
Mock Test-2
For Lecturer (Mathematics)

Name

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1. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ equals:
A. $\frac{\ln x}{1+\ln x}$ B. $\frac{\ln x}{(1+\ln x)^2}$ C. $\frac{1+\ln x}{\ln x}$ D. $\frac{(1+\ln x)^2}{\ln x}$
2. $\lim_{x \rightarrow 0} (\cot x)^{\sin x}$ equals:
A. 0 B. -1 C. 1 D. e
3. If a function f satisfies all axioms of Mean Value Theorem on $[a, b]$ and $|f'(x)| \leq M$ for all $x \in [a, b]$, then:
A. $|f(b) - f(a)| \leq M(a - b)$ B. $|f(b) - f(a)| \geq M(a - b)$ C. $|f(b) - f(a)| \leq M(b - a)$
D. $|f(a) - f(b)| \geq M(b - a)$
4. $\int \frac{dx}{x\sqrt{a^2+x^2}}$ equals:
A. $\frac{1}{a} \sinh^{-1}(x)$ B. $\frac{1}{a} \sinh^{-1}\left(\frac{-a}{x}\right)$ C. $-\frac{1}{a} \sinh^{-1}(x)$ D. $-\frac{1}{a} \sinh^{-1}\left(\frac{a}{x}\right)$
5. The maximum error formula in Simpson's rule is $Error \leq \frac{M(b-a)^5}{180n^4}$, where M equals:
A. $|f'(x)|$ B. $|f''(x)|$ C. $|f'''(x)|$ D. $|f^{(4)}(x)|$
6. For $a > 0$, $r = a \sin \theta$ represents a circle of radius:
A. $\frac{a}{2}$ B. a C. $2a$ D. a^2
7. Distance of the point $(3, -1, 2)$ from the plane $2x + y - z = 0$ is:
A. $\frac{3}{2}$ B. $\frac{4}{\sqrt{6}}$ C. $\frac{6}{2}$ D. $\frac{\sqrt{6}}{4}$
8. The acute angle between the planes $2x + y - z = 5$ and $x - y - 2z = -5$ is:
A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{6}$
9. The function $f(x, y, z) = \frac{\sqrt{x} + \sqrt{y} + \sqrt{z}}{x+y}$ is homogeneous of degree:
A. $\frac{1}{2}$ B. 2 C. $-\frac{1}{2}$ D. none of these
10. If D is the region in the first quadrant between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, ($0 < a < b$), then $\int \int_D \frac{dx dy}{x^2 + y^2}$ equals:
A. $\frac{\pi}{\ln b}$ B. $\frac{\pi}{2} \ln(b)$ C. $\frac{\pi}{2} \ln(a)$ D. $\frac{\pi}{2} \ln\left(\frac{b}{a}\right)$
11. The order of the differential equation $\frac{d^2 y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^3 - 4y + e^x = 0$ is:
A. 0 B. 1 C. 2 D. 3
12. Which of the following is a non-linear differential equation?
A. $(1-x)y' + 2y = e^x$ B. $y'' + \sin y = 0$ C. $y''' + y = 0$ D. $y'' + (\sin x)y = 5$

13. The differential equation $N(x, y)dx + M(x, y)dy = 0$ is exact if:
 A. $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ B. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ C. Both (A) and (B) D. None of these
14. The Wronskian $W(e^{3x}, e^{-3x})$ equals:
 A. 0 B. 6 C. -6 D. 1/6
15. If L^{-1} is the inverse Laplace transform, then $L^{-1}(\frac{t}{t^2+k^2})$ equals:
 A. $\sin ks$ B. $\cos ks$ C. e^s D. $\log(s)$
16. For a scalar point function ψ , $\nabla \times (\nabla \psi)$ equals:
 A. 0 B. ψ C. ψ^2 D. $\psi/2$
17. For a vector point function A , $\nabla(\nabla \cdot A) - \nabla^2 A$ equals:
 A. $\nabla(\nabla \cdot A)$ B. $\nabla \times (\nabla \times A)$ C. $\nabla(A \cdot \nabla)$ D. $\nabla \cdot (\nabla \times A)$
18. The volume of a tetrahedron with sides $\vec{a}, \vec{b}, \vec{c}$ is:
 A. $\frac{1}{2}|\vec{a} \times \vec{b} \cdot \vec{c}|$ B. $\frac{1}{6}|\vec{a} \times \vec{b} \cdot \vec{c}|$ C. $\frac{1}{2}|\vec{a} \times \vec{c} \cdot \vec{b}|$ D. $\frac{1}{2}|\vec{c} \times \vec{b} \cdot \vec{a}|$
19. For vectors \vec{a}, \vec{b} and \vec{c} , which of the following is true?
 A. $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ B. $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ implies $\vec{b} = \vec{c}$ C. $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ implies $\vec{b} = \vec{c}$
 D. None of these
20. If $\frac{d\vec{r}(t)}{dt} = 0$ on an interval $[a, b]$, then $\vec{r}(t)$ is ... on $[a, b]$.
 A. constant B. zero C. 1 D. smooth
21. If \vec{t}, \vec{n} and \vec{b} are tangent, normal and bi-normal vectors respectively, then for what value of X ,

$$\begin{bmatrix} \vec{t}' \\ \vec{n}' \\ \vec{b}' \end{bmatrix} = X \begin{bmatrix} \vec{t} \\ \vec{n} \\ \vec{b} \end{bmatrix} ?$$
 A. $X = \begin{bmatrix} 0 & \kappa & 0 \\ \kappa & 0 & -\tau \\ 0 & \tau & 0 \end{bmatrix}$ B. $X = \begin{bmatrix} 0 & -\kappa & 0 \\ \kappa & 0 & \tau \\ 0 & \tau & 0 \end{bmatrix}$ C. $X = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & -\tau \\ 0 & -\tau & 0 \end{bmatrix}$ D. $X = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix}$
22. The osculating plane to a curve is parallel to:
 A. \vec{t} and \vec{n} B. \vec{n} and \vec{b} C. \vec{b} only D. \vec{n} only
23. If for a curve, $\frac{\kappa}{\tau}$ is a constant, then curve is a:
 A. plane curve B. space curve C. helix D. straight line
24. The rotation index of a simple closed curve is:
 A. 0 B. 1 C. -1 D. ± 1
25. For any two sets A and B , $n(A - B)$ equals:
 A. $n(A) - n(A \cap B)$ B. $n(A) - n(A \cup B)$ C. $n(B) - n(A \cap B)$ D. $n(B) - n(A \cup B)$

26. Let R be a relation on a set A with n elements, then $R^1 \cup R^2 \cup \dots \cup R^N$ is ... closure of R .
A. reflexive B. symmetric C. transitive D. skew-symmetric
27. A proper subset of a countable set is:
A. finite B. co-finite C. infinite D. countable
28. If for any two functions f and g , $g \circ f$ is onto, then:
A. g is one-one B. g is onto C. f is one-one D. f in onto
29. The set $\{x \in \mathbb{Q} : x > 0 \text{ and } 2 < x^2 < 3\}$ has:
A. no supremum B. no infimum C. Both A and B D. None of these
30. The smallest positive divisor $d > 1$ of an integer n is:
A. prime B. composite C. coprime D. even
31. When 5^{48} is divided by 12, the remainder is:
A. 0 B. 1 C. 5 D. 10
32. When $1! + 2! + 3! + \dots + 899!$ is divided by 3, then the remainder is:
A. 0 B. 1 C. 2 D. None of these
33. If for any integers a, b, c, m and $k > 1$, $a^k \equiv b^k \pmod{m}$, then which of the following is not true in general?
A. $a^k + c \equiv b^k + c \pmod{m}$ B. $a^k c \equiv b^k c \pmod{m}$ C. $a \equiv b \pmod{m}$ D. $b^k \equiv a^k \pmod{m}$
34. Any two Sylow p -subgroups of a group G are:
A. commutative B. finite C. conjugate D. normal
35. For $n \geq 3$, A_n is generated by:
A. 2-cycles B. 3-cycles C. 4-cycles D. None of these
36. The order of an element $\frac{p}{q} + \mathbb{Z}$ in the group $\frac{\mathbb{Q}}{\mathbb{Z}}$ is:
A. p B. q C. infinite D. 1
37. How many subgroups does the group $\mathbb{Z}_3 + \mathbb{Z}_{16}$ have?
A. 6 B. 10 C. 12 D. 24
38. The rank of an $m \times n$ matrix is:
A. m B. n C. $\min(m, n)$ D. $\leq \min(m, n)$
39. Suppose the system $AX = 0$ has 20 unknowns and its solution space is spanned by 6 linearly independent vectors, then which of following can't be the order of A ?
A. 15×20 B. 6×20 C. 14×20 D. 16×20
40. Suppose that a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ has kernel spanned by one nonzero vector. Then what is the dimension of range of T ?
A. 0 B. 1 C. 2 D. 3

41. The linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $T(x, y) = (x - y, x + 2y)$ is:
A. one-one B. onto C. bijective D. None of these
42. For what value of k , the roots of the equation $x^3 - 6x^2 + kx + 64 = 0$ are in geometric progression?
A. -10 B. -18 C. -24 D. 12
43. If $f(1) = 2$ and $f(n - 1) = f(n) - \frac{1}{2}$ for all integers $n > 1$, then $f(101)$ equals:
A. 49 B. 50 C. 51 D. 52
44. If $|S| = n$, then the number of one-one functions from S onto S is:
A. $n!$ B. n^2 C. n^n D. 2^n
45. If V_1 and V_2 are 6 dimensional subspaces of a 10 vector space V , what is the smallest possible dimension that $V_1 \cap V_2$ can have?
A. 1 B. 2 C. 3 D. 4
46. A fair coin is tossed 8 times. What is the probability that more of the tosses will results in heads than will results in tails?
A. $\frac{93}{256}$ B. $\frac{23}{64}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$
47. For every set S and every metric d on S , which of the following is a metric on S ?
A. $4 + d$ B. $e^d - 1$ C. $d - \sqrt{d}$ D. \sqrt{d}
48. If $f(z)$ is an analytic function that maps the entire plane into the real axis, then the imaginary axis must be mapped onto:
A. the entire real axis B. a point C. a ray D. the empty set
49. Let $I \neq A \neq -I$, where I is the identity matrix. If $A = A^{-1}$, then the trace of A is:
A. -1 B. 0 C. 1 D. 2
50. For a positive integer m , $\Gamma(m + 1)$ equals:
A. $m + 1$ B. m C. $(m + 1)!$ D. $m!$

Best of Luck.