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Method of Undetermined Coefficients

This is an alternate method for finding particular integral i,e y_p, when $F(x)$ (i,e RHS of question) has terms like e^{ax}, $\sin ax$, $\cos ax$, x^m, and product of these terms. In this method we first find possible terms of 'y_p' by writing all the derivatives of the terms in $F(x)$. The Particular Integral 'y_p' is constructed according to the following table.

Table

If $F(x)$ is of the form	then take y_p as
i) a	Ax^K
ii) ax^n (n+ive integer)	$x^K(A_0x^n + A_1x^{n-1} + \cdots + A_{n-1}x + A_n)$
iii) ax^ne^{rx} (n+ive integer)	$x^K(A_0x^n + A_1x^{n-1} + \cdots + A_{n-1}x + A_n)e^{rx}$
iv) $a\cos ax$ or $a\sin ax$ (for both)	$x^K(A\cos ax + B\sin ax)$
v) $ax^ne^{rx}\cos ax$ or $ax^ne^{rx}\sin ax$ (for both)	$x^K[(A_0x^n + A_1x^{n-1} + \cdots + A_{n-1}x + A_n)e^{rx}\cos ax + (B_0x^n + B_1x^{n-1} + \cdots + B_{n-1}x + B_n)e^{rx}\sin ax]$

Note

i) In x^K, K is the smallest non-negative integer i,e 0,1,2,3... which will ensure that no term in y_p is already in C.F. i,e 'y_c'

ii) If $F(x)$ is sum of several terms write y_p for each term individually and then add up all of them.

iii) y_p is written on the basis of type of fn mentioned in $F(x)$

iv) From y_p fns are compared with y_c by giving values to $K = 0,1,2 \ldots$

v) If no term of y_c is in y_p, then put $K = 0$

vi) Start giving values to 'K' $= 0,1,2\ldots$ in y_p, and leave those values of 'K' which make y_p similar to terms in y_c

vii) The y_p and its derivatives i,e $y, y', y'' \ldots$ will be substituted in the eq. (1) and coeffts of like terms on the LHS & RHS will be equated to determine the U.C $A, B, \ldots$

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EXERCISE 10.3

Solve by the method of U.C. (Problem 1-9):

Q.1 $y'' - 4y' + 4y = 1\cdot e^{2x} \longrightarrow (1)$ $\quad F(x) = e^{2x}$

Sol. $\Rightarrow$ Auxilliary Eqn. is $D^2 - 4D + 4 = 0 \Rightarrow D = +2, +2$

$\therefore y_c = (c_1 + c_2 x)e^{2x} = c_1 e^{2x} + c_2 x' e^{2x}$

let us suppose that y_p of (1) is

$y_p = x^K A e^{2x}$ $\quad \therefore$ from (1) $F(x) = e^{2x}$ (see (iii) of table)

If we put $K=0$ in y_p we get Ae^{2x} similar to $c_1 e^{2x}$ in y_c, So leave $K=0$

Now put $K=1$ in y_p we get Axe^{2x} similar to $c_2 xe^{2x}$ in y_c, So leave $K=1$. So Put $K=2$

$\therefore y_p = Ax^2 e^{2x} \longrightarrow (2) \Rightarrow y'_p = 2Axe^{2x} + 2Ax^2 e^{2x}$

and $y''_p = 2Ae^{2x} + 4Axe^{2x} + 4Axe^{2x} + 4Ax^2 e^{2x}$

or $y''_p = 2Ae^{2x} + 8Axe^{2x} + 4Ax^2 e^{2x}$

put values of y_p, y'_p, y''_p in (1) and simplify

becomes $2Ae^{2x} + \cancel{8Axe^{2x}} + \cancel{4Ax^2e^{2x}} - \cancel{8Axe^{2x}} - \cancel{8Ax^2e^{2x}} + \cancel{4Ax^2e^{2x}} = e^{2x} \longrightarrow (3)$

Comparing coeff. of like powers we get

$2Ae^{2x} = 1e^{2x} \Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}$. put in (2)

from (2)

$\therefore y_p = \frac{1}{2}x^2 e^{2x}$

$\therefore$ Thus general sol. is $y = y_c + y_p$

or $y = (c_1 + c_2 x)e^{2x} + \frac{1}{2}x^2 e^{2x}$ ans.

Q.2 $y'' + 2y' + 5y = 6\sin 2x + 7\cos 2x \longrightarrow (1)$ $\quad F(x) = 6\sin 2x + 7\cos 2x$

Sol A. Eqn. is $D^2 + 2D + 5 = 0$

$\Rightarrow D = \frac{-2 \pm \sqrt{4-20}}{2\cdot 1} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$

$\Rightarrow y_c = e^{-x}[c_1 \cos 2x + c_2 \sin 2x] = c_1 e^{-x}\cos 2x + c_2 e^{-x}\sin 2x$

or $y_c = c_1 e^{-x}\cos 2x + c_2 e^{-x}\sin 2x$

Now let $y_p = x^K[A\sin 2x + B\cos 2x]$ (see (iv) of table)

Since no term of y_c is in y_p (due to absence of e^{-x} in y_p) thus put $K = 0$ in (2)

we have $y_p = A\sin 2x + B\cos 2x \longrightarrow (2)$

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$\Rightarrow y_p' = 2A\cos 2x - 2B\sin 2x$

and $y_p'' = -4A\sin 2x - 4B\cos 2x$

put values of y_p, y_p' and y_p'' in (1) we get

$-4A\sin 2x - 4B\cos 2x - 4A\cos 2x - 4B\sin 2x + 5A\sin 2x + 5B\cos 2x = 6\sin 2x + 7\cos 2x$

or $(A-4B)\sin 2x + (4A+B)\cos 2x = 6\sin 2x + 7\cos 2x$ — (3)

Comparing coeff of $\sin 2x$ & $\cos 2x$ we get

from (3) $A - 4B = 6$ & $4A + B = 7$ from (3)

$$\left.\begin{aligned} \text{or } 4A - 16B &= 24 \\ 4A + B &= 7 \end{aligned}\right\} \Rightarrow -17B = +17 \Rightarrow B = -1$$

Thus $A = 2$

So $y_p = 2\sin 2x - \cos 2x$

Thus G. Sol is $y = e^{-x}(c_1\cos 2x + c_2\sin 2x) + 2\sin 2x - \cos 2x$ Ans.

Q.3 $2y'' + 3y' + y - x^2 + 3\sin x = 0$ $F(x) = x^2 + 3\sin x$

Sol A. Eq is $2D^2 + 3D + 1 = 0$

$\Rightarrow D = \dfrac{-3 \pm \sqrt{9-8}}{2 \cdot 2} = \dfrac{-3 \pm 1}{4} = -\dfrac{1}{2}, -1$

$\Rightarrow y_c = c_1 e^{-x} + c_2 e^{-\frac{1}{2}x}$

For P.I of Eqn (1) we find P.I of

$$\left.\begin{aligned} 2y'' + 3y' + y &= x^2 \rightarrow (2) \\ \text{and } 2y'' + 3y' + y &= 3\sin x \rightarrow (3) \end{aligned}\right\}$$ and then (+) both results we get actual P.I of (1)

Now let y_p of (2) is

(from III of table) $y_p = x^k(Ax^2 + Bx + C) \rightarrow (4)$ (due to absence of e^{ax} in y_p)

since no term of y_c is in (4) k so put $k = 0$

$y_{p_1} = Ax^2 + Bx + C$

$\Rightarrow y_{p_1}' = 2Ax + B$ and $y_{p_1}'' = 2A$ put values of y_{p_1}, y_{p_1}'

and y_{p_1}'' in (2) we get

$2 \cdot 2A + 6Ax + 3B + Ax^2 + Bx + C = 1x^2 + 0x + 0$

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$\Rightarrow Ax^2 + (6A+B)x + (4A+3B+C) = x^2$ — (5)

Comparing coeff. we get

from (5) $A = 1$, $6A + B = 0 \Rightarrow 6 + B = 0 \Rightarrow B = -6$

and $4 \cdot 1 + 3(-6) + C = 0 \Rightarrow 4 - 18 + C = 0$ or $C = 14$.

So y_p of (2) is

$y_p = x^2 - 6x + 14$

Now let y_p of (3) is

(from given table) $y_p = x^k [C_1 \cos x + D \sin x] \rightarrow$ (6)

Since no term of y_c is in (6). Thus put $k=0$ in (6)

$\therefore y_p = C_1 \cos x + D \sin x \Rightarrow y_p' = -C_1 \sin x + D \cos x$

and $y_p'' = -C_1 \cos x - D \sin x$ put values in (3), we get

$-2C_1 \cos x - 2D \sin x - 3C_1 \sin x + 3D \cos x + C_1 \cos x + D \sin x = 3 \sin x$

or $(-3C_1 - D) \sin x + (-C_1 + 3D) \cos x = 3 \sin x + 0 \cos x$

Comparing coeff. we get

$-3C_1 - D = 3$ and $-C_1 + 3D = 0$

or $-9C_1 - 3D = 9$

and $-C_1 + 3D = 0$

$-10C_1 = 9 \Rightarrow C_1 = -9/10$. So $-D = 3 + 3C_1 = 3 - \frac{27}{10}$

or $-D = \frac{3}{10} \Rightarrow D = \frac{-3}{10}$

So y_p of Eq. (3) is

$y_p = -\frac{9}{10} \cos x - \frac{3}{10} \sin x$

Thus y_p of Eq. (1) is $= x^2 - 6x + 14 - \frac{9}{10} \cos x - \frac{3}{10} \sin x$

$\therefore$ G. Sol. of (1) is

$y = C_1 e^{-x} + C_2 e^{-\frac{1}{2}x} + x^2 - 6x + 14 - \frac{9}{10} \cos x - \frac{3}{10} \sin x$ Ans

x ———————— x

Q.4 $y'' + 2y' + y = e^x \cos x \rightarrow$ (1) $\quad F(x) = e^x \cos x$

Sol A. Eqn of (1) is

$D^2 + 2D + 1 = 0 \Rightarrow D = -1, -1$

$\Rightarrow y_c = (c_1 + c_2 x)e^{-x} = c_1 e^{-x} + c_2 x e^{-x}$

Now let y_p of (1) is

(from table) $y_p = x^k [A\cos x + B\sin x]e^x$

Since no term of y_c is in y_p so put $K=0$, in y_p

$\Rightarrow y_p = (A\cos x + B\sin x)e^x$

$\Rightarrow y_p' = (A\cos x + B\sin x)e^x + [-A\sin x + B\cos x]e^x$

$y_p' = (A+B)\cos x\, e^x + (B-A)\sin x \cdot e^x$

and $y_p'' = (A+B)\cos x \cdot e^x - (A+B)\sin x \cdot e^x + (B-A)\cos x \cdot e^x + (B-A)\sin x \cdot e^x$

or $y_p'' = 2B\cos x \cdot e^x - 2A\sin x \cdot e^x$

put values in (1) we get

$2B\cos x \cdot e^x - 2A\sin x \cdot e^x + 2A\cos x\, e^x + 2B\cos x\, e^x + 2B\sin x\, e^x - 2A\sin x\, e^x + A\cos x\, e^x + B\sin x\, e^x = e^x\cos x$

or $(3A+4B)\cos x \cdot e^x + (-4A+3B)\sin x \cdot e^x = e^x\cos x$

comparing coeff we get

$3A + 4B = 1$ and $-4A + 3B = 0$

$\Rightarrow 12A + 16B = 4$ & $-12A + 9B = 0$

$-12A + 9B = 0$

$25B = 4 \Rightarrow B = \frac{4}{25}$, $-4A = -3\left(\frac{4}{25}\right)$

$\Rightarrow A = \frac{3}{25}$

$\therefore y_p = \left(\frac{3}{25}\cos x + \frac{4}{25}\sin x\right)e^x$

$\therefore$ G. Sol. is

$y = (c_1 + c_2 x)e^{-x} + \left(\frac{3}{25}\cos x + \frac{4}{25}\sin x\right)e^x$ ans

Q.5 $y'' + y = 12\cos^2 x \rightarrow (1)$

Sol A. Eq of (1) is $D^2 + 1 = 0 \Rightarrow D = 0 \pm i$

$\Rightarrow y_c = e^{0x}[c_1\cos x + c_2\sin x] = c_1\cos x + c_2\sin x$

Now Since $\cos^2 x = \frac{1+\cos 2x}{2}$ So (1) will be

$\boxed{10\cdot 3-5}$ (31)

So ① can be written as

$$y'' + y = 12\left(\frac{1+\cos 2x}{2}\right)$$

or $y'' + y = 6 + 6\cos 2x \rightarrow ②$

Now for finding y_p of ②. We find saperately y_p's of

$y'' + y = 6 \rightarrow ③$ and $y'' + y = 6\cos 2x \rightarrow ④$

let y_p of ③ is $F(x) g(3) = 6$ Since no term of ③ is in y_c So

(from using table) $y_p = x^K A$

put $K=0$ in y_p we get $y_p = x^0 A = 1.A$

or $y_p = A \Rightarrow y_p' = 0$, $y_p'' = 0$ put in ③, we get

$0 + A = 6 \Rightarrow A = 6$

Thus $y_p = 6$ (i.e p.I. of ③)

Again let y_p of ④ is $y_p = x^K[B\cos 2x + D\sin 2x]$

Since no term of y_c is in y_p. So put $K=0$ in y_p, we get

$y_p = B\cos 2x + D\sin 2x \Rightarrow y_p' = -2B\sin 2x + 2D\cos 2x$

and $y_p'' = -4B\cos 2x - 4D\sin 2x$ put values of y_p'', y_p in ④, we get

place of y'' and y in ④, we get

$-4B\cos 2x - 4D\sin 2x + B\cos 2x + D\sin 2x = 6\cos 2x$

$\Rightarrow -3B\cos 2x - 3D\sin 2x = 6\cos 2x$ (by Comparing coeff.)

$\Rightarrow -3B = 6$ and $-3D = 0$

$\Rightarrow B = -2$ and $D = 0$

$\therefore y_p = -2\cos 2x + 0\sin 2x$

or $y_p = -2\cos 2x \Rightarrow y_p$ of ① $= 6 - 2\cos 2x$

Thus G. Sol. of eq. ① is

$y = c_1\cos x + c_2\sin x + 6 - 2\cos 2x$ Ans

Q.6 $y'' - 3y' + 2y = \cancel{x}\ 2x^2 + 2xe^x \rightarrow ①$

Sol A. Eqn of ① is $D^2 - 3D + 2 = 0$

$(D-1)(D-2) = 0 \Rightarrow D = 1, 2$

$\boxed{10\text{-}3\text{-}6}$ (32) (27)

$\Rightarrow y_c = c_1 e^{x} + c_2 e^{2x}$

For P.I. of (1), we find saperately P.I. of

$y'' - 3y' + 2y = 2x^2 \to (2)$ & $y'' - 3y' + 2y = 2xe^{x} \to (3)$

$F(x) = 2x^2$ $\quad$ $F(x) = 2xe^{x}$

Let yp of (2) is $= x^K[Ax^2 + Bx + C]$ P.I. of (2) is

Since no term of y_c is in yp of (2) $\therefore K=0$, So

$y_p = Ax^2 + Bx + C \Rightarrow y_p' = 2Ax + B$

and $y_p'' = 2A$ put values of y_p, y_p', y_p'' at the place of y, y', y'' respectively, we get

$2A - 6Ax - 3B + 2Ax^2 + 2Bx + 2C = 2x^2$

or $2Ax^2 + (-6A + 2B)x + (2A - 3B + 2C) = 2x^2$

Comparing Coeff of x^2, x & x^0 we get

$2A = 2 \Rightarrow \boxed{A=1}$, $-6A + 2B = 0$ or $-6 + 2B = 0 \Rightarrow \boxed{B=3}$

and $2A - 3B + 2C = 0$ or $2 - 3\times 3 + 2C = 0 \Rightarrow C = \frac{7}{2}$

So P.I. of (2) is $y_p = x^2 + 3x + \frac{7}{2}$

Now let P.I. of (3) is ... common factor e^x of y_p is al...

(general table) $y_p = x^K[Dx + E]e^x$

since if $K=0$ then Ee^x of y_p is similar to c_1e^x of y_c

So leave $K=0$ & put $K=1$

$\therefore y_p = x^1[Dx + E]e^x = [Dx^2 + Ex]e^x = Dx^2e^x + (E+2D)xe^x + Ee^x$

or $y_p' = \{Dx^2 + Ex\}e^x + \{2Dx + E\}e^x$

$y_p' = \{Dx^2 + (2D+E)x + E\}e^x$

$\Rightarrow y_p'' = \{Dx^2 + (2D+E)x + E\}e^x + \{2xD + 2D + E\}e^x$

or $y_p'' = \{Dx^2 + (4D+E)x + (2D+2E)\}e^x$

put values of y_p, y_p' & y_p'' in place of y, y', y'' in eqn (3), we get

$\{Dx^2 + (4D+E)x + (2D+2E)\}e^x - 3\{Dx^2 + (2D+E)x + E\}e^x + 2(Dx^2 + Ex)e^x = 2xe^x$

Comparing coeff of like powers

$3D - 3D = 0$ and $4D + E - 6D - 3E + 2E = 2 \Rightarrow -2D = 2 \Rightarrow D = -1$

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$2D+2E-3E=0$ or $-2-E=0 \Rightarrow E=-2$

$\therefore$ P.I. of (3) is

$y_p = (-x^2-2x)e^x$

Thus P.I. of (1) is

$y_p = x^2+3x+\frac{7}{2}-(x^2+2x)e^x$

So G. Sol is

$y = c_1e^x + c_2e^{2x} + x^2 + 3x + \frac{7}{2} - (x^2+2x)e^x$ is

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Q.7 $y''' + y' = 2x^2 + 4\sin x \rightarrow (1)$

Sol A. Eqn is $D^3 + D = 0$, $D(D^2+1)=0 \Rightarrow D = 0, \pm i$

$\Rightarrow y_c = c_1e^{0x} + c_2\cos x + c_3\sin x$

$y_c = c_1 + c_2\cos x + c_3\sin x$

Now for finding P.I. of (1). We find saperately P.I. of $y''' + y' = 2x^2 \rightarrow (2)$ And $y''' + y' = 4\sin x \rightarrow (3)$

Let P.I. of (2) is $y_p = x^k[Ax^2+Bx+C]$ $\therefore F(x)$ of (2) $=2x^2$

Since if we put $K=0$ then 'C' of y_p is similar to 'c_1' of y_c, being const so put $K=1$ in y_p.

$\therefore$ We get $y_p = x^1[Ax^2+Bx+C]$

or $y_p = Ax^3 + Bx^2 + Cx$

$\Rightarrow y_p' = 3x^2A + 2xB + C$, $y_p'' = 6xA + 2B$

and $y_p''' = 6A$ put values of y_p''' and y_p'

(2) We get $6A + 3x^2A + 2xB + C = 2x^2$

$\Rightarrow 3A = 2 \Rightarrow \boxed{A = \frac{2}{3}}$, $2B = 0 \Rightarrow \boxed{B=0}$

and $6A + C = 0 \Rightarrow C = -6\times\frac{2}{3} \Rightarrow \boxed{C=-4}$

$\Rightarrow y_p = \frac{2}{3}x^3 + 0x^2 + (-4)x$

or $\boxed{y_p = \frac{2}{3}x^3 - 4x}$

Now let P.I. of (3) is $y_p = x^k(D\cos x + E\sin x)$ $\therefore F(x)$ of (3) is $4\sin x$

If we put $K=0$ in y_p then $D\cos x$ is similar to $c_2\cos x$ of y_c & $E\sin x$ of y_p is similar to $c_3\sin x$ of y_c so leave $K=0$ & Put $K=1$

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$y_p = x^1[D\cos x + E\sin x] = Dx\cos x + Ex\sin x$

$\Rightarrow y_p' = -Dx\sin x + D\cos x + E\sin x + Ex\cos x$

$y_p' = (D+Ex)\cos x + (E-Dx)\sin x$

$y_p'' = E\cos x - (D+Ex)\sin x - D\sin x + (E-Dx)\cos x$

$y_p'' = -(2D+Ex)\sin x + (2E-Dx)\cos x$

$y''' = -(2D+Ex)\cos x - E\sin x + (Dx-2E)\sin x - D\cos x$

$= -(3D+Ex)\cos x + (Dx-3E)\sin x$

put values of y_p''' and y_p' in (3) we get

$-(3D+Ex)\cos x + (Dx-3E)\sin x + (D+Ex)\cos x + (E-Dx)\sin x = 4\sin x$

$-3D\cos x - \cancel{Ex\cos x} + \cancel{Dx\sin x} - \underline{3E\sin x} + D\cos x + \cancel{Ex\cos x}$
$+ \underline{E\sin x} - \cancel{Dx\sin x} = 4\sin x$

$-2D\cos x - 2E\sin x = 4\sin x$ comparing coeff of Sinx and Cosx, we get

$\Rightarrow -2D = 0 \Rightarrow D = 0$ | and $-2E = 4 \Rightarrow E = -2$

So $y_p = 0x\cos x - 2x\sin x = -2x\sin x$

Thus G. Sol. of (1) is

$y_1 = c_1 + c_2\cos x + c_3\sin x + \frac{2}{3}x^3 - 4x - 2x\sin x$

x ———————— x

Q8 $y''' + y'' + 3y' - 5y = 5\sin 2x + 10x^2 + 3x + 7$

Sol: A. Eqn is $D^3 + D^2 + 3D - 5 = 0$

1	1	1	3	-5
	0	1	2	5
	1	2	5	0

$\Rightarrow (D-1)(D^2+2D+5) = 0$

$\Rightarrow D = 1$ and $D = \frac{-2\pm\sqrt{4-20}}{2\cdot 1} = -1\pm 2i$

$\Rightarrow y_c = c_1e^x + e^{-x}(c_2\cos 2x + c_3\sin 2x)$

For P.I. of (1) we first find P.I. of

$y''' + y'' + 3y' - 5y = 5\sin 2x \longrightarrow (2)$

and $y''' + y'' + 3y' - 5y = 10x^2 + 3x + 7 \longrightarrow (3)$

Let y_p of (2) is $y_p = x^k[A\cos 2x + B\sin 2x]$

Since no term of y_c is in y_p so put $k = 0$ in y_p

we get $y_p = A\cos 2x + B\sin 2x$

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$\Rightarrow y'_p = -2A\sin 2x + 2B\cos 2x$

$y''_p = -4A\cos 2x - 4B\sin 2x$

$y'''_p = +8A\sin 2x - 8B\cos 2x$ put values of y'_p, y''_p, y'''_p and y_p in (2), we get

$8A\sin 2x - 8B\cos 2x - 4A\cos 2x - 4B\sin 2x - 6A\sin 2x + 6B\cos 2x$
$-5A\cos 2x - 5B\sin 2x = 5\sin 2x$

$2A\sin 2x - 9B\sin 2x - 9A\cos 2x - 2B\cos 2x = 5\sin 2x$

$\Rightarrow (2A - 9B)\sin 2x + (-9A - 2B)\cos 2x = 5\sin 2x$

$\Rightarrow 2A - 9B = 5$ and $-9A - 2B = 0$

or $18A - 81B = 45$ & $-18A - 4B = 0$

$$\begin{array}{r} 18A - 81B = 45 \\ -18A - 4B = 0 \\ \hline -85B = 45 \end{array} \Rightarrow B = -\frac{45}{85} \Rightarrow \boxed{B = -\frac{9}{17}}$$

$\Rightarrow -9A = +2\left(-\frac{9}{17}\right)$ or $9A = \frac{18}{17}$ or $\boxed{A = \frac{2}{17}}$

$\Rightarrow \boxed{y_p = \frac{2}{17}\cos 2x - \frac{9}{17}\sin 2x}$

Now let P.I. of (3) is

$y_p = (Cx^2 + Dx + E)x^K$

Since no term of y_c is in y_p, so put $K=0$ in y_p

$\Rightarrow y_p = Cx^2 + Dx + E \Rightarrow y'_p = 2Cx + D$

$y''_p = 2C$, $y'''_p = 0$, put values in (3), we get

$0 + 2C + 6Cx + 3D - 5Cx^2 - 5Dx - 5E = 10x^2 + 3x + 7$

or $-5Cx^2 + (6C - 5D)x + (3D - 5E + 2C) = 10x^2 + 3x + 7$

$\Rightarrow -5C = 10 \Rightarrow \boxed{C = -2}$, $6C - 5D = 3$ or $-12 - 5D = 3 \Rightarrow \boxed{D = }$

and $3D - 5E + 2C = 7$ or $-9 - 5E - 4 = 7$ or $\boxed{E = 4}$

$\therefore \boxed{y_p = -2x^2 - 3x + 4}$ (P.I. of (3))

So G. Sol. of (1) is

$y = c_1e^{x} + e^{-x}[c_2\cos 2x + c_3\sin 2x] + \frac{2}{17}\cos 2x - \frac{9}{17}\sin 2x - 2x^2 - 3x + 4$

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$y_p = x^k\left[E_1 \cos x + F_1 \sin x\right] e^{-x}$

If we put $K=0$ in y_p we get $[E_1\cos x + F_1 \sin x]e^{-x}$ similar to $(c_1\cos x + c_2 \sin x)$ of y_c. So leave $K=0$ & put $K=1$

we get $y_p = x\left(E_1 \cos x + F_1 \sin x\right) e^{-x}$

or $y_p = (E_1 x \cos x + F_1 x \sin x)e^{-x}$

Now as $C = F = E$ ($\because$ all's are constants)

Then $y_p = [(Cx)\cos x + (Cx)\sin x]e^{-x} \rightarrow (7)$

but we see that y_p of eqn (7) is already present y_p of eq (5)

Thus P.I of (1) is only the sum of Eq's (5) & (6)

i.e. $y_p = e^{-x}\left[(Ax^3 + Bx^2 + Cx)\cos x + (Ax^3 + Bx^2 + Cx)\sin x\right] + D_1 e^{-x}$ Ans

Q 10(ii) $y'' + 3y' + 2y = e^x(x^2+1)\sin 2x + 4e^x + 3e^{-x}\cos x \rightarrow (1)$

Sol: A. Eqn of (1) is $D^2 + 3D + 2 = 0 \Rightarrow D = -1, -2$

$\Rightarrow y_c = c_1 e^{-x} + c_2 e^{-2x}$

We first find P.I of each term of right saperately and then their sum for finding P.I of (1)

i.e. P.I of $y'' + 3y' + 2y = e^x(x^2+1)\sin 2x \rightarrow (2)$

$y'' + 3y' + 2y = 4e^x \rightarrow (3)$

and $y'' + 3y' + 2y = 3e^{-x}\cos x \rightarrow (4)$

let P.I of (2) is $y_p = e^x\left\{[Ax^2 + Bx + C]\sin 2x + (Ax^2 + Bx + C)\cos 2x\right\}x^k \rightarrow (5)$

$\rightarrow (6)$

P.I of (3) is $y_p = x^k D e^x \rightarrow (7)$

P.I of (4) is $y_p = x^k[E\cos x + F\sin x]e^{-x}$

Since no term of y_c is in (5), (6) and (7) So put $K=0$ in (5), (6) and (7), and then adding, We get

$y_p = e^x\left\{(Ax^2 + Bx + C)\sin 2x + (Ax^2 + Bx + C)\cos 2x\right\} + x^0 D e^x + x^0(E\cos x + F\sin x)e^{-x}$

or $y_p = e^x\left\{(Ax^2 + Bx + C)\sin 2x + (Ax^2 + Bx + C)\cos 2x\right\} + De^x + (E\cos x + F\sin x)e^{-x}$ Ans

Q.9 $y^{4} + 8y'' + 16y = \sin x \rightarrow (1)$

Sol. A. Eqn. is $D^4 + 8D^2 + 16 = 0$, $(D^2+4)^2 = 0 \Rightarrow (D^2+4)(D^2+4) = 0$

The roots are complex can repeated

$\Rightarrow D^2 = \pm 2i, \pm 2i \Rightarrow$

$\Rightarrow y_c = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x$

let P.I of (1) is $y_p = x^k[A\sin x + B\cos x]$ $\therefore k = 0$, in y_p

Since no term of y_c is in y_p

$\therefore y_p = x^0[A\sin x + B\cos x] \Rightarrow y_p = A\sin x + B\cos x$

$\Rightarrow y_p' = A\cos x - B\sin x$, $y_p'' = -A\sin x - B\cos x$

$y_p''' = -A\cos x + B\sin x$, $y_p'''' = A\sin x + B\cos x$

put values of y_p'''', y_p'', y_p at plance of y'''', y'' & y in (1)

$A\sin x + B\cos x - 8A\sin x - 8B\cos x + 16A\sin x + 16B\cos x = \sin x$

$9A\sin x + 9B\cos x = 1 \cdot \sin x \Rightarrow 9A = 1$, $9B = 0$

$\Rightarrow A = \frac{1}{9}$, $B = 0$ $\Rightarrow y_p = \frac{1}{9}\sin x + 0\cos x$ or $y_p = \frac{1}{9}\sin x$

Thus G. Sol. of (1) is $y = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x + \frac{1}{9}\sin x$ ans.

Q.10 Write the general term of P.I (without evaluating U.C)

(i) for $y'' + 2y' + 2y = 4e^{-x}x^2\sin x + 3e^{-x} + 2e^{-x}\cos x \rightarrow (1)$

Sol. A. Eqn. is $D^2 + 2D + 2 = 0 \Rightarrow D = \frac{-2 \pm \sqrt{4-8}}{2 \cdot 1} = -1 \pm i$

$\Rightarrow y_c = e^{-x}[c_1\cos x + c_2\sin x]$

> **NOTE:** In This Question To find undermined coefficients is not required. Here we have to find only general form of P.I

For find P.I of (1) we first find

P.I. of $y'' + 2y' + 2y = 4e^{-x}x^2\sin x \rightarrow (2)$

$y'' + 2y' + 2y = 3e^{-x} \rightarrow (3)$

$y'' + 2y' + 2y = 2e^{-x}\cos x \rightarrow (4)$

let P.I of (2) is

$y_p = x^k\{e^{-x}[Ax^2 + Bx + C]\sin x + e^{-x}[Dx^2 + Ex + F]\cos x\}$ (from table)

If we put K=0 in y_p we get $(e^{-x}C\cos x + e^{-x}C\sin x)$ which is similar to $e^{-x}(c_1\cos x + c_2\sin x)$ of y_c. Put $\boxed{K=1}$ in y_p

$y_p = x\{e^{-x}(Ax^2 + Bx + C)\sin x + e^{-x}(Dx^2 + Ex + F)\cos x\}$

or $y_p = e^{-x}(Ax^3 + Bx^2 + Cx)\sin x + e^{-x}(Dx^3 + Ex^2 + Fx)\cos x \rightarrow (5)$

let P.I of (3) is $y_p = x^k D_1 e^{-x}$ since no term of y_c is in y_p so K=0 $(\because x^0 = 1)$

$\Rightarrow y_p = D_1 e^{-x} \rightarrow (6)$

also let P.I of (4) is