Solution of Non Homogeneous Linear Diff Eq of order 'n'.

$$(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_{n-1} D + a_n) Y = F(x)$$

The solution consist of two parts

(i) Complementary Function (C.F) :- It is sol of Homogeneous L. Diff Eq i.e $(a_0 D^n + a_1 D^{n-1} + \cdots + a_n) Y = 0$. It is denoted by Y_c

(ii) Particular Integral (P.I) :- It is sol of $\dfrac{1}{a_0 D^n + a_1 D^{n-1} + \cdots a_n} F(x)$. It is denoted by Y_p

$\therefore$ General Sol $Y = Y_c + Y_p$

Properties of Differential Operator 'D' = $\frac{d}{dx}$

i) $D^n(ae^{bx}) = a(b)^n e^{bx}$ 'D is replaced by b'

ii) $F(D)(ae^{bx}) = aF(b)e^{bx}$ 'D is replaced by b'

iii) $D^n(e^{bx})U = e^{bx}(D+b)^n U$ b is added in D in D

iv) $F(D)e^{bx}U = e^{bx}F(D+b)U$ b is added in D

v) $\dfrac{1}{F(D)}ae^{bx} = \dfrac{ae^{bx}}{F(b)}$ 'D is replaced by b'

vi) $\dfrac{1}{F(D)}e^{bx}.U = e^{bx}\dfrac{1}{F(D+b)}.U$, b is added in D

vii) $\dfrac{1}{F(D^2)}\sin(ax) = \dfrac{\sin ax}{F(-a^2)}$ D^2 is replaced by $(-a^2)$

viii) $\dfrac{1}{F(D^2)}\cos(ax) = \dfrac{\cos ax}{F(-a^2)}$ only for D^2

ix) $\dfrac{1}{F(D)}\sin bx = \operatorname{Im}\dfrac{1}{F(D)}e^{ibx} = \operatorname{Im}\dfrac{e^{ibx}}{F(ib)}$

x) $\dfrac{1}{F(D)}\cos bx = \operatorname{Re}\dfrac{1}{F(D)}e^{ibx} = \operatorname{Re}\dfrac{e^{ibx}}{F(ib)}$

xi) $D^2\cos bx = (-b^2)\cos bx$ only for D^2

xii) $D^2\sin bx = (-b^2)\sin bx$

xiii) $\dfrac{1}{D^2}\cos bx = \dfrac{1}{-b^2}\cos bx$ only for D^2

xiv) $\dfrac{1}{D^2}\sin bx = \dfrac{1}{-b^2}\sin bx$

Imp Note

1) when, $\dfrac{1}{F(D)}(ae^{bx}) = \dfrac{ae^{bx}}{F(b)}$ if $F(b) = 0$

then $\dfrac{1}{F(D)}ae^{bx} = \dfrac{x}{F'(D)}(ae^{bx})$

$= \dfrac{x}{F'(b)}ae^{bx}$ if $F'(b) = 0$

then $\dfrac{x}{F'(D)}ae^{bx} = \dfrac{x^2}{F''(D)}ae^{bx}$

$= \dfrac{x^2}{F''(b)}ae^{bx}$ & so on. if $F''(b) = 0$

$$D(\sin x) = \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{1}{D}(\sin x) = \int \sin x\,dx = -\cos x$$

B. Series. If n is -ve or fraction.

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{\underline{2}}x^2 + \cdots$

We apply B. Series when $\frac{1}{F(x)}$ is other than Sin, Cos or e^{bx} see Q4, 5, 9,

Ex 10.2

Sol

① $(D^2+3D-4)y = 15e^x$

For C.F

$D^2+3D-4 = 0$ Characteristic Eq

$D = \frac{-3\pm\sqrt{9+4.1.4}}{2} = \frac{-3\pm\sqrt{25}}{2}$

$= \frac{-3\pm 5}{2} = 1, -4$

$y_c = c_1 e^x + c_2 e^{-4x}$

For P.I

$y_p = \frac{1}{D^2+3D-4} 15e^x$ (Failure Case) $\because \frac{15e^x}{1+3-4} = \infty$

$= \frac{x}{2D+3}(15e^x)$

$= \frac{x\,15e^x}{2(1)+3} = \frac{15xe^x}{5} = 3xe^x$

$y = y_c + y_p = c_1 e^x + c_2 e^{-4x} + 3xe^x$

Sol

② $(D^2-3D+2)y = e^x + e^{2x}$

For C.F $D^2-3D+2 = 0$

$D = \frac{3\pm\sqrt{9-4.1.2}}{2} = \frac{3\pm\sqrt{1}}{2}$

$= \frac{3\pm 1}{2} = 2, 1$

$y_c = c_1 e^{2x} + c_2 e^x$

For P.I

$y_p = \frac{1}{D^2-3D+2}(e^x + e^{2x})$

$= \frac{1}{D^2-3D+2} e^x + \frac{1}{D^2-3D+2} e^{2x}$

$\because$ Failure Case

$= \frac{x}{2D-3}(e^x) + \frac{x}{2D-3}(e^{2x})$

$= \frac{xe^x}{2(1)-3} + \frac{xe^{2x}}{2(2)-3}$

$y_p = -xe^x + xe^{2x}$

$y = y_c + y_p$

$= c_1 e^x + c_2 e^{2x} + xe^{2x} - xe^x.$

Sol

③ $(D^2-2D-3)y = 2e^x - 10\sin x$

For C.F $D^2-2D-3 = 0$ Characteristic Eq

$D = \frac{2\pm\sqrt{4+4.1.3}}{2} = \frac{2\pm\sqrt{16}}{2} = \frac{2\pm 4}{2} = 3, -1$

$y_c = c_1 e^{3x} + c_2 e^{-x}$

For P.I

$y_p = \frac{1}{D^2-2D-3}(2e^x - 10\sin x)$

$= \frac{1}{D^2-2D-3} 2e^x - \frac{1}{D^2-2D-3}(10\sin x)$

$= \frac{2e^x}{1^2-2(1)-3} - \frac{10\sin x}{-1^2-2D-3}$ D^2 is reple… by $-a^2$ … $D^2=-1^2$

$= \frac{2e^x}{-4} - \frac{10\sin x}{-4-2D}$

$= -\frac{e^x}{2} + \frac{5\sin x}{2+D}$

$= -\frac{e^x}{2} + \frac{5(2-D)\sin x}{(2-D)(2+D)}$ x & ÷ by (2- …

$= -\frac{e^x}{2} + \frac{5(2-D)\sin x}{4-D^2}$ D^2 by − … D^2 by − …

$= -\frac{e^x}{2} + \frac{5(2-D)\sin x}{4-(-1)}$

$= -\frac{e^x}{2} + \frac{\cancel{5}}{\cancel{5}}(2-D)\sin x$

$= -\frac{e^x}{2} + 2\sin x - D(\sin x)$

$y_p = -\frac{e^x}{2} + 2\sin x - \cos x$

$y = y_c + y_p$

$= c_1 e^{3x} + c_2 e^{-x} - \frac{e^x}{2} + 2\sin x - \cos x$

10.2.21

Q2 $y_p = \dfrac{e^x + e^{2x}}{D^2-3D+2}$

$= \dfrac{e^x}{(D-1)(D-2)} + \dfrac{1}{(D-1)(D-2)} e^{2x}$

$= \dfrac{e^x}{(D-1)(1-2)} + \dfrac{e^{2x}}{(2-1)(D-2)}$

$= \dfrac{-e^x}{D-1} + \dfrac{e^{2x}}{(D-2)}$

Failure case $= \dfrac{-xe^x}{1-0} + \dfrac{xe^{2x}}{1}$

Q1 $y_p = \dfrac{15e^x}{D^2+3D-4}$

$= \dfrac{15e^x}{(D+4)(D-1)}$

$= \dfrac{15e^x}{(1+4)(D-1)}$

$= \dfrac{15e^x}{5(D-1)}$

$= \dfrac{3e^x}{D-1}$

$= \dfrac{3xe^x}{1}$ Failure case

Q3 $\dfrac{2e^x - 10\sin x}{D^2-2D-3}$

$= \dfrac{2e^x}{(D-3)(D+1)} \; \dfrac{-10\sin x}{(D^2-2D-3)}$

$= \dfrac{2e^x}{(1-3)(1+1)} - \dfrac{10\sin x}{(-1^2)-2D-3}$

$= \dfrac{\cancel{2}e^x}{-\cancel{2}(2)} - \dfrac{10\sin x}{-4-2D}$

$= \dfrac{e^x}{-2} + \dfrac{5\sin x}{2+D}$

$+ \dfrac{5(2-D)\sin x}{(2^2-D^2)}$

$+ \dfrac{\cancel{5}(2\sin x - \cos x)}{2^2-(-1^2)}$

Q3 $\dfrac{1}{D^2-2D-3}\sin x = \operatorname{Im} \dfrac{1}{D^2-2D-3} e^{ix}$

D replaced by i

$= \operatorname{Im} \dfrac{1}{i^2-2i-3} e^{ix}$

$= \operatorname{Im} \dfrac{1}{-4-2i} e^{ix}$ $\because$ i in Denominator

$= \operatorname{Im} \dfrac{1}{-(4+2i)(4-2i)} (4-2i) e^{ix}$

$= \operatorname{Im} \dfrac{(4-2i)}{-(16+4)} [\cos x + i\sin x]$

$= \operatorname{Im} \dfrac{-2(2-i)}{20} (\cos x + i\sin x)$

$= \operatorname{Im} \dfrac{(2-i)}{10} (\cos x + i\sin x)$

$= \dfrac{-\operatorname{Im}}{10} \Big((2\cos x + 2i\sin x - i\cos x + 1\sin x\Big]$

$-\dfrac{1}{10}(2\sin x - \cos x)$

$\boxed{10.2\text{-}3}$

(5)

Q4 $(D^4-2D^3+D)y = x^4+3x+1$

$D(D^3-2D^2+1)y = 0$

Either $D=0$, OR $D^3-2D^2+0D+1=0$

$$\begin{array}{c|cccc} & 1 & -2 & 0 & 1 \\ 1 & \downarrow & 1 & -1 & -1 \\ \hline & 1 & -1 & -1 & \lfloor 0 \end{array}$$

$D^2-D-1=0$

$D = \frac{1\pm\sqrt{1+4\cdot1\cdot1}}{2} = \frac{1\pm\sqrt{5}}{2}$

$\therefore D = 0, 1, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

So $y_c = c_1e^{0x} + c_2e^{1\cdot x} + c_3e^{\left(\frac{1+\sqrt{5}}{2}\right)x} + c_4e^{\left(\frac{1-\sqrt{5}}{2}\right)x}$

For P.I $y_p = \frac{1}{D^4-2D^3+D}(x^4+3x+1)$

$= \frac{1}{D[D^3-2D^2+1]}(x^4+3x+1)$

$y_p = \frac{1}{D}\left[1+(D^3-2D^2)\right]^{-1}\{x^4+3x+1\}$

$\because (1+x)^n = 1+nx+\frac{n(n-1)}{\lfloor 2}x^2+\cdots$

$y_p = \frac{1}{D}\left[1+(-1)(D^3-2D^2)+\frac{(-1)(-1-1)}{\lfloor 2}(D^3-2D^2)^2+\cdots\right]\{x^4+3x+1\}$

$= \frac{1}{D}\left[1-D^3+2D^2+\frac{\cancel{2}}{\cancel{\lfloor 2}}D^6+4D^4-4D^5+\cdots\right]\{x^4+3x+1\}$

$= \frac{1}{D}\left[1+2D^2-D^3+4D^4-4D^5+D^6\right]\{x^4+3x+1\}$

$= \frac{1}{D}\left[x^4+3x+1+2D^2(x^4+3x+1)-D^3(x^4+3x+1)+4D^4(x^4+3x+1)-4D^5(x^4+3x+1)+D^6(x^4+3x+1)\right]$

$= \frac{1}{D}\left[x^4+3x+1+2(12x^2)-24x+4(24)-0+0\right]$

$= \frac{x^5}{5}+\frac{3x^2}{2}+x+24\left(\frac{x^3}{3}\right)-24\frac{x^2}{2}+96x$

$y_p = \frac{x^5}{5}+8x^3-21\frac{x^2}{2}+97x$

$y = y_c+y_p$

$= c_1+c_2e^x+c_3e^{\left(\frac{1+\sqrt{5}}{2}\right)x}+c_4e^{\left(\frac{1-\sqrt{5}}{2}\right)x}+\frac{x^5}{5}+8x^3-\frac{21x^2}{2}+97x$

(5) $(D^3-D^2+D-1)y = 4\sin x$

$D^3-D^2+D-1=0$

$$\begin{array}{c|cccc} & 1 & -1 & 1 & -1 \\ 1 & \downarrow & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & \lfloor 0 \end{array}$$

$D^2+1=0$

$D^2=-1$

$D=\pm i$

$\therefore$ Roots are $D = 1, \pm i$

So $y_c = c_1e^x + e^{0x}(c_2\cos x + c_3\sin x)$

$y_p = \frac{1}{D^3-D^2+D-1}(4\sin x)$

$= \frac{1}{D(D^2)-D^2+D-1}4\sin x = \frac{1}{D(-1^2)-(-1^2)+D-1}4\sin x$

$y_p = \frac{1}{-D+1+D-1} = \frac{1}{0} = \infty$ failure

$\therefore y_p = x\frac{1}{3D^2-2D+1}4\sin x$

$= x\frac{1}{3(-1^2)-2D+1}4\sin x = \frac{4x\sin x}{-2-2D}$

$= \frac{4x\sin x}{-2(1+D)} = \frac{-2x\sin x}{1+D}$

$= \frac{-2x(1-D)\sin x}{(1+D)(1-D)} = \frac{-2x(1-D)\sin x}{1-D^2}$

$= \frac{-2x(1-D)\sin x}{1-(-1^2)} = \frac{-2x\sin x+2x\cos x}{2}$

$y_p = \cancel{2}\left(\frac{-x\sin x+x\cos x}{\cancel{2}}\right)$

G.Sol $y = y_c+y_p$

$= c_1e^x+c_2\cos x+c_3\sin x+x\cos x$

10.2-24

⑥ $(D^3-2D^2-3D+10)y = 40\cos x$

$D^3-2D^2-3D+10=0$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -3 & 10 \\ & \downarrow & -2 & 8 & -10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$D^2-4D+5=0$

$D = \dfrac{4 \pm \sqrt{16-4.1.5}}{2} = \dfrac{4 \pm \sqrt{-4}}{2}$

$D = \dfrac{4 \pm i2}{2} = \dfrac{\cancel{2}(2 \pm i)}{\cancel{2}}$

$D = -2,\ 2 \pm i$

$y_c = c_1 e^{-2x} + e^{2x}(c_2 \cos x + c_3 \sin x)$

$y_p = \dfrac{40\cos x}{D^3-2D^2-3D+10}$

$= \dfrac{40\cos x}{D(D^2)-2D^2-3D+10}$

$= \dfrac{40\cos x}{D(-1^2)-2(-1^2)-3D+10}$

$= \dfrac{40\cos x}{-D+2-3D+10}$

$= \dfrac{40\cos x}{-4D+12}$

$= \dfrac{40\cos x}{-4(D-3)}$

$= -10\, \dfrac{(D+3)\cos x}{(D-3)(D+3)}$

$= \dfrac{-10\,(D+3)\cos x}{D^2-9}$

$= \dfrac{-10[D(\cos x)+3\cos x]}{(-1^2)-9}$

$y_p = \dfrac{\cancel{+10}}{\cancel{+10}}[-\sin x + 3\cos x]$

$y = y_c + y_p = c_1 e^{-2x} + e^{2x}(c_2 \cos x + c_3 \sin x) - \sin x + 3\cos x$

⑥ ⑦ $(D^2+4)y = 4\sin^2 x$

$D^2+4=0 \Rightarrow D^2=-4 \Rightarrow D = \pm 2i$

$y_c = e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$

$y_p = \dfrac{1}{D^2+4}(4\sin^2 x) = 2\,\dfrac{(2\sin^2 x)}{D^2+4}$

$= 2\,\dfrac{(1-\cos 2x)}{D^2+4} = \dfrac{2(1)}{D^2+4} - \dfrac{2\cos 2x}{D^2+4}$ — ①

$= \dfrac{2e^{0x}}{D^2+4} - \cancel{2}\dfrac{x(\cos 2x)}{\cancel{2}D+0}$ failure case $1 = e^{0x}$

$= \dfrac{2}{0^2+4} - \dfrac{x\cos 2x}{D}$

$y_p = \dfrac{1}{2} - x\,\dfrac{\sin 2x}{2}$

$y = y_c + y_p$

$y = (c_1 \cos 2x + c_2 \sin 2x) + \dfrac{1}{2} - \dfrac{x \sin 2x}{2}$

2nd Method from ①: $\dfrac{2}{D^2+4}(1) = 2(D^2+4)^{-1}(1) = 2.4^{-1}\left(1+\dfrac{D^2}{4}\right)^{-1}(1) = \dfrac{2}{4}\left[1+(-1)\left(\dfrac{D^2}{4}\right)+\ldots\right]$

$= \dfrac{1}{2}[1+0] = \dfrac{1}{2}$

⑧ $(D^3+D)y = 2x^2 + 3\sin x$

$D^3+D=0 \Rightarrow D(D^2+1)=0 \Rightarrow D=0,\ D^2=-1$

$D^2 = i^2$

$D = \pm i$

$\therefore D = 0, \pm i$

$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x$

$y_p = \dfrac{1}{(D^3+D)}[2x^2 + 3\sin x]$

$= \dfrac{1}{D^3+D}2x^2 + \dfrac{1}{D^3+D}3\sin x = \dfrac{1}{D(D^2+1)}2x^2 + \dfrac{1}{D(D^2+1)}3\sin$

$= \dfrac{1}{(D^2+1)}\dfrac{2x^3}{3} + \dfrac{1}{(D^2+1)}(-3\cos x) = (1+D^2)^{-1}\left(\dfrac{2}{3}x^3\right) + \dfrac{x(-3)}{2D}$ ∴ failure

$= \left[1+(-1)(D^2)+\dfrac{(-1)(-1-1)(D^2)^2}{\lfloor 2}+\ldots\right]\left(\dfrac{2}{3}x^3\right) + \left[x\left(-\dfrac{3}{2}\right)\dfrac{\cos x}{D}\right]$

$= (1-D^2+D^4+\ldots)\left(\dfrac{2}{3}x^3\right) + \left[-\left(\dfrac{3x}{2}\right)(\sin x)\right]$

$= \dfrac{2}{3}x^3 - \dfrac{2}{\cancel{3}}(\cancel{3})(2x) + 0 - \dfrac{3x}{2}\sin x$

$= \dfrac{2}{3}x^3 - 4x - \dfrac{3x}{2}\sin x$

$y = y_c + y_p = c_1 + c_2 \cos x + c_3 \sin x + \dfrac{2}{3}x^3 - 4x - \dfrac{3x}{2}\sin x$ Ans

10.2-5

(9) $(D^4+D^2)y = 3x^2+6\sin x-2\cos x$

$D^4+D^2=0$

$D^2(D^2+1)=0$

$D^2=0,\ D^2+1=0$

$D=0,0,\ D^2=-1$

$D=\pm i$

$\therefore D=0,0,\pm i$

$y_c=(c_1+c_2x)e^{0x}+e^{0x}(c_3\cos x+c_4\sin x)$

$y_c=c_1+c_2x+c_3\cos x+c_4\sin x$

$y_p=\dfrac{1}{D^4+D^2}(3x^2+6\sin x-2\cos x)$

$=\dfrac{1}{D^4+D^2}(3x^2)+\dfrac{1}{D^4+D^2}6\sin x-\dfrac{1}{D^4+D^2}2\cos x$

$=\dfrac{1}{D^2(D^2+1)}3x^2+\dfrac{1}{D^2(D^2+1)}6\sin x-\dfrac{1}{D^2(D^2+1)}2\cos x$

$=\dfrac{1}{(D^2+1)}3\left(\dfrac{x^4}{12}\right)+\dfrac{6}{(D^2+1)}(-\sin x)-\dfrac{2}{(D^2+1)}(-\cos x)$

failure case $=(1+D^2)^{-1}\left(\dfrac{x^4}{4}\right)-\dfrac{6x\sin x}{2D}+\dfrac{\cancel{2}x\cos x}{\cancel{2}D}$

$=(1+D^2)^{-1}\left(\dfrac{x^4}{4}\right)-\dfrac{(3x)}{D}\sin x+\dfrac{x}{D}(\cos x)$

B.S. $=\left[1+(-1)(D^2)+\dfrac{(-1)(-1-1)}{\underline{2}}(D^2)^2+\cdots\right]\dfrac{x^4}{4}-3x(-\cos x)+x\sin x$

$=\left(1-D^2+\dfrac{2}{2}D^4+\cdots\right)\dfrac{x^4}{4}+3x\cos x+x\sin x$

$=\left(\dfrac{x^4}{4}-\dfrac{12x^2}{4}+\dfrac{24}{4}+\cdots\right)+3x\cos x+x\sin x$

$y_p=\dfrac{x^4}{4}-3x^2+6+3x\cos x+x\sin x$

$y=y_c+y_p$

$y=c_1+c_2x+c_3\cos x+c_4\sin x+\dfrac{x^4}{4}-3x^2+6+3x\cos x+x\sin x,$

(11) $(D^3-D^2+3D+5)y=e^x\sin 2x$

$D^3-D^2+3D+5=0$

$$\begin{array}{r|rrrr} & 1 & -1 & 3 & 5\\ -1 & \downarrow & -1 & 2 & -5\\ \hline & 1 & -2 & 5 & 0\end{array}$$

$D^2-2D+5=0$

$D=\dfrac{2\pm\sqrt{4-4\cdot1\cdot5}}{2}$

$=\dfrac{2\pm\sqrt{-16}}{2}=\dfrac{2\pm4i}{2}$

$=\dfrac{2(1\pm2i)}{2}=1\pm2i$

$\therefore D=-1,\ 1\pm2i$

$y_c=c_1e^{-x}+e^{x}(c_2\cos 2x+c_3\sin 2x)$

$y_p=\dfrac{1}{D^3-D^2+3D+5}e^x\sin 2x$

$=e^x\dfrac{1}{(D+1)^3-(D+1)^2+3(D+1)+5}\sin 2x$

$=e^x\dfrac{1}{[D^3+1+3D^2+3D]-[D^2+1+2D]+3D+3+5}\sin 2x$ ——(1)

$=e^x\dfrac{1}{D^3+2D^2+4D+8}\sin 2x$

$=e^x\dfrac{1}{D(D^2)+2D^2+4D+8}\sin 2x$ D is replaced $-a^2$

$=e^x\dfrac{1}{D(-2^2)+2(-2^2)+4D+8}\sin 2x$

$=e^x\dfrac{1}{D(-4)-8+4D+8}$ failure case

$\therefore y_p=xe^x\dfrac{1}{3D^2+4D+4}\sin 2x$

$=xe^x\dfrac{1}{3(-2^2)+4D+4}\sin 2x$

$=xe^x\dfrac{1}{4D-8}\sin 2x$ $=\dfrac{xe^x}{4}\dfrac{1}{(D-2)}\sin\ldots$

$=\dfrac{xe^x}{4}\dfrac{(D+2)\sin 2x}{D^2-4}$ $=\dfrac{xe^x}{4}\dfrac{(D+2)}{(-2^2-\ldots}$

$=\dfrac{xe^x}{4(-8)}[2\cos 2x+2\sin 2x]$

$y_p=-\dfrac{xe^x\,\cancel{2}}{\cancel{32}\,16}(\cos 2x+\sin 2x)$

$y=y_c+y_p$

$=c_1e^{-x}+e^x(c_2\cos 2x+c_3\sin 2x)-\dfrac{xe^x}{16}(\cos 2x+\sin 2x)$

Ex 2.6

(13) $(D^2-7D+12)y = e^{2x}(x^3-5x^2)$

Characteristic Eq. $D^2-7D+12=0$

$$D = \frac{+7 \pm \sqrt{49-4\cdot1\cdot12}}{2} = \frac{7\pm\sqrt{1}}{2}$$

$$= \frac{7+1}{2},\ \frac{7-1}{2} = 4,3$$

$$y_c = c_1 e^{3x} + c_2 e^{4x}$$

$$y_p = \frac{e^{2x}(x^3-5x^2)}{D^2-7D+12}$$

$$= e^{2x}\frac{1}{(D+2)^2-7(D+2)+12}(x^3-5x^2)$$

$$= e^{2x}\frac{1}{D^2+4+4D-7D-14+12}(x^3-5x^2)$$

$$= e^{2x}\frac{1}{D^2-3D+2}(x^3-5x^2)$$

$$= \frac{e^{2x}}{2}\left[\frac{1}{1+\left(\frac{D^2-3D}{2}\right)}\right](x^3-5x^2)$$

$$= \frac{e^{2x}}{2}\left[1+\left(\frac{D^2-3D}{2}\right)\right]^{-1}(x^3-5x^2)$$

$$= \frac{e^{2x}}{2}\left[1+(-1)\left(\frac{D^2-3D}{2}\right)+\frac{(-1)(-1-1)}{\lfloor 2}\left(\frac{D^2-3D}{2}\right)^2+\frac{(-1)(-1-1)(-1-2)}{\lfloor 3}\left(\frac{D^2-3D}{2}\right)^3\right](x^3-5x^2)$$

$$= \frac{e^{2x}}{2}\left[1+\frac{3D-D^2}{2}+\frac{D^4+9D^2-6D^3}{4}+\frac{27D^3}{8}\right](x^3-5x^2)$$

D^4 & higher neglected

$$= \frac{e^{2x}}{2}\left[1+\frac{3}{2}D-\frac{D^2}{2}+\frac{9D^2}{4}-\frac{6D^3}{4}+\frac{27D^3}{8}\right]$$

$$= \frac{e^{2x}}{2}\left[1+\frac{3}{2}D+\frac{7D^2}{4}+\frac{15}{8}D^3\right](x^3-5x^2)$$

$$= \frac{e^{2x}}{2}\left[x^3-5x^2+\frac{3}{2}(3x^2-10x)+\frac{7}{4}(6x-10)+\frac{15}{8}(6)\right]$$

$$= \frac{e^{2x}}{2}\left[x^3-5x^2+\frac{9x^2}{2}-15x+\frac{21x}{2}-\frac{35}{2}+\frac{45}{4}\right]$$

$$y_p = \frac{e^{2x}}{2}\left[x^3-\frac{x^2}{2}-\frac{9x}{2}-\frac{25}{4}\right] = \frac{e^{2x}}{8}\left[4x^3-2x^2-18x-25\right]$$

$$y = c_1e^{3x}+c_2e^{4x}+\frac{e^{2x}}{8}(4x^3-2x^2-18x-25)$$

(10) $(D^2-2D+4)y = e^x\cos x$

$D^2-2D+4=0$ Characteristic Eq

$$D = \frac{2\pm\sqrt{4-4\cdot1\cdot4}}{2} = \frac{2\pm\sqrt{4-16}}{2}$$

$$= \frac{2\pm\sqrt{-12}}{2} = \frac{2\pm i2\sqrt{3}}{2} = \frac{\cancel{2}(1\pm i\sqrt{3})}{\cancel{2}}$$

$$y_c = e^{x}(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)$$

$$y_p = \frac{1}{D^2-2D+4}e^x\cos x$$

$$= e^x\frac{1}{(D+1)^2-2(D+1)+4}\cos x$$

$$= e^x\frac{1}{D^2+2D+1-2D-2+4}\cos x$$

$$= e^x\frac{1}{D^2+3}\cos x$$

$$y_p = \frac{e^x\cos x}{-1^2+3} = \frac{e^x\cos x}{2}$$

$$y = y_c+y_p$$

$$= e^x(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)+\frac{e^x\cos x}{2}$$

10.2-7

(14) $(D^4 + 8D^2 - 9)y = 9x^3 + 5\cos 2x$

Characteristic eq

$D^4 + 8D^2 - 9 = 0$

$$\begin{array}{r|rrrrr} & 1 & 0 & 8 & 0 & -9 \\ +1 & \downarrow & +1 & +1 & 9 & 9 \\ \hline & 1 & +1 & 9 & 9 & 0 \\ -1 & \downarrow & -1 & 0 & -9 & \\ \hline & 1 & 0 & 9 & 0 & \end{array}$$

$D^2 + 9 = 0 \qquad D^2 = -9$

$D = -1, 1, \pm 3i$

$y_c = c_1 e^{x} + c_2 e^{-x} + e^{0x}(c_3 \cos 3x + c_4 \sin 3x)$

$y_p = \dfrac{9x^3 + 5\cos 2x}{D^4 + 8D^2 - 9}$

$= \dfrac{9x^3}{D^4+8D^2-9} + \dfrac{5\cos 2x}{D^4+8D^2-9}$

$= \dfrac{1}{(-9)\left[1 - \left(\frac{D^4+8D^2}{9}\right)\right]}\,9x^3 + \dfrac{5\cos 2x}{(-2^2)^2 + 8(-2^2) - 9}$

$= -\left[1 - \dfrac{(D^4+8D^2)}{9}\right]^{-1} x^3 + \dfrac{5\cos 2x}{16 - 32 - 9}$

$= -\left[1 - (-1)\left(\dfrac{D^4+8D^2}{9}\right)\right] x^3 + \dfrac{5\cos 2x}{-25}$

$= -\left[x^3 + 0 + \dfrac{8}{9}(6x)\right] - \dfrac{\cos 2x}{5}$

$y_p = -x^3 - \dfrac{16}{3}x - \dfrac{\cos 2x}{5}$

$y = c_1 e^{x} + c_2 e^{-x} + e^{0x}(c_3 \cos 3x + c_4 \sin 3x)$

(12) $(D^3 - 7D - 6)y = e^{2x} + xe^{2x}$

$D^3 - 7D - 6 = 0$ Characteristic eq

$$\begin{array}{r|rrrr} & 1 & 0 & -7 & -6 \\ -1 & \downarrow & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$D^2 - D - 6 = 0$

$D = \dfrac{1 \pm \sqrt{1 + 4\cdot 1\cdot 6}}{2} = \dfrac{1 \pm 5}{2} = 3, -2$

$\therefore D = -1, -2, 3$

So $y_c = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x}$

$y_p = \dfrac{1}{D^3 - 7D - 6}(e^{2x} + xe^{2x})$

$= \dfrac{1}{D^3 - 7D - 6}(1+x)e^{2x}$

$= e^{2x}\dfrac{1}{(D+2)^3 - 7(D+2) - 6}(1+x)$

$= \dfrac{e^{2x}}{D^3 + 2^3 + 3\cdot 2\cdot D(D+2) - 7D - 14 - 6}(1+x)$

$= \dfrac{e^{2x}}{D^3 - 12 + 6D^2 + 12D - 7D}(1+x)$

$= \dfrac{e^{2x}}{D^3 + 6D^2 + 5D - 12}(1+x)$

$= \dfrac{e^{2x}}{-12}\left[\dfrac{1}{1 + \left(\frac{D^3+6D^2+5D}{-12}\right)}\right](1+x)$

$= \dfrac{e^{2x}}{-12}\left[1 - \left(\dfrac{D^3+6D^2+5D}{12}\right)\right]^{-1}(1+x)$ Apply B.Se

$= \dfrac{e^{2x}}{-12}\left[1 - (-1)\left(\dfrac{D^3+6D^2+5D}{12}\right)\right](1+x)$

$= \dfrac{e^{2x}}{-12}\left[(1+x) + \left(\dfrac{D^3+6D^2+5D}{12}\right)(1+x)\right]$

$= \dfrac{e^{2x}}{-12}\left[(1+x) + \dfrac{1}{12}\left[D^3(1+x) + 6D^2(1+x) \ldots\right.\right.$

$= \dfrac{e^{2x}}{-12}\left[(1+x) + \dfrac{1}{12}(0 + 0 + 5(0+1))\right]$

$= \dfrac{e^{2x}}{-12}\left(1 + x + \dfrac{5}{12}\right)$

$y_p = \dfrac{e^{2x}}{-12}\left[\dfrac{12+5}{12} + x\right] = \dfrac{e^{2x}}{-12}\left[\ldots\right.$

$y = y_c + y_p$

$y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x} + \dfrac{e^{2x}}{-12}\left[\dfrac{17}{12} + x\right]$ Ans

Ex 10.2 - 8

(10)

(15) $(D^4+3D^2-4)y = \sinh x - \cos^2 x$

$D^4+3D^2-4=0$

	1	0	3	0	−4
1	↓	1	1	4	4
	1	1	4	4	$\underline{0}$
−1	↓	−1	0	−4	
	1	0	4	$\underline{0}$	

$D^2+4=0$

$D^2=-4 \qquad D=\pm 2i$

$\therefore D = 1, -1, \pm 2i$

$y_c = c_1 e^{x} + c_2 e^{-x} + c_3 \cos 2x + c_4 \sin 2x.$

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$$y_p = \frac{\sinh x - \cos^2 x}{D^4+3D^2-4}$$

$$= \frac{\sinh x}{D^4+3D^2-4} - \frac{\cos^2 x}{D^4+3D^2-4}$$

$$= \frac{\frac{e^{x}-e^{-x}}{2}}{(D+1)(D-1)(D^2+4)} - \frac{\left(\frac{1+\cos 2x}{2}\right)}{D^4+3D^2-4}$$

$$= \left[\frac{1}{2(D+1)(D-1)(D^2+4)}e^{x} - \frac{1}{2(D+1)(D-1)(D^2+4)}e^{-x}\right] - \left[\frac{1}{2(D^4+3D^2-4)} + \frac{\cos 2x}{2(D^4+3D^2-4)}\right]$$

$$= \left(\frac{e^{x}}{2(1+1)(D-1)(1+4)} - \frac{e^{-x}}{2(D+1)(-1-1)\{(-1)^2+4\}}\right) - \left(\frac{1}{2(-4)\{1-(D^4+3D^2)\}} + \frac{\cos 2x}{2(D^2-1)(D^2+4)}\right)$$

$$= \left(\frac{e^{x}}{20(D-1)} + \frac{e^{-x}}{20(D+1)}\right) - \left(\frac{1}{-8}\{1-(D^4+3D^2)\}^{-1} + \frac{\cos 2x}{2(-2^2-1)(D^2+4)}\right)$$

$$= \left(\frac{xe^{x}}{20} + \frac{xe^{-x}}{20}\right) - \left(-\frac{1}{8} + \frac{x}{(-10)(2D)}\cos 2x\right)$$

$$= \left(\frac{x}{20}(e^{x}+e^{-x})\right) - \left(-\frac{1}{8} + \frac{x}{-20}\,\frac{\sin 2x}{2}\right)$$

$\because \frac{e^{x}+e^{-x}}{2} = \cos$

$\frac{e^{x}-e^{-x}}{2} =$

$$= \left(\frac{x\cosh x}{10} + \frac{1}{8} + \frac{x\sin 2x}{40}\right)$$

$y = y_c + y_p = c_1 e^{x} + c_2 e^{-x} + (c_3\cos 2x + c_4 \sin 2x) + \frac{x\cosh x}{10} + \frac{1}{8} + \frac{x\sin 2x}{40}$

Exc 0.2.9

(2d) $y''' + 3y'' + 7y' + 5y = 16e^{-x}\cos 2x$, $\quad y(0) = 2$, $y'(0) = -4$, $y''(0) = -2$

D Characteristic Eq is

$D^3 + 3D^2 + 7D + 5 = 0$

$$\begin{array}{c|cccc} -1 & 1 & 3 & 7 & 5 \\ & \downarrow & -1 & -2 & -5 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

$\therefore D = -1$

& $D^2 + 2D + 5 = 0$

$D = \dfrac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 5}}{2} = \dfrac{-2 \pm \sqrt{-16}}{2}$

$D = \dfrac{-2 \pm i4}{2} = -1 \pm 2i$

$y_c = c_1 e^{-x} + e^{-x}(c_2 \cos 2x + c_3 \sin 2x)$

$y_p = \dfrac{16e^{-x}\cos 2x}{D^3 + 3D^2 + 7D + 5}$

$= 16e^{-x} \dfrac{1}{(D-1)^3 + 3(D-1)^2 + 7(D-1) + 5} \cos 2x$

$= 16e^{-x} \dfrac{\cos 2x}{[D^3 - 1 - 3D(D-1)] + [3D^2 + 3 - 6D] + 7D - 7 + 5}$

$= \dfrac{16e^{-x}\cos 2x}{D^3 + 4D}$

$= \dfrac{16e^{-x}\cos 2x}{D(D^2+4)} = \dfrac{16e^{-x}\cos 2x}{D(-2^2+4)}$ Failure Case

$= \dfrac{16e^{-x} x \cos 2x}{3D^2 + 4}$ $\quad$ x by N by x & take Derivative of D

$= \dfrac{+16e^{-x} x \cos 2x}{3(-2^2) + 4}$

$= +\dfrac{16}{8} e^{-x} x \cos 2x$

$y_p = -2e^{-x} x \cos 2x$

$y = c_1 e^{-x} + e^{-x}(c_2 \cos 2x + c_3 \sin 2x) - 2e^{-x} x \cos 2x$

$\boxed{\text{Ex. 2}}$ 10

$$y = c_1 e^{-x} + e^{-x}(c_2 \cos 2x + c_3 \sin 2x) - 2e^{-x} x \cos 2x$$

$$y' = -c_1 e^{-x} - e^{-x}(c_2 \cos 2x + c_3 \sin 2x) + e^{-x}(c_2 \sin 2x\,(2) + 2c_3 \cos 2x) - 2(-1)e^{-x} x \cos 2x - 2e^{-x}\cos 2x - 2e^{-x} x(-2\sin 2x)$$

$$= -c_1 e^{-x} - e^{-x}(c_2 \cos 2x + c_3 \sin 2x) + e^{-x}(-2c_2 \sin 2x + 2c_3 \cos 2x) + 2e^{-x} x \cos 2x - 2e^{-x}\cos 2x + 4xe^{-x}\sin 2x$$

$$y'' = c_1 e^{-x} + e^{-x}(c_2 \cos 2x + c_3 \sin 2x) - e^{-x}(c_2(-2)\sin 2x + 2c_3 \cos 2x) - e^{-x}(-2c_2 \sin 2x + 2c_3 \cos 2x) + e^{-x}(-4c_2 \cos 2x - 4c_3 \sin 2x) + 2e^{-x}\cos 2x - 2e^{-x}(-2\sin 2x)$$
$$+ 2e^{-x}x(-2\sin 2x) + 2(-1)e^{-x} x \cos 2x + 2e^{-x}\cos 2x + 4xe^{-x}(\cos 2x)2.$$
$$+ 4e^{-x}\sin 2x - 4xe^{-x}\sin 2x$$

Apply $y(0) = 2 \Rightarrow 2 = c_1 + c_2$ —— ①

$y'(0) = -4 \Rightarrow -4 = -c_1 - c_2 + 2c_3 + 0 - 2$

$-2 = -(c_1 + c_2 - 2c_3)$ —— ⑪

$y''(0) = -2 \Rightarrow -2 = c_1 + c_2 - 2c_3 - 2c_3 - 4c_2 + 2 + 0 + 2 + 0 + 0 - 0 + 0$

$-2 = c_1 + c_2 - 4c_3 - 4c_2 + 4$

$-6 = c_1 - 3c_2 - 4c_3$ —— ⑪⑪

Put ① in ⑪ $\quad 2 = 2 - 2c_3 \Rightarrow 0 = -2c_3 \Rightarrow \boxed{c_3 = 0}$

Put $c_3 = 0$ in ⑪ $\quad 2 = c_1 + c_2 - 0$

Put $c_3 = 0$ in ⑪⑪ $\quad -6 = c_1 - 3c_2 - 0$

$8 = 4c_2 \Rightarrow c_2 = \frac{8}{4} = \boxed{2}$

Put $c_2 = 2$ in ① $\quad 2 = c_1 + 2 \Rightarrow \boxed{c_1 = 0}$

The required sol is $y = 0 + e^{-x}(2\cos 2x + 0) - 2e^{-x} x \cos 2x$

$$y = 2e^{-x}\cos 2x - 2xe^{-x}\cos 2x$$

10.2-11

(16) $(D^2-8D+15)Y = 9xe^{2x}$ $Y(0)=5$, $Y'(0)=10$ (11)

$D^2-8D+15 = 0$:. Characteristic Eq

$D = \frac{8\pm\sqrt{64-4.1.15}}{2} = \frac{8\pm\sqrt{64-60}}{2}$

$= \frac{8\pm\sqrt{4}}{2} = \frac{8\pm 2}{2} = 5, 3$

$Y_c = c_1e^{3x} + c_2e^{5x}$

$Y_p = \frac{1}{D^2-8D+15}9xe^{2x}$

$= e^{2x}\frac{1}{(D+2)^2-8(D+2)+15}9x$

$= e^{2x}\frac{1}{D^2+4+4D-8D-16+15}9x$

$= e^{2x}\frac{1}{D^2-4D+3}9x$

$= \frac{e^{2x}}{\cancel{3}}\frac{1}{\left(\frac{D^2-4D}{3}+1\right)}\cancel{9}^{3}x$

$= e^{2x}\left[1+\left(\frac{D^2-4D}{3}\right)\right]^{-1}3x$ Apply B.Series

$= e^{2x}\left[1+(-1)\left(\frac{D^2-4D}{3}\right)+\cdots\right]3x$

$= e^{2x}\left[3x-\frac{1}{3}(D^2-4D)3x\right]$

$= e^{2x}\left[3x-(0-4)\right]$

$Y_p = 3xe^{2x}+4e^{2x}$

$Y = Y_c+Y_p = c_1e^{3x}+c_2e^{5x}+3xe^{2x}+4e^{2x}$ — (I)

Diff (I) $Y' = 3c_1e^{3x}+5c_2e^{5x}+3e^{2x}+6xe^{2x}+8e^{2x}$ — (II)

$Y(0)=5$ ⇒ Put $x=0$, $Y=5$ in (I): $5 = c_1e^0+c_2+0+4e^0$

$5 = c_1+c_2+4$

$1 = c_1+c_2$ — (III)

$Y'(0)=10$ ⇒ Put $x=0$, $Y'=10$ in (II): $10 = 3c_1+5c_2+11$

$-1 = 3c_1+5c_2$ — (IV)

Solve (III) & (IV)

$3 = \cancel{3c_1}+3c_2$
$-1 = \cancel{3c_1}+5c_2$
$4 = -2c_2$

$5 = 5c_1+\cancel{5c_2}$
$-1 = 3c_1+\cancel{5c_2}$
$6 = 2c_1$

Hence from (I) $c_1=3$, $c_2=-2$

$Y = 3e^{3x} \ldots e^{5x} \ldots 3xe^{2x}+4e^{2x}$

(17) $y''-4y'+13y = 8\sin3x$ $y(0)=1$, $y'(0)=2$

$(D^2-4D+13)Y = 8\sin3x$

$D^2-4D+13 = 0$ Characteristic Eq

$D = \frac{4\pm\sqrt{16-4.1.13}}{2} = \frac{4\pm\sqrt{16-52}}{2} = \frac{4\pm\sqrt{-36}}{2}$

$D = \frac{4\pm6i}{2} = \frac{\cancel{2}(2\pm3i)}{\cancel{2}} = 2\pm3i$

$Y_c = e^{2x}(c_1\cos3x+c_2\sin3x)$

$Y_p = \frac{1}{D^2-4D+13}8\sin3x$ D^2 is replaced by $-a^2$ i.e -3^2

$= 8\frac{(1)}{-3^2-4D+13}\sin3x$

$= 8\frac{1}{4-4D}$

$= \frac{8}{4}\frac{1}{1-D}\sin3x$ x & ÷ by 1+D

$= 2\frac{(1+D)}{1-D^2}\sin3x$

$= 2\frac{(1+D)\sin3x}{1-(-3^2)}$

$= \frac{2}{10}\left[\sin3x+\cos3x\,(3)\right]$

$Y_p = \frac{1}{5}\left[\sin3x+3\cos3x\right]$

$Y = Y_c+Y_p$

$Y = e^{2x}(c_1\cos3x+c_2\sin3x)+\frac{1}{5}[\sin3x+3\cos\ldots$ — (I)

$Y' = 2e^{2x}(c_1\cos3x+c_2\sin3x)+e^{2x}(-3c_1\sin3x+3c_2\ldots$

$+\frac{1}{5}[\cos3x\,(3)+9\,(-\sin3x)]$ — (II)

$Y(0)=1$, Put $x=0$, $Y=1$ in (I) ⇒ $1 = c_1+\frac{3}{5}$ ⇒ $c_1 = 1-\frac{3}{5}$, $c_1 = \frac{5-3}{5} =$

$Y'(0)=2$, Put $x=0$, $Y'=2$ in (II) ⇒ $2 = 2c_1+3c_2+\frac{3}{5}$

$2 = 2\left(\frac{2}{5}\right)+3c_2+\frac{3}{5}$

$2-\frac{4}{5}-\frac{3}{5} = 3c_2$

$\frac{10-7}{5} = 3c_2$ ⇒ $c_2 = \frac{3}{5}\cdot\frac{1}{3}$

So $Y = e^{2x}\left[\frac{2}{5}\cos3x+\frac{1}{5}\sin3x\right]+\frac{1}{5}[\sin3x+\ldots$

$= \frac{1}{5}\left[e^{2x}(\cos3x+\sin3x)+\sin3x+3\cos\ldots\right.$

10.2.12

(18) $y'' - 4y = 2 - 8x \quad y(0) = 0, \quad y'(0) = 5$

$(D^2 - 4)y = 2 - 8x$

$D^2 - 4 = 0$ Characteristic Eq.

$D^2 = 4$

$D = \pm 2$

$\therefore y_c = c_1 e^{2x} + c_2 e^{-2x}$

$y_p = \frac{1}{D^2 - 4}(2 - 8x)$

$= \frac{1}{-4\left(1 + \frac{D^2}{-4}\right)}(2 - 8x)$

$= \frac{1}{-4}\left(1 - \frac{D^2}{4}\right)^{-1}(2 - 8x)$ Apply B. Series

$= -\frac{1}{4}\left[1 - (-1)\frac{D^2}{4} + \cdots\right](2 - 8x)$

$= -\frac{1}{4}\left[1 + \frac{D^2}{4}\right](2 - 8x)$

$= -\frac{1}{4}\left[2 - 8x + \frac{D^2(2 - 8x)}{4}\right]$

$= -\frac{1}{4}(2 - 8x) + \frac{1}{4}(0 - 0)$

$= -\frac{2}{4} + \frac{8x}{4}$

$y_p = -\frac{1}{2} + 2x$

$y = y_c + y_p$

$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{2} + 2x$ ——(I)

$y' = 2c_1 e^{2x} + (-2)c_2 e^{-2x} - 0 + 2$ ——(II)

$y(0) = 0 \Rightarrow 0 = c_1 + c_2 - \frac{1}{2}$

Put $x = 0, y = 0$ in (I)

$\frac{1}{2} = c_1 + c_2$ ——(III)

$y'(0) = 5 \Rightarrow 5 = 2c_1 - 2c_2 + 2$

Put $x = 0, y' = 5$ in (II)

$3 = 2c_1 - 2c_2$ ——(IV)

Solving (III) & (IV)

$\frac{1}{\cancel{2}}(\cancel{2}) = 2c_1 + \cancel{2c_2}$

$3 = 2c_1 - \cancel{2c_2}$

$4 = 4c_1$

$\boxed{1 = c_1}$

$\frac{1}{\cancel{2}}(\cancel{2}) = \cancel{2c_1} + 2c_2$

$3 = \cancel{2c_1} - 2c_2$
$\quad - \quad\; +$

$-2 = 4c_2$

$-\frac{2}{4} = c_2$

$\boxed{-\frac{1}{2} = c_2}$

from (I)

$y = e^{2x} - \frac{1}{2}e^{-2x} - \frac{1}{2} + 2x$ Ans.

(19) $y'' + y = x\sin x \quad y(0) = 1, \quad y'(0) = 2$

$(D^2 + 1)y = x\sin x$

$D^2 + 1 = 0$ Characteristic Eq

$D^2 = -1$

$D = \pm 2$

$\therefore y_c = c_1\cos x + c_2\sin x$

$y_p = \frac{1}{D^2 + 1}x\sin x$

$= \frac{1}{-1^2 + 1}x\sin x$ failure case

$\therefore y_p = \frac{x}{2D}(x\sin x)$

$= \frac{x}{2} \cdot \frac{1}{D}(\underset{I}{x}\,\underset{II}{\sin x})$ I.B.P

$= \frac{x}{2}\left[x \cdot (-\cos x) - \int 1 \cdot (-\cos x)\,dx\right]$

$= \frac{x}{2}\left[-x\cos x + \int \cos x\,dx\right]$

$= \frac{x}{2}\left[-x\cos x + \sin x\right]$

$y_p = -\frac{x^2\cos x}{2} + \frac{x\sin x}{2}$ ——(I)

$y = y_c + y_p$

$y = c_1\cos x + c_2\sin x - \frac{x^2\cos x}{2} + \frac{x\sin x}{2}$

$y' = -c_1\sin x + c_2\cos x - \frac{1}{2}\left[2x\cos x - x^2\sin x\right] + \frac{1}{2}\left[1 \cdot \sin x + x\cos x\right]$ ——(II)

$y(0) = 1 \Rightarrow \boxed{1 = c_1}$

$x = 0, y = 1$ in (I)

$y'(0) = 2 \Rightarrow \boxed{2 = c_2}$

$x = 0, y' = 2$ in (II)

$\therefore y = 1\cos x + 2\sin - \frac{x^2\cos x}{2} + \frac{x\sin x}{2}$

Ans.