

Q.No.1 i) $a_1 = 5$ and other three

consecutive terms are 23, 26, 29

Since $a_1 = 5$ & $d = 26 - 23 = 3$

Now $a_2 = a_1 + d = 5 + 3 = 8$

$a_3 = a_2 + d = 8 + 3 = 11$

$a_4 = a_3 + d = 11 + 3 = 14$

hence 5, 8, 11, 14 are first four terms of A.P

-----x-----

Q.No.2 (ii) $a_5 = 17$ and $a_9 = 37$

Consider a_1 be the first term and 'd' be the common difference.

Since $a_5 = 17$

$$\Rightarrow a_1 + (5-1)d = 17$$

$$\Rightarrow a_1 + 4d = 17 \text{ --- (i)}$$

also $a_9 = 37$

$$\Rightarrow a_1 + (9-1)d = 37$$

$$\Rightarrow a_1 + 8d = 37 \text{ --- (ii)}$$

Subtracting (i) and (ii)

$$a_1 + 4d = 17$$

$$a_1 + 8d = 37$$

$$\hline -4d = -20$$

$$\Rightarrow d = 5$$

putting value of d in (i)

$$a_1 + 4(5) = 17$$

$$\Rightarrow a_1 + 20 = 17$$

$$\Rightarrow a_1 = 17 - 20 = -3$$

$$\Rightarrow a_1 = -3$$

$$\text{So } a_2 = a_1 + d = -3 + 5 = 2$$

$$a_3 = a_2 + d = 2 + 5 = 7$$

$$a_4 = a_3 + d = 7 + 5 = 12$$

hence -3, 2, 7, 12 are first four terms of A.P.

iii) $3a_7 = 7a_4$ & $a_{10} = 33$

Suppose a_1 be the first term and d be the common difference

Since $3a_7 = 7a_4$

$$\Rightarrow 3(a_1 + 6d) = 7(a_1 + 3d)$$

$$\Rightarrow 3a_1 + 18d = 7a_1 + 21d$$

$$\Rightarrow 3a_1 + 18d - 7a_1 - 21d = 0$$

$$\Rightarrow -4a_1 - 3d = 0$$

$$\Rightarrow 4a_1 + 3d = 0 \text{ --- (i)}$$

also $a_{10} = 33$

$$\Rightarrow a_1 + 9d = 33 \text{ --- (ii)}$$

Multiplying eq. (ii) by 4 & subtracting from (i)

$$4a_1 + 3d = 0$$

$$\hline -4a_1 + 36d = 132$$

$$-33d = -132$$

$$\Rightarrow d = \frac{-132}{-33} = 4$$

putting value of d in (ii)

$$a_1 + 9(4) = 33$$

$$\Rightarrow a_1 + 36 = 33$$

$$\Rightarrow a_1 = 33 - 36 \Rightarrow a_1 = -3$$

$$\text{So } a_2 = a_1 + d = -3 + 4 = 1$$

$$a_3 = a_2 + d = 1 + 4 = 5$$

$$a_4 = a_3 + d = 5 + 4 = 9$$

hence -3, 1, 5, 9 are the first four terms of A.P.

-----x-----

Q.No.2 $a_{n-3} = 2n - 5$

$$\Rightarrow a_{n-3} = 2n - 6 + 1$$

$$= 2(n-3) + 1$$

Replacing $n-3$ by n .

$$a_n = 2n + 1$$

Answer

QNo.3 Suppose a_1 be the first term and d be common difference of A.P.

Since $a_5 = 16$

$\Rightarrow a_1 + 4d = 16$ — (i)

also $a_{20} = 46$

$\Rightarrow a_1 + 19d = 46$ — (ii)

Subtracting (i) & (ii)

$$\begin{array}{r} a_1 + 4d = 16 \\ a_1 + 19d = 46 \\ \hline -15d = -30 \end{array}$$

$\Rightarrow d = 2$

putting value of d in (i)

$a_1 + 4(2) = 16$

$\Rightarrow a_1 + 8 = 16$

$\Rightarrow a_1 = 16 - 8 \Rightarrow a_1 = 8$

Now

$$\begin{aligned} a_{12} &= a_1 + 11d \\ &= 8 + 11(2) \\ &= 8 + 22 = 30 \end{aligned}$$

Answer

QNo.4

$x, 1, 2-x, 3-2x, \dots$

here $a_1 = 1$

and $d = a_2 - a_1 = 1 - x$

Since

$$\begin{aligned} a_{13} &= a_1 + 12d \\ &= 1 + 12(1-x) \\ &= 1 + 12 - 12x \end{aligned}$$

~~$a_{13} = 12 - 11x$~~ Answer

QNo.5 Same as QNo.3

QNo.6

$5, 2, -1, \dots, -85$

here $a_1 = 5$

$d = a_2 - a_1 = 2 - 5 = -3$

$a_n = -85, n = ?$

Since

$a_n = a_1 + (n-1)d$

$\Rightarrow -85 = 5 + (n-1)(-3)$

$\Rightarrow -85 = 5 - 3n + 3$

$\Rightarrow 3n = 5 + 3 + 85$

$\Rightarrow 3n = 93$

$\Rightarrow \boxed{n = 31}$ Answer

QNo.7 Same as above

QNo.8 $a_1 = 11, a_n = 68$

$d = 3, n = ?$

Since

$a_n = a_1 + (n-1)d$

$\Rightarrow 68 = 11 + (n-1)3$

Now solve yourself as above

QNo.9

Since $a_n = 3n - 1$

put $n = 1$

$a_1 = 3(1) - 1 = 3 - 1 = 2$

put $n = 2$

$a_2 = 3(2) - 1 = 6 - 1 = 5$

put $n = 3$

$a_3 = 3(3) - 1 = 9 - 1 = 8$

put $n = 4$

$a_4 = 3(4) - 1 = 12 - 1 = 11$

Thus

$2, 5, 8, 11, \dots$

is the required A.P

Q.No.10 17, 13, 9,

$$a_1 = 17, \quad d = 13 - 17 = -4$$

i) Suppose -19 be the n th term of A.P. i.e $a_n = -19$

$$\text{Since } a_n = a_1 + (n-1)d$$

$$\Rightarrow -19 = 17 + (n-1)(-4)$$

$$\Rightarrow -19 = 17 - 4n + 4$$

$$\Rightarrow 4n = 17 + 4 + 19$$

$$= 40$$

$$\Rightarrow n = 10$$

Thus -19 is the 10th term of A.P.

ii) Suppose 2 be the n th term of A.P. i.e $a_n = 2$

$$\text{Since } a_n = a_1 + (n-1)d$$

$$\Rightarrow 2 = 17 + (n-1)(-4)$$

$$\Rightarrow 2 = 17 - 4n + 4$$

$$\Rightarrow 4n = 17 + 4 - 2$$

$$= 19$$

$$\Rightarrow n = \frac{19}{4} \text{ which is}$$

a rational. therefore 2 is not the term of A.P.

Q.No.11

Let a_1 be the first term and d be the common difference

$$\text{Now } a_p = l$$

$$\Rightarrow a_1 + (p-1)d = l$$

$$a_q = m$$

$$\Rightarrow a_1 + (q-1)d = m$$

$$a_r = n$$

$$\Rightarrow a_1 + (r-1)d = n$$

$$i) \text{ L.H.S} = l(q-r) + m(r-p) + n(p-q)$$

$$= [a_1 + (p-1)d](q-r) + [a_1 + (q-1)d](r-p)$$

$$+ [a_1 + (r-1)d](p-q)$$

$$= (a_1 + pd - d)(q-r) + (a_1 + qd - d)(r-p)$$

$$+ (a_1 + rd - d)(p-q)$$

$$= a_1q + pqd - qd - a_1r - prd + rd$$

$$+ a_1r + qrd - qr - a_1p - pd + pq$$

$$+ a_1p + prd - pd - a_1q - qrd + qd$$

$$= 0 = \text{R.H.S proved}$$

$$ii) \text{ L.H.S} = p(m-n) + q(n-l) + r(l-m)$$

$$= p[a_1 + (q-1)d - a_1 - (r-1)d]$$

$$+ q[a_1 + (r-1)d - a_1 - (p-1)d]$$

$$+ r[a_1 + (p-1)d - a_1 - (q-1)d]$$

$$= p[qd - d - rd + d]$$

$$+ q[rd - d - pd + d]$$

$$+ r[pd - d - qd + d]$$

$$= pqd - prd + qrd - prd$$

$$+ prd - qrd = 0 = \text{R.H.S}$$

$$\text{proved}$$

Q.No.12

$$\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \dots$$

We first find the n th term of 4, 7, 10,

$$a_1 = 4, \quad d = 7 - 4 = 3$$

$$\text{So } a_n = a_1 + (n-1)d$$

$$= 4 + (n-1)3$$

$$= 4 + 3n - 3 = 3n + 1$$

hence n th term of

$$\text{given sequence is } \left(\frac{3n+1}{3}\right)^2$$

P.T.O

Q No. 13 Since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. therefore

$$d = \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{1}{b} + \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \frac{1+1}{b} = \frac{c+a}{ac}$$

$$\Rightarrow \frac{2}{b} = \frac{a+c}{ac}$$

$$\Rightarrow \frac{b}{2} = \frac{ac}{a+c}$$

$$\Rightarrow b = \frac{2ac}{a+c} \quad \text{proved}$$

Q No. 14 Since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. therefore

$$d = \frac{1}{b} - \frac{1}{a} \quad \text{--- (i)}$$

also

$$d = \frac{1}{c} - \frac{1}{b} \quad \text{--- (ii)}$$

Comparing (i) and (ii)

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{1}{b} + \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$$

$$\Rightarrow \frac{1+1}{b} = \frac{a+c}{ac}$$

$$\Rightarrow \frac{2}{b} = \frac{a+c}{ac}$$

$$\Rightarrow \frac{b}{2} = \frac{ac}{a+c}$$

$$\Rightarrow b = \frac{2ac}{a+c}$$

putting value of b in eq (i)

$$d = \frac{1}{\frac{2ac}{a+c}} - \frac{1}{a} = \frac{a+c}{2ac} - \frac{1}{a}$$

$$= \frac{a+c-2c}{2ac} = \frac{a-c}{2ac}$$

hence the common difference is $\frac{a-c}{2ac}$

END.

Available online at <http://www.mathcity.org>
