

**Formula for the sum**

i)  $\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

ii)  $\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

iii)  $\sum_{k=1}^n k^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2$

iv)  $\sum_{k=1}^n (1) = 1 + 1 + 1 + \dots + 1 \text{ (n times)} = n$

**Question # 1**

$1 \times 1 + 2 \times 4 + 3 \times 7 + \dots$

$1 + (k-1)3$   
 $= 1 + 3k - 3$   
 $= 3k - 2$

If  $T_k$  denotes the  $k$ th term of the series then

$T_k = k(3k - 2) = 3k^2 - 2k$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$\begin{aligned} S_n &= \sum_{k=1}^n (3k^2 - 2k) \\ &= 3 \sum_{k=1}^n k^2 - 2 \sum_{k=1}^n k = 3 \left( \frac{n(n+1)(2n+1)}{6} \right) - 2 \left( \frac{n(n+1)}{2} \right) \\ &= \frac{n(n+1)(2n+1)}{2} - n(n+1) \\ &= \frac{n(n+1)}{2} (2n+1 - 2) = \frac{n(n+1)(2n-1)}{2} \quad \text{Answer} \end{aligned}$$

**Question # 2**

$1 \times 3 + 3 \times 6 + 5 \times 9 + \dots$

$1 + (k-1)2$   
 $= 1 + 2k - 2$   
 $= 2k - 1$

If  $T_k$  denotes the  $k$ th term of the series then

$T_k = (2k - 1)(3k) = 6k^2 - 3k$

$3 + (k-1)3$   
 $= 3 + 3k - 3$   
 $= 3k$

**Now do yourself as above.**

**Question # 3 & 4**

**Do yourself as Question # 1**

**Question # 5**

$1^2 + 3^2 + 5^2 + \dots$

$1 + (k-1)2$   
 $= 1 + 2k - 2$   
 $= 2k - 1$

If  $T_k$  denotes the  $k$ th term of the series then

$T_k = (2k - 1)^2 = 4k^2 - 4k + 1$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$\begin{aligned} S_n &= \sum_{k=1}^n (4k^2 - 4k + 1) \\ &= 4 \sum_{k=1}^n k^2 - 4 \sum_{k=1}^n k + \sum_{k=1}^n (1) = 4 \left( \frac{n(n+1)(2n+1)}{6} \right) - 4 \left( \frac{n(n+1)}{2} \right) + n \\ &= \frac{2n(n+1)(2n+1)}{3} - 2(n(n+1)) + n \end{aligned}$$

$$\begin{aligned}
 &= n \left( \frac{2(n+1)(2n+1)}{3} - 2(n+1) + 1 \right) = n \left( \frac{2(2n^2 + 2n + n + 1)}{3} - 2n - 2 + 1 \right) \\
 &= n \left( \frac{2(2n^2 + 3n + 1)}{3} - 2n - 1 \right) = n \left( \frac{4n^2 + 6n + 2 - 6n - 3}{3} \right) \\
 &= n \left( \frac{4n^2 - 1}{3} \right) = \frac{n}{3}(4n^2 - 1) \quad \text{Answer}
 \end{aligned}$$

**Question # 6**

$$2^2 + 5^2 + 8^2 + \dots\dots\dots$$

$$\begin{aligned}
 &2 + (k-1)3 \\
 &= 2 + 3k - 3 \\
 &= 3k - 1
 \end{aligned}$$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$T_k = (3k - 1)^2 = 9k^2 - 6k + 1$$

Let  $S_n$  be the sum of first  $n$  term of the series then

**Now do yourself as above**

**Question # 7**

$$2 \times 1^2 + 4 \times 2^2 + 6 \times 3^2 + \dots\dots\dots$$

$$\begin{aligned}
 &2 + (k-1)2 \\
 &= 2 + 2k - 2 \\
 &= 2k
 \end{aligned}$$

If  $T_k$  denotes the  $k$ th term of the series then

$$T_k = (2k)(k)^2 = 2k^3$$

$$\begin{aligned}
 &1 + (k-1)1 \\
 &= 1 + k - 1 \\
 &= k
 \end{aligned}$$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$\begin{aligned}
 S_n &= \sum_{k=1}^n (2k^3) = 2 \sum_{k=1}^n k^3 \\
 &= 2 \left( \frac{n(n+1)}{2} \right)^2 = 2 \frac{n^2(n+1)^2}{4} = \frac{n^2(n+1)^2}{2} \quad \text{Answer}
 \end{aligned}$$

**Question #8**

$$3 \times 2^2 + 5 \times 3^2 + 7 \times 4^2 + \dots\dots\dots$$

$$\begin{aligned}
 &3 + (k-1)2 \\
 &= 3 + 2k - 2 \\
 &= 2k + 1
 \end{aligned}$$

If  $T_k$  denotes the  $k$ th term of the series then

$$\begin{aligned}
 T_k &= (2k + 1)(k + 1)^2 = (2k + 1)(k^2 + 2k + 1) \\
 &= 2k^3 + 4k^2 + 2k + k^2 + 2k + 1 \\
 &= 2k^3 + 5k^2 + 4k + 1
 \end{aligned}$$

$$\begin{aligned}
 &2 + (k-1)1 \\
 &= 2 + k - 1 \\
 &= k + 1
 \end{aligned}$$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$\begin{aligned}
 S_n &= \sum_{k=1}^n (2k^3 + 5k^2 + 4k + 1) \\
 &= 2 \sum_{k=1}^n k^3 + 5 \sum_{k=1}^n k^2 + 4 \sum_{k=1}^n k + \sum_{k=1}^n (1) \\
 &= 2 \left( \frac{n(n+1)}{2} \right)^2 + 5 \left( \frac{n(n+1)(2n+1)}{6} \right) + 4 \frac{n(n+1)}{2} + n \\
 &= 2 \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1) + n \\
 &= n \left( \frac{n^2 + 2n + 1}{2} + \frac{5(2n^2 + 2n + n + 1)}{6} + 2(n+1) + 1 \right) \\
 &= n \left( \frac{n^3 + 2n^2 + n}{2} + \frac{5(2n^2 + 3n + 1)}{6} + 2n + 2 + 1 \right)
 \end{aligned}$$

$$\begin{aligned}
 &= n \left( \frac{n^3 + 2n^2 + n}{2} + \frac{10n^2 + 15n + 5}{6} + 2n + 3 \right) \\
 &= n \left( \frac{3n^3 + 6n^2 + 3n + 10n^2 + 15n + 5 + 12n + 18}{6} \right) \\
 &= \frac{n}{6} (3n^3 + 16n^2 + 30n + 23) \quad \text{Answer}
 \end{aligned}$$

**Question # 9**

$$2 \times 4 \times 7 + 3 \times 6 \times 10 + 4 \times 8 \times 13 + \dots$$

If  $T_k$  denotes the  $k$ th term of the series then

$$\begin{aligned}
 T_k &= 2(k+1)(k+1)(3k+4) = 2(k+1)^2(3k+4) \\
 &= 2(k^2 + 2k + 1)(3k+4) \\
 &= 3k^3 + 6k^2 + 3k + 4k^2 + 8k + 4 \\
 &= 3k^3 + 10k^2 + 11k + 4
 \end{aligned}$$

**Now do yourself.**

$$\begin{aligned}
 &2 + (k-1)1 \\
 &= 2 + k - 1 \\
 &= k + 1 \\
 &7 + (k-1)3 \\
 &= 7 + 3k - 3 \\
 &= 3k + 4 \\
 &4 + (k-1)2 \\
 &= 4 + 2k - 2 \\
 &= 2k + 2 = 2(k+1)
 \end{aligned}$$

**Question # 10**

$$1 \times 4 \times 6 + 4 \times 7 \times 10 + 7 \times 10 \times 14 + \dots$$

If  $T_k$  denotes the  $k$ th term of the series then

$$\begin{aligned}
 T_k &= (3k-2)(3k+1)2(2k+1) = 2(3k-2)(3k+1)(2k+1) \\
 &= 2(3k-2)(6k^2 + 2k + 3k + 1) = 2(3k-2)(6k^2 + 5k + 1) \\
 &= 2(18k^3 + 15k^2 + 3k - 12k^2 - 10k - 2) \\
 &= 2(18k^3 + 3k^2 - 7k - 2)
 \end{aligned}$$

**Now do yourself.**

$$\begin{aligned}
 &1 + (k-1)3 \\
 &= 1 + 3k - 3 \\
 &= 3k - 2 \\
 &4 + (k-1)3 \\
 &= 4 + 3k - 3 \\
 &= 3k + 1 \\
 &6 + (k-1)4 \\
 &= 6 + 4k - 4 \\
 &= 4k + 2 = 2(2k+1)
 \end{aligned}$$

**Question # 11**

$$1 + (1+2) + (1+2+3) + \dots$$

If  $T_k$  denotes the  $k$ th term of the series then

$$T_k = 1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2} = \frac{1}{2}(k^2 + k)$$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$\begin{aligned}
 S_n &= \sum_{k=1}^n \left( \frac{1}{2}(k^2 + k) \right) = \frac{1}{2} \sum_{k=1}^n k^2 + \frac{1}{2} \sum_{k=1}^n k \\
 &= \frac{1}{2} \left( \frac{n(n+1)(2n+1)}{6} \right) + \frac{1}{2} \left( \frac{n(n+1)}{2} \right) \\
 &= \frac{n(n+1)}{4} \left( \frac{(2n+1)}{3} + 1 \right) = \frac{n(n+1)}{4} \left( \frac{2n+1+3}{3} \right) \\
 &= \frac{n(n+1)}{4} \left( \frac{2n+4}{3} \right) = \frac{n(n+1)(2n+4)}{12} \quad \text{Answer}
 \end{aligned}$$

**Question # 12**

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$$

If  $T_k$  denotes the  $k$ th term of the series then

$$\begin{aligned}
 T_k &= 1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6} \\
 &= \frac{1}{6}k(2k^2 + 2k + k + 1) = \frac{1}{6}k(2k^2 + 3k + 1)
 \end{aligned}$$

$$= \frac{1}{6}(2k^3 + 3k^2 + k) = \frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k$$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$\begin{aligned} S_n &= \sum_{k=1}^n \left( \frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k \right) = \frac{1}{3} \sum_{k=1}^n k^3 + \frac{1}{2} \sum_{k=1}^n k^2 + \frac{1}{6} \sum_{k=1}^n k \\ &= \frac{1}{3} \left( \frac{n^2(n+1)^2}{4} \right) + \frac{1}{2} \left( \frac{n(n+1)(2n+1)}{6} \right) + \frac{1}{6} \left( \frac{n(n+1)}{2} \right) \\ &= \frac{n(n+1)}{12} (n(n+1) + (2n+1) + 1) = \frac{n(n+1)}{12} (n^2 + n + 2n + 1 + 2) \\ &= \frac{n(n+1)(n^2 + 3n + 3)}{12} \quad \text{Answer} \end{aligned}$$

**Question # 13**

$$2 + (2 + 5) + (2 + 5 + 8) + \dots$$

If  $T_k$  denotes the  $k$ th term of the series then

$$\therefore S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

$$T_k = 2 + 5 + 8 + \dots + \text{up to } k \text{ terms}$$

$$= \frac{k}{2} [2(2) + (k-1)(3)] = \frac{k}{2} [4 + 3k - 3] = \frac{k}{2} [3k + 1] = \frac{3}{2}k^2 + \frac{1}{2}k$$

**Now do yourself**

**Question # 14 (i)**

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + (2n-1)^2 - (2n)^2$$

If  $T_k$  denotes the  $k$ th term of the series then

$$T_k = (2k-1)^2 - (2k)^2 = 4k^2 - 4k + 1 - 4k^2 = -4k + 1$$

**Now do yourself**

**Question # 14 (ii)**

$$1^2 - 3^2 + 5^2 - 7^2 + \dots + (4n-3)^2 - (4n-1)^2$$

If  $T_k$  denotes the  $k$ th term of the series then

$$\begin{aligned} T_k &= (4k-3)^2 - (4k-1)^2 = (16k^2 - 24k + 9) - (16k^2 - 8k + 1) \\ &= 16k^2 - 24k + 9 - 16k^2 + 8k - 1 = -16k + 8 \end{aligned}$$

**Now do yourself**

**Question # 14 (iii)**

$$\frac{1^2}{1} + \frac{1^2 + 2^2}{2} + \frac{1^2 + 2^2 + 3^2}{3} + \dots \text{ to } n \text{ terms}$$

If  $T_k$  denotes the  $k$ th term of the series then

$$\begin{aligned} T_k &= \frac{1^2 + 2^2 + 3^2 + \dots + k^2}{k} \\ &= \frac{k(k+1)(2k+1)/6}{k} = \frac{k(k+1)(2k+1)}{6k} = \frac{(k+1)(2k+1)}{6} \\ &= \frac{2k^2 + 2k + k + 1}{6} = \frac{2k^2 + 3k + 1}{6} = \frac{2}{6}k^2 + \frac{3}{6}k + \frac{1}{6} \\ &= \frac{1}{3}k^2 + \frac{1}{2}k + \frac{1}{6} \end{aligned}$$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$S_n = \sum_{k=1}^n \left( \frac{1}{3}k^2 + \frac{1}{2}k + \frac{1}{6} \right) = \frac{1}{3} \sum_{k=1}^n k^2 + \frac{1}{2} \sum_{k=1}^n k + \frac{1}{6} \sum_{k=1}^n (1)$$

$$\begin{aligned}
&= \frac{1}{3} \left( \frac{n(n+1)(2n+1)}{6} \right) + \frac{1}{2} \left( \frac{n(n+1)}{2} \right) + \frac{n}{6} \\
&= \frac{n(n+1)(2n+1)}{18} + \frac{n(n+1)}{4} + \frac{n}{3} = \frac{n}{2} \left( \frac{(n+1)(2n+1)}{9} + \frac{n+1}{2} + \frac{1}{3} \right) \\
&= \frac{n}{2} \left( \frac{(2n^2 + 2n + n + 1)}{9} + \frac{n+1}{2} + \frac{1}{3} \right) = \frac{n}{2} \left( \frac{2n^2 + 3n + 1}{9} + \frac{n+1}{2} + \frac{1}{3} \right) \\
&= \frac{n}{2} \left( \frac{4n^2 + 6n + 2 + 9n + 9 + 6}{18} \right) = \frac{n}{2} \left( \frac{4n^2 + 15n + 17}{18} \right) \\
&= \frac{n}{36} (4n^2 + 15n + 17) \quad \text{Answer}
\end{aligned}$$


---

**Question # 15 (i)**

Since  $T_n = 3n^2 + n + 1$

Therefore  $T_k = 3k^2 + k + 1$

**Now do yourself**

---

**Question # 15 (ii)**

**Do yourself**

---

**Question # 16 (i)**

**Do yourself as below ( Q # 16 (ii) )**

---

**Question # 16 (ii)**

Since  $T_n = n^3 + 2n + 3$

Therefore  $T_k = k^3 + 2k + 3$

Let  $S_n$  denotes the sum of first  $n$  terms of the series then

$$\begin{aligned}
S_n &= \sum_{k=1}^n (k^3 + 2k + 3) = \sum_{k=1}^n k^3 + 2 \sum_{k=1}^n k + 3 \sum_{k=1}^n (1) \\
&= \frac{n^2(n+1)^2}{4} + 2 \frac{n(n+1)}{2} + 3n = n \left( \frac{n(n+1)^2}{4} + n + 1 + 3 \right) \\
&= n \left( \frac{n(n^2 + 2n + 1)}{4} + n + 4 \right) = n \left( \frac{n^3 + 2n^2 + n}{4} + n + 4 \right) \\
&= n \left( \frac{n^3 + 2n^2 + n + 4n + 16}{4} \right) \\
&= \frac{n}{4} (n^3 + 2n^2 + 5n + 16)
\end{aligned}$$

Now for sum of first  $2n$  terms put  $n = 2n$

$$\begin{aligned}
S_{2n} &= \frac{2n}{4} ((2n)^3 + 2(2n)^2 + 5(2n) + 16) \\
&= \frac{n}{2} (8n^3 + 8n^2 + 10n + 16) = \frac{2n}{2} (4n^3 + 4n^2 + 5n + 8) \\
&= n(4n^3 + 4n^2 + 5n + 8) \quad \text{Answer}
\end{aligned}$$


---