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Exercise 5.3 (Solutions)

Textbook of Algebra and Trigonometry for Class XI
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Question # 1 $\dfrac{9x-7}{(x^2+1)(x+3)}$

Resolving it into partial fraction.

$$\frac{9x-7}{(x^2+1)(x+3)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x+3)}$$

Multiplying both sides by $(x^2+1)(x+3)$.

$$9x-7=(Ax+B)(x+3)+C(x^2+1)\ \ldots\ldots\ldots\ldots (i)$$

Put $x+3=0 \Rightarrow x=-3$ in equation (i).

$$9(-3)-7=\big(A(-3)+B\big)(0)+C\left((-3)^2+1\right) \quad \Rightarrow -27-7=0+C(9+1)$$

$$\Rightarrow -34=10C \quad \Rightarrow C=-\frac{34}{10} \quad \Rightarrow \boxed{C=-\frac{17}{5}}$$

Now equation (i) can be written as

$$9x-7=A(x^2+3x)+B(x+3)+C(x^2+1)$$

Comparing the coefficients of x^2, x and x^0.

$$0=A+C\ \ldots\ldots\ldots\ldots (ii)$$
$$9=3A+B\ \ldots\ldots\ldots\ldots (iii)$$
$$-7=+3B+C\ \ldots\ldots\ldots\ldots (iv)$$

Putting value of C in equation (ii)

$$0=A-\frac{17}{5} \qquad \Rightarrow \boxed{A=\frac{17}{5}}$$

Now putting value of A in equation (iii)

$$9=3\left(\frac{17}{5}\right)+B \quad \Rightarrow 9=\frac{51}{5}+B \quad \Rightarrow 9-\frac{51}{5}=B \quad \Rightarrow \boxed{B=-\frac{6}{5}}$$

Hence

$$\frac{9x-7}{(x^2+1)(x+3)}=\frac{\frac{17}{5}x-\frac{6}{5}}{x^2+1}+\frac{-\frac{17}{5}}{(x+3)}$$

$$=\frac{\frac{17x-6}{5}}{x^2+1}-\frac{\frac{17}{5}}{(x+3)} \qquad =\frac{17x-6}{5(x^2+1)}-\frac{17}{5(x+3)} \quad \textit{Answer}$$

Question # 2 $\dfrac{1}{(x^2+1)(x+1)}$

Now Consider

$$\frac{1}{(x^2+1)(x+1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x+1}$$

Multiplying both sides by $(x^2+1)(x+1)$.

$$1=(Ax+B)(x+1)+C(x^2+1)\ldots\ldots\ldots\ldots (i)$$

Put $x+1=0 \Rightarrow x=-1$ in equation (i)

$$1=0+C\left((-1)^2+1\right) \qquad \Rightarrow 1=2C \quad \Rightarrow \boxed{C=\frac{1}{2}}$$

Now eq. (i) can be written as

$$1 = A(x^2+x) + B(x+1) + C(x^2+1)$$

Comparing the coefficients of x^2, x and x^0.

$0 = A + C$ ………………. (ii)

$0 = A + B$ ………………. (iii)

$1 = A + C$ ………………. (iv)

Putting value of C in equation (ii)

$$0 = A + \frac{1}{2} \Rightarrow \boxed{A = -\frac{1}{2}}$$

Putting value of A in equation (iii)

$$0 = -\frac{1}{2} + B \Rightarrow \boxed{B = \frac{1}{2}}$$

Hence $$\frac{1}{(x^2+1)(x+1)} = \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} + \frac{\frac{1}{2}}{x+1} = \frac{\frac{-x+1}{2}}{x^2+1} + \frac{\frac{1}{2}}{x+1}$$

$$= \frac{-x+1}{2(x^2+1)} + \frac{1}{2(x+1)} = \frac{1-x}{2(x^2+1)} + \frac{1}{2(x+1)} \qquad \textit{Answer}$$

Question # 3 $\dfrac{3x+7}{(x^2+4)(x+3)}$

Resolving it into partial fraction.

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+3}$$

$$\left[\begin{array}{c} \textit{Now do yourself, you will get} \\ A = 2/13,\ B = 33/13 \textit{ and } C = -2/13 \end{array}\right]$$

Question # 4 $\dfrac{x^2+15}{(x^2+2x+5)(x-1)}$

Resolving it into partial fraction.

$$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{Ax+B}{x^2+2x+5} + \frac{C}{x-1}$$

$$\Rightarrow x^2+15 = (Ax+B)(x-1) + C(x^2+2x+5) \ldots\ldots\ldots\ldots (i)$$

Put $x-1=0 \Rightarrow x=1$ in equation (i)

$$(1)^2+15 = \big(A(1)+B\big)(0) + C\big((1)^2+2(1)+5\big) \Rightarrow 1+15 = 0 + C(1+2+5)$$

$$\Rightarrow 16 = 8C \Rightarrow \frac{16}{8} = C \Rightarrow \boxed{C = 2}$$

Now equation (i) can be written as

$$x^2+15 = A(x^2-x) + B(x-1) + C(x^2+2x+5)$$

Comparing the coefficients of x^2, x and x^0.

$1 = A + C$ ……………………. (ii)

$0 = -A + B + 2C$ ………….. (iii)

$15 = -B + 5C$ ………………. (iv)

Putting value of C in equation (ii).

$$1 = A + 2 \Rightarrow 1-2 = A \Rightarrow \boxed{A = -1}$$

Putting value of A and C in equation (iii)

$$0=-(-1)+B+2(2) \quad \Rightarrow 0=1+B+4 \quad \Rightarrow 0=B+5 \quad \Rightarrow \boxed{B=-5}$$

Hence $$\frac{x^2+15}{(x^2+2x+5)(x-1)}=\frac{(-1)x-5}{x^2+2x+5}+\frac{2}{x-1}$$
$$=\frac{-x-5}{x^2+2x+5}+\frac{2}{x-1} \quad \textit{Answer}$$

Question # 5 $$\frac{x^2}{(x^2+4)(x+2)}$$

Resolving it into partial fraction.

$$\frac{x^2}{(x^2+4)(x+2)}=\frac{Ax+B}{x^2+4}+\frac{C}{x+2}$$

$$\left[\begin{array}{c}\textit{Now do yourself, you will get}\\ A=\tfrac{1}{2},\ B=-1 \textit{ and } C=-\tfrac{1}{2}\end{array}\right]$$

Question # 6 $$\frac{x^2+1}{x^3+1}=\frac{x^2+1}{(x+1)(x^2-x+1)} \qquad \because x^3+1=(x+1)(x^2-x+1)$$

Now consider

$$\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$$

$$\left[\begin{array}{c}\textit{Now do yourself, you will get}\\ A=\tfrac{2}{3},\ B=\tfrac{1}{3} \textit{ and } C=\tfrac{1}{3}\end{array}\right]$$

Question # 7 $$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$$

Consider

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}=\frac{Ax+B}{x^2+3}+\frac{C}{x+1}+\frac{D}{x-1}$$

$$\Rightarrow x^2+2x+2=(Ax+B)(x+1)(x-1)+C(x^2+3)(x-1)+D(x^2+3)(x+1)\ldots\ldots\ldots\ldots(i)$$

Put $x+1=0 \quad \Rightarrow x=-1$ in equation (i)

$$(-1)^2+2(-1)+2=0+C\big((-1)^2+3\big)\big((-1)-1\big)+0 \quad \Rightarrow 1-2+2=C(4)(-2)$$

$$\Rightarrow 1=-8C \qquad \Rightarrow \boxed{C=-\frac{1}{8}}$$

Now put $x-1=0 \quad \Rightarrow x=1$ in equation (i)

$$\Rightarrow (1)^2+2(1)+2=0+0+D\big((1)^2+3\big)\big((1)+1\big) \quad \Rightarrow 1+2+2=D(4)(2)$$

$$\Rightarrow 5=8D \qquad \Rightarrow \boxed{D=\frac{5}{8}}$$

Equation (i) can be written as

$$x^2+2x+2=(Ax+B)(x^2-1)+C(x^3-x^2+3x-3)+D(x^3+x^2+3x+3)$$
$$\Rightarrow x^2+2x+2=A(x^3-x)+B(x^2-1)+C(x^3-x^2+3x-3)+D(x^3+x^2+3x+3)$$

Comparing the coefficients of x^3, x^2, x and x^0.

$0=A+C+D$ ………………..…. (ii)

$1=B-C+D$ ……………….…. (iii)

$2=-A+3C+3D$ ……………. (iv)

$2 = -B - 3C + 3D$ .... (v)

Putting values of C and D in (ii)

$$0 = A - \frac{1}{8} + \frac{5}{8} \quad \Rightarrow 0 = A + \frac{1}{2} \quad \Rightarrow \boxed{A = -\frac{1}{2}}$$

Putting values of C and D in (iii)

$$1 = B - \left(-\frac{1}{8}\right) + \frac{5}{8} \quad \Rightarrow 1 = B + \frac{1}{8} + \frac{5}{8} \quad \Rightarrow 1 = B + \frac{3}{4}$$

$$\Rightarrow 1 - \frac{3}{4} = B \quad \Rightarrow \boxed{B = \frac{1}{4}}$$

Hence

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)} = \frac{-\frac{1}{2}x + \frac{1}{4}}{x^2+3} + \frac{-\frac{1}{8}}{x+1} + \frac{\frac{5}{8}}{x-1}$$

$$= \frac{\frac{-2x+1}{4}}{x^2+3} + \frac{-\frac{1}{8}}{x+1} + \frac{\frac{5}{8}}{x-1} = \frac{-2x+1}{4(x^2+3)} + \frac{-1}{8(x+1)} + \frac{5}{8(x-1)}$$

$$= \frac{1-2x}{4(x^2+3)} - \frac{1}{8(x+1)} + \frac{5}{8(x-1)} \quad \textit{Answer}$$

Question # 8 $\quad \dfrac{1}{(x-1)^2(x^2+2)}$

Resolving it into partial fraction.

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2}$$

$$\Rightarrow 1 = A(x-1)(x^2+2) + B(x^2+2) + (Cx+D)(x-1)^2 \text{.............}(i)$$

Put $x-1=0 \Rightarrow x=1$ in equation (i)

$$1 = 0 + B\left((1)^2+2\right) + 0 \quad \Rightarrow 1 = 3B \quad \Rightarrow \boxed{B = \frac{1}{3}}$$

Now equation (i) can be written as

$$1 = A(x^3 - x^2 + 2x - 2) + B(x^2+2) + (Cx+D)(x^2-2x+1)$$

$$\Rightarrow 1 = A(x^3 - x^2 + 2x - 2) + B(x^2+2) + C(x^3 - 2x^2 + x) + D(x^2 - 2x + 1)$$

Comparing the coefficients of x^3, x^2, x and x^0.

$0 = A + C$ .... (ii)

$0 = -A + B - 2C + D$ .... (iii)

$0 = 2A + C - 2D$ .... (iv)

$1 = -2A + 2B + D$ .... (v)

Multiplying eq. (iii) by 2 and adding in (iv)

$$\begin{aligned} 0 &= -2A + 2B - 4C + 2D \\ 0 &= 2A \qquad\quad + C - 2D \\ \hline 0 &= \qquad 2B - 3C \end{aligned}$$

Putting value of B in above

$$0 = 2\left(\frac{1}{3}\right) - 3C \quad \Rightarrow 0 = \frac{2}{3} - 3C \quad \Rightarrow 3C = \frac{2}{3} \quad \Rightarrow \boxed{C = \frac{2}{9}}$$

Putting value of C in eq. (ii)

$$0 = A + \frac{2}{9} \quad \Rightarrow \boxed{A = -\frac{2}{9}}$$

Putting value of A and B in eq. (v)

$$1=-2\left(-\frac{2}{9}\right)+2\left(\frac{1}{3}\right)+D \Rightarrow 1=\frac{4}{9}+\frac{2}{3}+D \Rightarrow 1-\frac{4}{9}-\frac{2}{3}=D \Rightarrow \boxed{D=-\frac{1}{9}}$$

Hence

$$\frac{1}{(x-1)^2(x^2+2)}=\frac{-2/9}{x-1}+\frac{1/3}{(x-1)^2}+\frac{(2/9)x+(-1/9)}{x^2+2}$$

$$=\frac{-2/9}{x-1}+\frac{1/3}{(x-1)^2}+\frac{\frac{2x-1}{9}}{x^2+2} = \frac{-2}{9(x-1)}+\frac{1}{3(x-1)^2}+\frac{2x-1}{9(x^2+2)}$$

Question # 9 $\dfrac{x^4}{1-x^4}$

$$=-1+\frac{1}{1-x^4}=-1+\frac{1}{(1-x^2)(1+x^2)}$$

$$=-1+\frac{1}{(1-x)(1+x)(1+x^2)}$$

$$\begin{array}{r|l} & -1 \\ 1-x^4 & x^4 \\ & \underset{-}{x^4}\ \underset{+}{-1} \\ \hline & 1 \end{array}$$

Now consider

$$\frac{1}{(1-x)(1+x)(1+x^2)}=\frac{A}{1-x}+\frac{B}{1+x}+\frac{Cx+D}{1+x^2}$$

$$\left[\begin{array}{c}\textit{Now find values of } A,B,C \textit{ and } D \textit{ yourself.}\\ \textit{You will get } A=1/4,\ B=1/4,\ C=0 \textit{ and } D=1/2\end{array}\right]$$

So

$$\frac{1}{(1-x)(1+x)(1+x^2)}=\frac{1/4}{1-x}+\frac{1/4}{1+x}+\frac{(0)x+1/2}{1+x^2}$$

$$=\frac{1}{4(1-x)}+\frac{1}{4(1+x)}+\frac{1}{2(1+x^2)}$$

Hence

$$\frac{x^4}{1-x^4}=-1+\frac{1}{4(1-x)}+\frac{1}{4(1+x)}+\frac{1}{2(1+x^2)} \quad \textit{Answer}$$

Question # 10 $\dfrac{x^2-2x+3}{x^4+x^2+1}$

$$=\frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)}$$

$$\because\ x^4+x^2+1=x^4+2x^2+1-x^2$$
$$=(x^2+1)^2-x^2$$
$$=(x^2+1+x)(x^2+1-x)$$
$$=(x^2+x+1)(x^2-x+1)$$

Now Consider

$$\frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2-x+1}$$

$$\Rightarrow x^2-2x+3=(Ax+B)(x^2-x+1)+(Cx+D)(x^2+x+1)\ldots\ldots\ldots\ldots(i)$$

$$\Rightarrow x^2-2x+3=A(x^3-x^2+x)+B(x^2-x+1)+C(x^3+x^2+x)+D(x^2+x+1)$$

Comparing the coefficients of x^3, x^2, x and x^0.

$0=A+C$ ………………………..… (ii)

$1=-A+B+C+D$……………. (iii)

$-2=A-B+C+D$ ……………..… (iv)

$$3 = B + D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .... \; (v)$$

Subtracting (ii) and (iv)

$$\begin{array}{rl} 0 = & A \qquad +C \\ -2 = & A - B \; + C + D \\ \hline 2 = & \quad B \qquad - D \end{array}$$

$$\Rightarrow 2 = B - D \quad \ldots\ldots\ldots\ldots\ldots \; (vi)$$

Adding (v) and (vi)

$$\begin{array}{l} 3 = B + D \\ 2 = B - D \\ \hline 5 = 2B \end{array}$$

$$\Rightarrow \boxed{B = \frac{5}{2}}$$

Putting value of B in (v)

$$3 = \frac{5}{2} + D \qquad \Rightarrow 3 - \frac{5}{2} = D \qquad \Rightarrow \boxed{D = \frac{1}{2}}$$

Putting value of B and D in (iii)

$$1 = -A + \frac{5}{2} + C + \frac{1}{2} \qquad \Rightarrow 1 - \frac{5}{2} - \frac{1}{2} = -A + C$$

$$\Rightarrow -2 = -A + C \quad \ldots\ldots\ldots\ldots\ldots\ldots \; (vii)$$

Adding (ii) and (vii)

$$\begin{array}{rl} 0 = & A + C \\ -2 = & -A + C \\ \hline -2 = & \quad 2C \end{array} \qquad \Rightarrow \boxed{C = -1}$$

Putting value of C in equation (ii)

$$0 = A - 1 \qquad \Rightarrow \boxed{A = 1}$$

Hence

$$\frac{x^2 - 2x + 3}{(x^2 + x + 1)(x^2 - x + 1)} = \frac{(1)x + \frac{5}{2}}{x^2 + x + 1} + \frac{(-1)x + \frac{1}{2}}{x^2 - x + 1}$$

$$= \frac{\frac{2x+5}{2}}{x^2 + x + 1} + \frac{\frac{-2x+1}{2}}{x^2 - x + 1}$$

$$= \frac{2x+5}{2(x^2 + x + 1)} + \frac{-2x+1}{2(x^2 - x + 1)}$$

$$= \frac{2x+5}{2(x^2 + x + 1)} + \frac{1-2x}{2(x^2 - x + 1)} \qquad \textit{Answer}$$

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