UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II)

Mathematics A-Course (Paper-III)

Roll No: ________

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions, selecting **TWO** questions form **Section-A,** and **THREE** from **Section-B.**

SECTION – A

1- a) Find the limit of the sequence $\left\{\frac{\ln n}{n}\right\}$ as $n \to \infty$ *5*

b) Investigate the behaviour of the Euler's series. $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \cdots\cdots\frac{1}{n^2} + \cdots$ *5*

2- a) Apply any appropriate test to determine the convergence or divergence of the series $\sum_{1}^{\infty} \frac{\arctan n}{n^2}$ *5*

b) Using the Alternating Test, determine the convergence of the series $\sum_{1}^{\infty} (-1)^{n-1} \frac{n+4}{n^2+n}$ *5*

3- a) Test the series $\sum_{1}^{\infty} \frac{(-1)^n .n!}{1.3.5\ldots\ldots(2n-1)}$ for *5*

i) ABSOLUTE CONVERGENCE
ii) CONDITIONAL CONVERGENCE
iii) DIVERGENCE

b) Find the radius of convergence and the interval of convergence of the series *5*

$$\sum_{n=1}^{\infty} \frac{n!\, x^n}{(2n)!}$$

SECTION – B

4- a) Find the differential equation of all circles that pass through the origin. *5*

b) Solve the differential equation $y\sqrt{1+x^2}\, dx + x\sqrt{1+y^2}\, dy = 0$ *5*

5- a) Solve the differential equation $\frac{dy}{dx} = \frac{1}{e^y - x}$ *5*

b) Find an orthogonal trajectory for the curve $r^2 = a \sin 2\theta$ *5*

6- a) Find the singular solution if any of the equation $x^2p^2 + y\,p\,(2x+y) + y^2 = 0$ by making the substitutions $y = u,\ xy = v$ *5*

b) Find the general solution of the equation $(D^2 - 5D + 6)\,y = \sin 3x$ *5*

7- a) Solve the Cauchy Euler equation $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 4\,[\cos \ln(x+1)]^2$ *5*

b) Solve the differential equation using Reduction of Order $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{1}{(1+e^x)^2}$. *5*

8- a) Compute $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+5)}\right\}$ *5*

b) Use the Laplace transform method to solve the following initial value problem

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = e^t \qquad y(0) = 1,\ y'(0) = 0$ *5*