Question # 1

The sequence of the deposits is

10, 15, 20, ……………. to 9 terms

Here $a_1 = 10$, $d = 15 - 10 = 5$, $n = 9$

Since $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow S_9 = \frac{9}{2}[2(10) + (9-1)(5)] = \frac{9}{2}[20 + 40] = \frac{9}{2}(60) = 270$

Hence the total amount he deposits is Rs. 270.

Question # 2

The sequence of the trees from top to base row is

1, 2, 3, ……………

Let n be the total number of tress in base row then

$a_1 = 1$, $d = 2 - 1 = 1$, $n = n$, $S_n = 378$

Now $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow 378 = \frac{n}{2}[2(1) + (n-1)(1)] \quad \Rightarrow 756 = n[2 + n - 1]$

$\Rightarrow 756 = n(n+1) \quad \Rightarrow 756 = n^2 + n$

$\Rightarrow n^2 + n - 756 = 0$

So $n = \dfrac{-1 \pm \sqrt{(-1)^2 - 4(1)(-756)}}{2(1)}$

$\Rightarrow n = \dfrac{-1 \pm \sqrt{1 + 3024}}{2} = \dfrac{-1 \pm \sqrt{3025}}{2} = \dfrac{-1 \pm 55}{2}$

So $n = \dfrac{-1 + 55}{2} = \dfrac{54}{2} = 27$ or $n = \dfrac{-1 - 55}{2} = \dfrac{-56}{2} = -28$

Since n can never be negative therefore $n = 27$

Now $a_n = a_1 + (n-1)d$

$\Rightarrow a_{27} = (1) + (27-1)(1) = 1 + 26 = 27$

Thus the numbers of trees in the base row are 27.

Question # 3

Let the first installment be x then the sequence of installment will be

$x,\ x - 10,\ x - 20, \ldots\ldots\ldots\ldots\ldots\ldots$

Here $a_1 = x$, $d = -10$, $n = 14$ and $S_n = 1100 + 230 = 1330$

Now $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow 1330 = \frac{14}{2}[2x + (14-1)(-10)] \quad \Rightarrow 1330 = 7[2x - 130] \quad \Rightarrow 1330 = 14x - 910$

$\Rightarrow 1330 + 910 = 14x \quad \Rightarrow 2240 = 14x \quad \Rightarrow x = \frac{2240}{14} = 160$

Hence the first installment is 160.

Question # 4

The sequence of the strikes is

1, 2, 3, …………., 12

Here $a_1 = 1$, $d = 2 - 1 = 2$, $n = 12$, $a_n = 12$

Now $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow S_n = \frac{12}{2}[2(1) + (12-1)(1)] = \frac{12}{2}[2+11] = \frac{12}{2}[13] = 78$

Hence clock strikes 78 hours in twelve strikes.

Question # 5

The sequence of the savings is

12, 16, 20, ……………

Total Savings = 2100

So here $a_1 = 12$, $d = 16 - 12 = 4$, $S_n = 2100$, $n = ?$

Since $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow 2100 = \frac{n}{2}[2(12) + (n-1)(4)]$ $\Rightarrow 4200 = n[24 + 4n - 4]$

$\Rightarrow 4200 = n[4n + 20]$ $\Rightarrow 4200 = 4n^2 + 20n$

$\Rightarrow 4n^2 + 20n - 4200 = 0$ $\Rightarrow 4(n^2 + 5n - 1050) = 0$

$\Rightarrow n^2 + 5n - 1050 = 0$

$$\Rightarrow n = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-1050)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 4200}}{2}$$

$$= \frac{-5 \pm \sqrt{4225}}{2} = \frac{-5 \pm 65}{2}$$

So $n = \frac{-5-65}{2} = \frac{-70}{2} = -35$ or $n = \frac{-5+65}{2} = \frac{60}{2} = 30$

As n can never be negative therefore $n = 30$

Thus student will save Rs. 2100 in 30 weeks.

Question # 6

The sequence of the falls is

9, 27, 45, ………………

(i) $a_1 = 9$, $d = 27 - 9 = 18$, $a_5 = ?$

Since $a_5 = a_1 + 4d = 9 + 4(18) = 9 + 72 = 81$

Hence in fifth second the object will fall 81 meters.

(ii) Here $a_1 = 9$, $d = 27 - 9 = 18$, n = 5, $S_5 = ?$

Since $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\Rightarrow S_5 = \frac{5}{2}[2(9) + (5-1)(18)] = \frac{5}{2}[18 + 72] = \frac{5}{2}(90) = 225$

Thus up to 5^{th} second the object will fall 225 meters.

Question # 7

Here $a_1 = 6000$, $a_{11} = 12000$, $n = 11$

Now $S_n = \frac{n}{2}[a_1 + a_n]$

$\Rightarrow S_{11} = \frac{11}{2}[6000+12000] = \frac{11}{2}(18000) = 99000$

Hence he will receive Rs. 99000 in past eleven years.

Question # 8

Since the sum of angels of 3 sided polygon (triangle) = $a_1 = p$

Sum of angels of 4 sided polygon (quadrilateral) = $a_2 = 2p$

Sum of the angels of 5 sided polygon (pentagon) = $a_3 = 3p$

So

The sum of interior angels of 16 side polygon = $a_{14} = ?$

Here $a_1 = p$, $d = a_2 - a_1 = 2p - p = p$, $n = 14$

Since $a_n = a_1 + (n-1)d$

$\Rightarrow a_{14} = p + (14-1)(p) = p + 13p = 14p$

Hence sum of interior angels of 16 side polygon is $14p$.

Question # 9

Let a_1 denotes the prize money for the last position

Then $a_1 = 4000$, $S_n = 60000$, $n = 8$, $a_n = ?$

Since $S_n = \frac{n}{2}(a_1 + a_n)$

$\Rightarrow 60000 = \frac{8}{2}(4000 + a_n)$ $\Rightarrow 60000 = 4(4000 + a_n)$

$\Rightarrow 60000 = 16000 + 4a_n$ $\Rightarrow 60000 - 16000 = 4a_n$ $\Rightarrow 44000 = 4a_n$

$\Rightarrow a_n = \frac{44000}{4} = 11000$

Hence the team at 1st place will get 11000 Rs.

Question # 10

Balls in the first layer = 8 + 7 + 6 +……… + 2 + 1

$= \frac{8}{2}[2(8) + (8-1)(-1)] = 4(16-7) = 36$

Balls in the second layer = 7 + 6 + 5 + ………+ 2 + 1

$= \frac{7}{2}[2(7) + (7-1)(-1)] = \frac{7}{2}[14-6] = \frac{7}{2}[8] = 28$

Balls in the third layer = 6 + 5 + 4 +3 + 2 + 1 = 21
Balls in the fourth layer = 5 + 4 + 3 + 2 +1 = 15
Balls in the fifth layer = 4 + 3 + 2 + 1 = 10
Balls in the sixth layer = 3 + 2 + 1 = 6
Balls in the seventh layer = 2 + 1 = 3
Balls in the eighth layer = 1
Hence the number of balls in pyramid

= 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 120

The End