Question # 1 (i) $3\sqrt{5}$ and $5\sqrt{5}$

Here $a = 3\sqrt{5}$ and $b = 5\sqrt{5}$, so

$$\text{A.M.} = \frac{a+b}{2} = \frac{3\sqrt{5}+5\sqrt{5}}{2} = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$$

(ii) Same as (i) and (ii)

(iii) $1-x+x^2$ and $1+x+x^2$

Here $a = 1-x+x^2$ and $b = 1+x+x^2$

$$\text{A.M.} = \frac{a+b}{2} = \frac{1-x+x^2+1+x+x^2}{2} = \frac{2+2x^2}{2} = 1+x^2$$

Question # 2 Since 5, 8 are two A.Ms between a and b.

Therefore a, 5, 8, b are in A.P.

Here $a_1 = a$ and $d = 8-5 = 3$

Now $a_2 = a_1 + d \Rightarrow 5 = a+3 \Rightarrow 5-3 = a \Rightarrow \boxed{a=2}$

Also $a_4 = a_1 + 3d \Rightarrow b = 2+3(3) \Rightarrow \boxed{b=11}$

Question # 3 Let A_1, A_2, A_3, A_4, A_5 and A_6 are six A.Ms between 2 and 5.

Then $2, A_1, A_2, A_3, A_4, A_5, A_6, 5$ are in A.P.

Here $a_1 = 2$ and $a_8 = 5$

$\Rightarrow a_1 + 7d = 5 \Rightarrow 2 + 7d = 5$

$\Rightarrow 7d = 5-2 \Rightarrow 7d = 3 \Rightarrow d = 3/7$

So $A_1 = a_2 = a_1 + d = 2 + 3/7 = 17/7$

$A_2 = a_3 = a_1 + 2d = 2 + 2\left(3/7\right) = 2 + 6/7 = 20/7$

$A_3 = a_4 = a_1 + 3d = 2 + 3\left(3/7\right) = 2 + 9/7 = 23/7$

$A_4 = a_5 = a_1 + 4d = 2 + 4\left(3/7\right) = 2 + 12/7 = 26/7$

$A_5 = a_6 = a_1 + 5d = 2 + 5\left(3/7\right) = 2 + 15/7 = 29/7$

$A_6 = a_7 = a_1 + 6d = 2 + 6\left(3/7\right) = 2 + 18/7 = 32/7$

Hence $\frac{17}{7}, \frac{20}{7}, \frac{23}{7}, \frac{26}{7}, \frac{29}{7}, \frac{32}{7}$ are six A.Ms between 2 and 5.

Question # 4 Suppose A_1, A_2, A_3 and A_4 are four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Then $\sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$ are in A.P.

Here $a_1 = \sqrt{2}$ and $a_6 = \frac{12}{\sqrt{2}}$

$$\Rightarrow a_1 + 5d = \frac{12}{\sqrt{2}} \Rightarrow \sqrt{2} + 5d = \frac{12}{\sqrt{2}} \Rightarrow 5d = \frac{12}{\sqrt{2}} - \sqrt{2}$$

$$\Rightarrow 5d = \frac{12-2}{\sqrt{2}} \quad \Rightarrow 5d = \frac{10}{\sqrt{2}} \quad \Rightarrow d = \frac{2}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^2}{\sqrt{2}} \Rightarrow d = \sqrt{2}$$

Now $A_1 = a_2 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$A_2 = a_3 = a_1 + 2d = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

$A_3 = a_4 = a_1 + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

$A_4 = a_5 = a_1 + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$

Hence $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$, $5\sqrt{2}$ are four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Question # 5 & 6 Same as Question # 3 & 4.

Question # 7

Since we know that A.M. $= \frac{a+b}{2}$ ………….. (i)

But we have given A.M. $= \frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ ………….. (ii)

Comparing (i) and (ii)

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$$

$$\Rightarrow 2(a^n + b^n) = (a+b)(a^{n-1} + b^{n-1}) \qquad Cross \times ing$$

$$\Rightarrow 2a^n + 2b^n = a^n + a^{n-1}b + ab^{n-1} + b^n$$

$$\Rightarrow 2a^n + 2b^n - a^n - b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow a^n + b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow a^n - a^{n-1}b = ab^{n-1} - b^n$$

$$\Rightarrow a^{n-1+1} - a^{n-1}b = ab^{n-1} - b^{n-1+1} \qquad \because n = n-1+1$$

$$\Rightarrow a^{n-1}(a-b) = b^{n-1}(a-b)$$

$$\Rightarrow a^{n-1} = b^{n-1}$$

$$\Rightarrow \frac{a^{n-1}}{b^{n-1}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0 \qquad \because \left(\frac{a}{b}\right)^0 = 1$$

$$\Rightarrow n-1 = 0 \quad \Rightarrow \boxed{n=1}$$

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