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University of Sargodha

B.A/B. Sc 1st Annual Examination 2012

Applied Math **Paper: B**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. a. Prove by using vectors that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ (8)

b. Prove that the necessary and sufficient condition for a vector $\underline{a}$ to have a constant magnitude is $\underline{a}.\frac{d\underline{a}}{dt} = 0$. (9)

Q.2. a. Show by using vectors, that the medians of the triangle are concurrent. (8)

b. Prove that if ϕ is scalar and A_{ij} is a second order tensor, then $C_{ij} = \phi A_{ij}$ is also a second order tensor. (9)

Q.3. a. State and prove the $(\lambda-\mu)$ −theorem. (8)

b. A system of forces acts on a plane in the form of an equilateral triangle of side $2a$. The moments of forces about three vertices are G_1, G_2, G_3 respectively. Find the magnitude of the resultant. (9)

Q.4. a. The smallest force which can support a body of weight w on a smooth inclined plane, is of magnitude P. show that the horizontal force necessary to support the same body on the plane is of magnitude $\frac{PW}{\sqrt{W^2-P^2}}$ (8)

b. A triangular lamina ABC, right angled at A, rests with its plane vertical, and with the sides AC, AB supported by smooth fixed pegs D, E in a horizontal line. Prove that the inclination θ of AC to the horizontal is given by $AC\cos\theta - AB\sin\theta = 3DE\cos 2\theta$ (9)

Section- II

Q.5. a. Two uniform solid spheres, composed of the same material and whose diameters are 6 in. and 12 in. respectively, are firmly united. Find the c.m of the combined body. (8)

b. A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and on a smooth peg at a distance b from the wall. Show that, in the position of equilibrium, the beam is inclined to the wall at angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$. (8)

Q.6. a. A uniform ladder rests in limiting equilibrium with one end on a rough horizontal plane, and the other against a smooth vertical wall. A man ascends the ladder. Show that he cannot go more than half way up. (8)

b. The least force which will move a weight up an inclined plane is of magnitude P. Show that the least force, acting parallel to the plane, which will move the weight upwards is $P\sqrt{1+\mu^2}$ where μ is the co-efficient of friction. (8)

Q.7. a. Find the position of the centriod of a quadrant of an elliptic lamina. (8)

b. A particle moves in a plane in such a way that at any time t, its distance from a fixed point O is $r = at + bt^2$ and the line connecting O and P makes angle $\theta = ct^{\frac{3}{2}}$ with a fixed line OA. Find the radial and transverse components of velocity and acceleration of the particle at $t = 1$. (8)

Q.8. a. A particle is projected vertically upwards. After a time t, anther particle is sent up from the same point with the same velocity and meets the first at height h during the downward flight of the first. Find the velocity of projection. (8)

b. A point describes simple harmonic motion in such a way that its velocity and acceleration at point P are u and f respectively and the corresponding quantities at another point Q are v and g. Find the distance PQ. (8)

Section- III

Q.9. a. Prove that the speed required to project a particle form a height h to fall a horizontal distance a from the point of projection is at least $\sqrt{g\left(\sqrt{a^2+h^2}-h\right)}$ (8)

b. A projectile having horizontal range r, reaches a maximum height H. Prove that it must have been launched with (a) an initial speed equal to $\sqrt{\frac{g(R^2+16H^2)}{8H}}$ and (b) at an angle with the horizontal given by $Sin^{-1}\left(\frac{4H}{\sqrt{R^2+16H^2}}\right)$. (9)

Q.10. a. A particle describes the curve $r^n = A\cos n\theta + B\sin n\theta$ under the force F to the pole, show that $F \propto \frac{1}{r^{2n+3}}$. (8)

b. If a particle be describing an ellipse about a centre of force in the center, show that the sum of the reciprocates of its angular velocities about foci is constant. (9)

Q.11. a. Find the condition that two straight lines $\frac{x-\alpha}{c_1} = \frac{y-\beta}{c_2} = \frac{z-\gamma}{c_3}$ and $\frac{x-\alpha'}{d_1} = \frac{y-\beta'}{d_2} = \frac{z-\gamma'}{d_3}$ may be coplanar. Also find an equation of the plane containing them. (8)

b. Show that the shortest distance between the straight lines. (9)
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ and equations of the straight line perpendicular to both are $11x+2y-7z+6 = 0 = 7x+y-5z+7$

Q.12. a. A sphere of radius K passes through the origin and meets the axes in A, B, C. Prove that the centriod of the triangle ABC lies on the sphere $9(x^2+y^2+z^2) = 4k^2$ (8)

b. Find the direction of Qibla of the Badshai Mosque, Lahore, latitude = $31°35.4'$ and longtitude $= 74°18.7'E$. (9)

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University of Sargodha

B.A/B. Sc 1st Annual Examination 2011

Applied Math **Paper: B**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. (a) Find the position vector of a point which divides the join of two points with position vectors $\underline{a}$ and $\underline{b}$ in the ratio $\lambda:\mu$. (8)

(b) Show that $div\, r^7 \underline{r} = 10r^7$ (9)

Q.2. (a) Show that the four points $4\underline{i}+5\underline{j}+\underline{k}, -\underline{j}-\underline{k}, 3\underline{i}+9\underline{j}+4\underline{k}$ *and* $-4\underline{i}+4\underline{j}+4\underline{k}$ are coplanar. (8)

(b) Prove that the inner product of two tensors is also a tensor. (9)

Q.3. (a) Prove that a system of coplanar forces is in equilibrium if the algebraic sum of moments of all the forces about three different points O, A, B not in the same straight line, are separately zero. (8)

(b) If forces $p\overrightarrow{AB}, q\overrightarrow{CB}, r\overrightarrow{CD}, s\overrightarrow{AD}$ acting along the sides of a plane quadrilateral are in equilibrium, show that $pr = qs$. (9)

Q.4. (a) A circular disc of weight W and radius a is suspended horizontally by two vertical strings each of length ℓ attached to the ends of a diameter. A horizontal couple applied to the disc turns it through an angle θ. Find the moment of the couple. (8)

(b) A cylinder of radius r, whose axis is fixed horizontally touches a vertical wall along a generating line. A flat beam of uniform material, of length 2ℓ and weight W, rests with its extremities in contact with wall and the cylinder, making an angle of 45° with the vertical. Show that, in absence of friction $\frac{\ell}{r}=\frac{\sqrt{5}-1}{\sqrt{10}}$, that the pressure on the wall is $\frac{1}{2}W$ and the reaction of cylinder is $\frac{1}{2}\sqrt{5}W$. (9)

Section- II

Q.5. (a) In a uniform circular disc of 8″ radius a circular hole of 2″ radius is cut, the centre of the hole being 3″ from the centre of the disc. Find the centre of mass of the remainder of the disc. (8)

(b) Four equal heavy uniform rods are freely jointed to form a rhombus $ABCD$ which is freely suspended from A, and kept in shape of a square by an inextensible string connecting A and C. show that tension in the string is $2W$, where W is the weight of one rod. (8)

Q.6. (a) A uniform ladder rests in limiting equilibrium with one end on a rough floor whose co-efficient of friction is μ and the other against a smooth vertical wall. Show that its inclination to the vertical is $\tan^{-1}(2\mu)$ (8)

b. A uniform rod of weight W is placed with lower end on a rough horizontal floor and its upper end against an equally rough vertical wall. The rod makes an angle α with the wall and is just prevented from slipping down by a horizontal force P applied at its mid point. Prove that $P = W\tan(\alpha - 2\lambda)$ where λ is angle of friction and $\lambda < \frac{\alpha}{2}$ (8)

P.T.O

Q.7. (a) Two uniform solid spheres, composed of the same material and whose diameters are 6 inches and 12 inches respectively, are firmly united. Find the centre of mass of the combined body. (8)

(b) Find the radial and transverse components of velocity and acceleration. (8)

Q.8. (a) A particle is projected vertically upwards with a velocity $\sqrt{2gh}$ and another is let fall from a height h at the same time. Find the height of the point where they meet each other. (8)

(b) A particle moves in a straight line with an acceleration KV^3. If its initial velocity is μ, find the velocity and time spent when the particle has travelled a distance x. (8)

Section- III

Q.9. (a) Find the range of a rifle bullet when α is the elevation of projection and v_0 the speed. Show that, if the rifle is fired with the same elevation and speed from a car travelling with speed V towards a target, the range will be increased by $\frac{2v_oV}{g}\sin\alpha$ (8)

(b) What is the maximum range possible for a projectile fired from a cannon having muzzle velocity v_0, and prove that the height reached $\frac{v_0^2}{4}$. (9)

Q.10. (a) A particle describes the curve $r^n\cos n\theta = a^n$ under force F to the pole, show that $F \propto r^{2n-3}$. (8)

(b) The law of force is MU^5 and a particle is projected from an apse at distance a. find the orbit when the velocity of projection is $\frac{\sqrt{M}}{a^2}$ (9)

Q.11. (a) Show that the straight lines (8)

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ *and* $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar.

(b) Show that the shortest distance between the lines $x+a=2y=-12z$ and $x=y+2a=6(z-a)$ is 2a. (9)

Q.12. (a) A particle P moves such that the square of its distance from the origin is proportional to its distance from a fixed plane. Show that P always lies on a sphere. (8)

(b) Prove that for a place on the equator the direction of Qibla is inclined at arctan(tan$\phi_\circ$ *cosec* ℓ) north of west or north of east according as its classical longitude ℓ is east or west ($\phi_\circ$ is latitude of Khana-e-Ka'aba) (9)

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University of Sargodha

B.A / B.Sc 2nd Annual Exam 2010

Applied Math **Paper: B**

Time Allowed: 3 Hours **Maximum Marks: 100**

Note: **Attempt six questions in all, selecting two questions from each section.**

Section- I

Q.1. a. Prove that in a quadrilateral the straight lines joining the mid points of the opposite sides bisect each other. (8)

b. Find the directional derivative of $\phi = x^2 + y^2 + z^2$ at point (2, 0, 3) in the direction of $2\underline{i} - \underline{j}$. (9)

Q.2. a. Show that the necessary and sufficient condition for a vector function f of a scalar variable t to have constant magnitude is $\underline{f}.\frac{d\underline{f}}{dt} = 0$ (8)

b. Prove that if a tensor is anti-symmetric in one co-ordinate system, then it will be anti-symmetric in every other co-ordinate system. (9)

Q.3. a. Three forces P, Q, R acting at a point, are in equilibrium, and the angle between P and Q is double the angle between P and R. Prove that $R^2 = Q(Q - P)$ (8)

b. If forces $\ell\overrightarrow{AB}, m\overrightarrow{BC}, \ell\overrightarrow{CD}, m\overrightarrow{DA}$ acting along the sides of a quadrilateral are equivalent to a couple, show that either $\ell = m$ or $ABCD$ is a parallelogram. (9)

Q.4. a. A uniform square lamina of side $2a$ rests in a vertical plane with two of its sides in contact with two smooth pegs distant b apart, and in the same horizontal line. Show that, if $\frac{a}{\sqrt{2}} < b < a$, a non symmetrical position of equilibrium is possible in which $b(\sin\theta + \cos\theta) = a$ where θ is inclination of a side of the square to the horizontal. (8)

b. A triangular lamina ABC, right angled at A , rests with its plane vertical, and with the side AB, AC supported by smooth fixed pegs D,E in a horizontal line. Prove that the inclination θ of AC to the horizontal is given by $AC\cos\theta - AB\sin\theta = 3DE\cos 2\theta$. (9)

Section- II

Q.5. a. Weights of 1, 2, 3, 4 lb. are placed at the corners A, B, C,D of a square of side 8 inches. Find the distances of the centre of gravity of the set from AB and AD. (8)

b. A hexagon ABCDEF, consisting of six equal heavy rods, of weight W, freely jointed together, hangs in a vertical plane with AB horizontal, and the frame is kept in the form of a regular hexagon by a light rod connecting the mid points of CD and EF. Show that the thrust in the light rod is $2\sqrt{3}W$. (8)

Q.6. a. A rod, 4 ft. long, rests on a rough floor against the smooth edge of a table of height 3ft. if the rod is on the point of sliding when inclined at an angle of 60° to the horizontal, find the coefficient of friction. (8)

b. A uniform ladder rests in limiting equilibrium with one end on a rough horizontal plane, and the other against a smooth vertical wall. A man ascends the ladder. Show that he cannot go more than half way up. (8)

Q.7. a. Find the centroid of the surface formed by the revolution of the cardiod $r = a(1 + \cos\theta)$ about the initial line. (8)

b. Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with uniform speed v when the particle is at $(0, b)$. (8)

P.T.O

Q.8. a. A particle moving along a straight line starts from rest and is accelerated uniformly till it attains a velocity v. The motion is then retarded and the particle comes to rest after traversing a total distance X. if the acceleration is f, find the retardation and the total time taken by the particle from rest to rest. (8)

b. A particle describes simple harmonic motion with frequency N. if the greatest velocity is v, find the amplitude and the maximum value of acceleration. Also show that the velocity v at a distance x from the centre of motion is given by $v = 2\pi N\sqrt{a^2 - x^2}$ where a is the amplitude. (8)

Section- III

Q.9. a. Find the equation of parabola of safety of a projectile. Find its focus and directrix. (8)

b. From a gun placed on a horizontal plane, which can fire a shell with speed $\sqrt{2gH}$, it is required to throw a shell over a wall of height h, and the elevation of the gun cannot exceed $\alpha < 45°$. Show that this will be possible only when $h < H\sin^2\alpha$, and that, if this condition be satisfied, the gun must be fired from within a strip of the plane whose breadth is $4\cos\alpha\sqrt{H(H\sin^2\alpha - h)}$. (9)

Q.10. a. A particle of unit mass describes an ellipse under the action of central force Mr. Show that the normal component of acceleration at any instant is $\frac{abM^{\frac{3}{2}}}{v}$, where v is the velocity at that instant and a, b the semi-axes of the ellipse. (8)

b. A particle moves under a central repulsive force $\frac{\mu}{r^3}$ and is projected from an apse at a distance a with velocity v. show that the equation to the path is $r\cos p\theta = a$, and that the angle θ described in time t is $\frac{1}{p}\tan^{-1}\frac{pvt}{a}$ where $p^2 = \frac{\mu + a^2v^2}{a^2v^2}$. (9)

Q.11. a. If a, b, c are the intercepts of a plane on co-ordinate axes and r is the distance of the origin from the plane, prove that $\frac{1}{r^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$. (8)

b. Show that the shortest distance between the straight lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ and equations of the straight line perpendicular to both are $11x + 2y - 7z + 6 = 0 = 7x + y - 5z + 7$ (9)

Q.12. a. Find an equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,\ 2x + 3y + 4z - 8 = 0$ is a great circle. (8)

b. Find the direction of Qibla for Peshawar with given data: (9)

Place	Latitude	Longitude
Khana-e-Kaaba	$21°25.2'N$	$39°49.2'E$
Peshawar	$34°1'N$	$71°40'E$