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University of Sargodha

B.A/B.Sc 1st Annual Examination 2007

Applied Mathematics **Paper- A**

Time Allowed: 3 Hours

Maximum Marks: 100

Note: **Attempt six questions in all. selecting two questions from each section.**

CH#9
CH:10
2Q. Attempt

Section- I

Q.1. (a) Solve the differential equation. $(2x+y+1)dx+(4x+2y-1)dy=0$ (9.3) Example 15 (9) CH=9 page 393 Method

(b) Solve the differential equation. $xp^2+(y-1-x^2)p-x(y-1)=0$ Q.7 9.8 (8) Method

Q.2. (a) Solve the initial value problem. $(x^2+1)\dfrac{dy}{dx}+4xy=x$, $y(2)=1$ Q16 EX:9.6 (9) method

(b) A newly built fish farm is stocked with 400 fish at time $t=0$ (month), thereafter the population increases at the rate of $\sqrt{p}$ per month, when these are p fish in the farm, what is the fish population at time 't'? (8) Q6 10.11 method

Q.3. (a) Solve the differential equation. $(3y+4xy^2)dx+(2x+3x^2y)dy=0$ Q.9 EX:9.5 (9) Method

(b) Solve by method of U.C $y''-3y'+2y=2x^2+2xe^x$ Q6 EX:10.3 (8) Method

Q.4. (a) Find a particular solution of following. $\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+4y=\dfrac{e^{2x}}{1+x}$ (9) Q:5 EX:10

(b) Apply the Power Series Method to solve the differential equation. $x(1-x)y'=y$ (8) Q:9 EX: 10.9

Section- II

Q.5. (a) Compute the inverse Laplace Transformation of the following. $\dfrac{s^3+3s^2-s-3}{(s^2+2s+5)^2}$ (8) Q13 EX 11.2

(b) Find a positive root of a non linear equation, using Bisection Method. $f(x)=x^3-x^2-2x+1$. (8) (Numerical Any.

Q.6. (a) Use the Laplace Transformation Method to solve the following initial value Problem. $\dfrac{d^2y}{dt^2}-2\dfrac{dy}{dt}=20e^{-t}\cot t, y(0)=0=y'(0)$ (8) Q. 9 EX.11.3

(b) Find the positive root of the equation. $x^3+x^2-x-3=0$ Using the Regula-Falsi Method (8) Numerical Any.

Q.7. (a) Find the positive root of the equation. $e^x=2x+21$ by using Newton's Raphson Method with $x_0=3$ (8) Numerical Any

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Sect. I Q. 1 - 4 : 2
Sect. II Q. 5 - 8 : 2
Sect. III Q. 9 - 11 : 1
6

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(b) Prove that $\Delta\left(\dfrac{f_n}{g_n}\right) = \dfrac{g_n \Delta f_n - f_n \Delta g_n}{g_n g_{n+1}}$ (8)

Numerical Any:

Q.8. (a) Find the first three derivatives of $f(x)$ from the following table at $x = 19$ (8)

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	0.544	0.296	2.432	4.00

Numerical Ans

(b) Use the Simpson's Rule to approximate the given integral. (8)

$\int_2^3 \frac{\sin x}{x} . dx$ with $n = 6$

EX: 5.5 Calculus.

Section- III

Q.9. (a) Find the maximum value of $z = 9x_1 + x_2$ subject to the Constraints. (9)

$$2x_1 + x_2 \le 8$$
$$4x_1 + 3x_2 \le 14$$
$$x_1, x_2 \ge 0$$

Graphs

(b) Minimize $z = x_2 - x_1$ (8)

such that

$$-x_1 + 2x_2 \le 2$$
$$x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 3$$

$x_1, x_2 \ge 0$ Using simplex method.

Q.10. (a) If A and B are any two events defined in a sample space S, then (9)

$P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(b) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or by 8? (8)

Probability

Q.11. (a) If A and B are two independent events in a sample space S, then show that (9)

i. A and $\overline{B}$ are independent. ii. $\overline{A}$ and $\overline{B}$ are independent.

(b) An event has the probability $p = 3/8$. Find the complete Binomial Distribution for $n = 5$ trials. (8)

Q.12. (a) A man draw two balls from a bag containing 3 white and 5 black balls. If he receives Rs.70 for every white ball he draws and Rs.7 for every black ball, find his expectation. (9)

(b) Write down the properties of Binomial Distribution. (8)

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University of Sargodha

B.A / B.S : 1st Annual Exam 2010

Applied Math **Paper-A**

Time Allowed: 3 Hours **Maximum Marks: 100**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. a. Solve the following differential equation: $\frac{dy}{dx}=\frac{y-x+1}{y-x+5}$ (8) CH#9 (method)

b. Find the general solution of the following non-homogeneous differential equation: $(D^3+D^2-4D-4)y=e^{2x}\cos 3x$ (9) CH#10 method

Q.2. a. Solve the differential equation: $(x-1)^3\frac{dy}{dx}+4(x-1)^2y=x+1$ (8) CH#10 method

b. Solve the differential equation: $4x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+3y=\sin(\ln(-x))$ (9) CH#10 method

where $x<0$.

Q.3. a. Solve the differential equation: $\frac{d^2y}{dx^2}+y=\csc x$ (8) CH#10 method

b. Solve $2y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2=1$ (9) CH#10 method

Q.4. a. Find an equation of orthogonal trajectory of the curve of the family $xy=c$. (8) CH#9 method

b. Find the series solution of the differential equation $y''-x^2y=0$ around the point $x=0$. (9) CH#10

Section- II

Q.5. a. Compute the Laplace transform of $e^{-t}\sin 2t$ (8) CH#11 (method)

b. Compute the inverse Laplace transform of $\frac{1}{(s-1)(s^2+4)}$ (8) CH#11 (method)

Q.6. a. Use Simpson's rule, with $n=8$, to approximate the value of $\int_0^1\frac{dx}{1+x^2}$ (8) CH#5 (cal

b. Find the area under the semicircle $y=\sqrt{4-x^2}$ and above the x-axis using trapezoidal rule, for $n=8$. (8) CH#5 (cal

Q.7. a. Use the bisection method to find the positive root, correct to three places of decimal, of the equation $x-e^{\frac{1}{x}}=0$ within [1,2]. (8) Numer. Ana.

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b. Compute the positive root, correct to four places of decimal, of the equation $x^2 \pm 4\sin x = 0$ using Newton-Raphson method. (8)

Numerical Ans

Q.8. a. Use method of False position to find the root of the equation $x^3 - 4x^2 + x - 10 = 0$ accurate to three places of decimal within [4,5]. (8)

Numerical Ans

b. Find the 2^{nd} degree Lagrange interpolation polynomial for $f(x) = \dfrac{1}{x}$, choosing the points $x_0 = 2,\ x_1 = 2.5,\ x_2 = 4$. Also approximate $f(3) = \dfrac{1}{3}$ (8)

Numerical Any

Section - II

Q.9. a. Find the maximal value of the object function $z = x + 3y$; subject to the constraints. $y \le x + 1,\ x + y \ge 2,\ 2y \ge x - 1,\ x \ge 0,\ y \ge 0$ (9)

Graphic

b. Use the simplex method to find the maximum value of the object function $c = 4x \cancel{+ y + 3z}$, ~~where~~ where x, y and z are non-negative variables satisfying the constraints $x + y + z \le 4,\ 3x + y + 2z \le 7$ *and* $x + 2y + 4z \le 9$ (8)

Q.10. a. Three horses A, B and C are in a race; A is twice as likely to win as B and B is twice as likely to win as C. What is the probability that A or B wins. (9)

Probability

b. An urn contains four balls which are known to be either; i. all white or ii. Two white and two black. A ball is drawn at random and is found to be white. What is the probability that all balls are white? (8)

Q.11. a. Find the value of K, so that the function $f(x)$ defined as follows may be a density function (8)

(R.V)

$$f(x) = \begin{cases} kx, & 0 \le x \le 2 \\ 0, & elsewhere \end{cases}$$

b. If $f(x) = \dfrac{6 - |7 - x|}{36}$ for $x = 2, 3, \ldots\ldots, 12$, then find mean and variance of the random variable X. (9)

(Bin)

Q.12. a. For the binomial distribution the probability density is given by $f(x) = \binom{n}{x} p^x q^{n-x}$, $x = 0,1,2,\ldots\ldots\ldots\ldots,n$ where the random variable X assumes a value x. Prove the relation $\mu_{r+1} = pq\left(nr\mu_{r-1} + \dfrac{d\mu_r}{dp}\right)$. (9)

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b. Show that the mean of negative binomial distribution is less than its variance. (8)

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University of Sargodha

B.A/B. Sc 1st Annual Examination 2012.

Applied Math **Paper: A**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. a. Solve the initial value problem $\frac{dy}{dx} = \frac{x(x^2+1)}{4y^3}$ $y(0) = \frac{-1}{\sqrt{2}}$ (8)

b. Solve the equation. $(D^2 + 6D + 9)y = 0$ $y(0) = 2$ $y'(0) = -3$ (9)

Q.2. a. Solve differential equation. $\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$ (8)

b. Solve by the method of U.C $y'' - 4y' + 4y = e^{2x}$ (9)

Q.3. a. Solve $(1+x^2)\frac{dy}{dx} + 4xy = \frac{1}{(1+x^2)^2}$ (8)

b. Solve $x^2\frac{d_2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$ (9)

Q.4. a. Find orthogonal trajectories of family of cardiods. $r = a(1+cos\theta)$ (8)

b. Find a series solution of differential equation around indicated point (9)

$y'' - x^2y = 0$ around $x = 0$

Section-II

Q.5. a. Compute the Laplace transformation of $cos^2 a\,t$ (8)

b. Compute the inverse Laplace transformation of $\frac{9S-67}{S^2-16S+49}$ (8)

Q.6. a. Using Newton Raphson method find a root of $f(x) = x^3 - 2x - 5 = 0$ (8)

b. Solve the transcendental equation $f(x) = e^{-x} - sin\left(\frac{\pi x}{2}\right) = 0$ to a positive real root by Bisection method. (8)

Q.7. a. Use the trapezoidal rule with $n = 4$ to approximate. $I = \int_0^4 \sqrt{x^2+1}\,dx$ (8)

b. Use Simpson's rule to approximate the Integral $\int_1^2 \ln x\,dx$ with $n = 4$. (8)

Q.8. a. Find the first and second order derivatives of the function from the following data at $x = 2$. (8)

x	1	2	3	4	5
$f(x)$	3	10	29	66	127

b. Find a bound on the error in approximating the given integral using: (8)

i. Trapezoidal rule ii. Simpson's rule. $\int_{-1}^2 x^5 dx$ with $n = 10$

Section- III

Q.9. a. Minimize $z = 2x_1 + x_2$ subject to the conditions (9)

$$x_1 + x_2 \geq 1$$
$$x_1 - x_2 \geq -1$$
$$x_1 + 2x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

b. Use the simplex method to find the maximum value of object function $z = 10x_1 + 11x_2$ with the condition (8)

$$3x_1 + 4x_2 \leq 9$$
$$5x_1 + 2x_2 \leq 8$$
$$x_1 + 2x_2 \leq 1$$
$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

Q.10. a. A set of eight cards contains one joker. A and B are two players and A choose 5 cards at random, B taking the remaining 3 cards. What is the probability that A has the joker? (8)

b. A pair of fair dice is thrown. If the two numbers appearing are different, find the probability that sum is (i) 6 (ii) sum is 4 or less. (9)

Q.11. a. If $f(x) = \frac{1}{n}(x = 1,2,3 \ldots\ldots\ldots\ldots n)$ then find $E(x)$ and $Var(x)$ (8)

b. Suppose that the life length (in hours) of a certain radio tube is continuous random variable x with probability density function $f(x) = \frac{100}{x^2}$ $x > 100$ (9)

And zero elsewhere. What is the probability that a tube will last less than 200 hours, if it is known that tube is still functioning after 150 hours of service?

Q.12. a. An event has the probability $P = 3/8$, Find the complete Binomial distribution for $n = 5$ trials? (9)

b. Let X be random variable having a binomial distribution with parameters $n = 25$ and $P = 0.2$ evaluate $P[X < \mu - 2\sigma]$ (8)

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University of Sargodha

B.A/B. Sc 1st Annual Exam 2013.

Subject: Applied Math Paper: A

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Attempt any two questions from each section.**

Section- I

Q.1. a. Solve the initial value problem $\frac{dy}{dx}=\frac{2x}{y+x^2y}$, $y(0) = -2$ (8)

b. Solve $(x-y)dx+(x+y)dy=0$ (9)

Q.2. a. Solve the differential equation $(3x^2y+2)dx+(x^3+y)dy=0$ (8)

b. Solve the equation $\frac{dy}{dx}+\frac{xy}{1-x^2}=xy^{\frac{1}{2}}$ (9)

Q.3. a. Find the orthogonal trajectories of the family of cardiods $r=a(1+cos\theta)$ (8)

b. Solve $(D^2-5D+6)Y=\sin 3x$ (9)

Q.4. a. Find the general solution of $(D^2+3D-4)Y=15e^x$ (8)

b. Solve by the method of U.C $y''-3y'+2y=x^2e^x$ (9)

Section- II

Q.5 a. Compute the Laplace transformation of e^{at} where a is a constant and $s \neq a$. (8)

b. Find the inverse Laplace transformation of $\frac{3S+17}{S^2+8S+25}$ (8)

Q.6. a. Solve the equation $f(x)=e^x-3x=0$ by bisection method. (8)

b. Using Newton Raphson method, evaluate to two decimal places the root of the equation which lies between 0 and 1, the function is $f(x)=e^x-3x=0$ (8)

Q.7. a. Evaluate $\int_1^3 \frac{1}{x^2}dx$ by using trapezoidal rule for five points. (8)

b. Apply 5 points Simpson's rule to evaluate $\int_0^1 \frac{1}{1+x^2}dx$ (8)

Q.8. a. Find first and second derivatives of the function from the following data at $x = 2$. (8)

x	1	2	3	4	5
f(x)	3	10	29	66	127

b. Evaluate the integral $\int_0^1 \frac{1}{1+x^2}dx$ by using rectangular rule for $n = 4$. (8)

Section- III

Q.9. a. Maximize $z=10x_1+11x_2$ subject to the conditions (8)
$3x_1+4x_2 \le 9$, $5x_1+2x_2 \le 8$, $x_1-2x_2 \le 1$ where $x_1, x_2 \ge 0$

b. Use Simplex method to find the maximum value of object function $z=3x_1+2x_2$ with the condition (9)
$x_1+2x_2 \le 6$, $2x_1+x_2 \le 8$, $-x_1+x_2 \le 1$, $x_2 \le 2$ where $x_1, x_2 \ge 0$

Q.10. a. An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or by 8? (8)

b. A card is drawn at random from a deck of ordinary playing cards. What is the probability that it is a diamond, a face card or a king. (9)

Q.11. a. A man tosses two fair dice. What is the conditional probability that the sum of the two dice will be 7, given that: (8)
i. the sum is odd ii. the sum is greater than 6 iii. the two dice had same outcome.

b. A pair of fair dice is thrown twice. What is the probability of getting totals of 5 and 11? (9)

Q.12. a. A certain event is believed to follow the binomial distribution. In 1024 samples of 5, the result was observed once 405 times and twice 270 times. Find p and q. (8)

b. An event has the probability $P=\frac{3}{8}$. Find the complete binomial distribution for $n = 5$ trials. (9)