}$$

Also we have given

$$a+b = 20 \text{ ------ (ii)}$$

Comparing (i) & (ii)

$$2(\sqrt{ab}+2) = 20 \Rightarrow 2\sqrt{ab}+4=20$$

$$\Rightarrow 2\sqrt{ab} = 20-4 = 16$$

$$\Rightarrow \sqrt{ab} = \frac{16}{2} = 8$$

$$\Rightarrow ab = 64 \quad \text{by squaring.}$$

$$\Rightarrow b = \frac{64}{a} \text{ ------ (iii)}$$

putting in (ii)

$$a+\frac{64}{a} = 20$$

$$\Rightarrow a^2+64 = 20a$$

$$\Rightarrow a^2-20a+64=0$$

$$\Rightarrow a^2-16a-4a+64=0$$

$$\Rightarrow a(a-16)-4(a-16)=0$$

$$\Rightarrow (a-16)(a-4)=0$$

$$a-16=0, \quad a-4=0$$

$$a=16, \quad a=4$$

putting in (iii)

$$b=\frac{64}{16}=4, \quad b=\frac{64}{4}=16$$

hence 16, 4 OR 4, 16 are required numbers.

Q No. 8 Let a, and b be two ~~give~~ required number.

then $A\cdot M = 5$

$$\Rightarrow \frac{a+b}{2} = 5$$

$$\Rightarrow a+b = 10 \text{ ------ (i)}$$

Also $G\cdot M = 4$

$$\Rightarrow \sqrt{ab} = 4$$

$$\Rightarrow ab = 16 \quad \text{on squaring}$$

$$\Rightarrow b = \frac{16}{a} \text{ ------ (ii)}$$

putting in (i)

$$a+\frac{16}{a} = 10$$

$$\Rightarrow a^2+16 = 10a$$

$$\Rightarrow a^2-10a+16=0$$

$$\Rightarrow a^2-8a-2a+16=0$$

$$\Rightarrow a(a-8)-2(a-8)=0$$

$$\Rightarrow (a-8)(a-2)=0$$

$$a-8=0, \quad a-2=0$$

$$a=8, \quad a=2$$

putting in (ii)

$$b=\frac{16}{8}=2, \quad b=\frac{16}{2}=8$$

hence 8, 2 OR 2, 8 are required numbers.

END