UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)

Mathematics B-Course (Paper-II)

Roll No: _____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

Section-A

1. a) Prove that $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$ 5

b) Show that $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = (\vec{a} \cdot \vec{b} \times \vec{c})^2$ 5

2. a) If $\vec{u}(t)$ is a unit vector, Show that $\vec{u} \cdot (\vec{u} + \frac{d^2\vec{u}}{dt^2}) + \left(\frac{d\vec{u}}{dt}\right)^2 = 1.$ 5

b) Integrate : $\frac{d^2\vec{r}}{dt^2} = -n^2 \vec{r}$. 5

3. a) If $\vec{a}$ is a constant vector and $\vec{v} = \vec{a} \times \vec{r}$, show that div $\vec{v} = 0$. 5

b) Evaluate: $\nabla \times \left(\frac{\vec{r}}{r^2}\right)$. 5

Section-B

4- a) P is any point in the plane of triangle ABC and D, E, F are middle points of its sides, prove that forces $\vec{AP}$, $\vec{BP}$, $\vec{CP}$, $\vec{PD}$, $\vec{PE}$, $\vec{PF}$ are in equilibrium. 5

b) A couple of moment G acts on a square board ABCD of side a. Replace the couple by forces acting along AB, BD and CA. 5

5- a) A triangular lamina ABC, right-angled at A, rests with its plane vertical, and with the sides AB, AC supported by smooth fixed pegs D, E in a horizontal line. Prove that the inclination θ of AC to the horizontal is given by AC Cos θ – AB Sin θ = 3 DE Cos 2 θ. 5

b) Two equal beams AB, AC each of weights W, connected by a hinge at A, are placed in a vertical plane with their extremities B, C resting on a horizontal plane; they are kept from falling by strings connecting B and C with the middle points of the opposite sides. Show that the tension of either string is $\frac{w}{8}\sqrt{1 + 9\,\text{Cot}^2\theta}$, where θ is the inclination of either beam to the horizontal. 5

6- a) A uniform ladder rests in limiting equilibrium with one end on a rough floor whose co-efficient of friction is μ and with the other against a smooth vertical wall. Show that its inclination to the vertical is $\tan^{-1}(2\mu)$. 5

b) A thin uniform rod passes over one peg and under another, the coefficient of friction between each peg and the rod being μ. The distance between the pegs is α, and the straight line joining them makes an angle β with the horizontal. Show that the equilibrium is not possible unless the length of the rod is greater than $\frac{\alpha}{\mu}(\mu + \tan\beta)$. 5

7- a) Find the C.G of the curved surface of a hollow right circular cone. 5

b) Find the C.G of uniform lamina forming a quadrant of an ellipse bounded by its semi-axes. 5

8- a) A particle of mass 20lb is supported on a smooth plane inclined at 60° to the horizontal by a force of magnitude x poundals which makes an angle of 30° with the plane. Find x and also the reaction of the plane on the particle. 5

b) Six equal uniform rods freely jointed at their extremities form a tetrahedron. If this tetrahedron is placed with one face on a smooth horizontal table, prove that the thrust along a horizontal rod is $\frac{W}{2\sqrt{6}}$, where W is the weight of a rod. 5