UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)

Mathematics A-Course (Paper-II)

Roll No: _____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

Section-A

1. a) Evaluate $\left(\frac{\sqrt{3} - i}{\sqrt{3} + i}\right)^6$ *5*

b) Prove that $64 (\text{Cos}^8\theta + \text{Sin}^8\theta) = \text{Cos } 8\theta + 28 \text{ Cos } 4\theta + 35$ *5*

2. a) Show that $\text{Cot}^{-1} Z = \frac{1}{2i} \ell\text{og}\left(\frac{Z + i}{Z - i}\right)$. *5*

b) Prove that $a^i = \text{Cos } (\ell\text{n } a) + i \text{ Sin } (\ell\text{n } a) \qquad a > 0.$ *5*

3. a) Evaluate the sum of infinite series $n \text{ Sin } \theta + \frac{n(n+1)}{2!} \text{Sin } 2\theta + \frac{n(n+1)(n+2)}{3!} \text{Sin } 3\theta + \ldots..$ *5*

b) Separate into real and imaginary parts $\text{Sin}^{-1} (\text{Cos } \theta + i \text{ Sin } \theta)$. *5*

Section-B

4. a) Find the condition that straight line $\ell x + my + n = 0$ may touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, also find the coordinates of point of contact. *5*

b) Show that in any conic the sum of the reciprocals of the segments of any focal chord is constant. *5*

5. a) Find the measure of angle of intersection between two curves. $r = \frac{a\theta}{1 + \theta}$, $r = \frac{a}{1 + \theta^2}$. *5*

b) Show that the pedal equation of the curve
$x = ae^{\theta} (\text{Sin } \theta - \text{Cos } \theta)$, $y = ae^{\theta} (\text{Sin } \theta + \text{Cos } \theta)$ is $r = \sqrt{2}\, p$. *5*

6. a) Find length of point P (1, 6, 3) from the line $\frac{x}{1} = \frac{y - 1}{2} = \frac{z - 2}{3}$. Also find foot of perpendicular. *5*

b) Find equation of plane through the points (1, 0, 1) and (2, 2, 1) and perpendicular to the plane $x - y - z + 4 = 0$. *5*

7. a) Find the shortest distance between lines $\frac{x - 3}{1} = \frac{y - 5}{-2} = \frac{z - 7}{1}$ and $\frac{x + 1}{7} = \frac{y + 1}{-6} = \frac{z + 1}{1}$ find equations of straight lines perpendicular to both lines and also find its point of intersection with given straight lines. *5*

b) Express the given equation $x^2 + y^2 + 2z = 16$ into cylindrical and spherical coordinates. *5*

8. a) Find an equation of the sphere for which the circle
$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y - 4z - 8 = 0$ is a great circle. *5*

b) Show that an equation to the right circular cone with vertex 0, axis OZ and semi vertical angle α is
$x^2 + y^2 = z^2 \tan^2 \alpha$ *5*