UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II)

Mathematics A-Course (Paper-III)

Roll No: _____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions, selecting **TWO** questions form **Section-A,** and **THREE** from **Section-B.**

SECTION – A

1- a) Given the nth-term of the sequence. Determine whether it converges or diverges.

If convergent find its limit $a_n = \frac{2^n}{(2n)!}$ *5*

b) Determine whether the following series converges or diverges. If it converges find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} = \frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} + \cdots\cdots\cdots\cdots$$ *5*

2- a) Test the series for convergence or divergence $\sum_{0}^{\infty} \frac{2^n + n}{(n+1)!}$ *5*

b) Use the Alternating Series Test to determine whether the given series converges *5*

$$\sum_{2}^{\infty} (-1)^n \frac{1}{n \ell_n n}$$

3- a) Test the series $\sum_{1}^{\infty} \frac{(-1)^n (n+2)}{n(n+1)}$ for *5*

i) Absolute Convergence
ii) Conditional Convergence
iii) Divergence

b) Find radius of convergence and interval of the convergence for the series *5*

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} (x+1)^{2n}}{(n+1)^2 5^n}$$

SECTION – B

4- a) Verify that $y = C_1 \cos x$ and $y = C_2 \sin x$ are solutions of $\frac{d^2y}{dx^2} + y = 0$. Find a particular solution of the equation satisfying the boundary conditions $y(0) = 1, y(\pi/2) = 2$ *5*

b) Solve the differential equation $(xy + 2x + y + 2)\,dx + (x^2 + 2x)\,dy = 0$ *5*

5- a) Solve the differential equation $x\frac{dy}{dx} + (1 + x \cot x)\, y = x$ *5*

b) Find an equation of orthogonal trajectory of the curve of the following family $y = (x - c)^2$ *5*

6- a) Find the singular solution if any of the eq. $xp^2 - 2yp + 4x = 0$ *5*

b) Find the general solution of $(D^3 + 1)\, y = 1 + e^{-x} + e^{2x}$ *5*

7- a) Solve the Cauchy-Euler equation $4x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 3y = \sin \ell n(-x)$ $x < 0$. *5*

b) Solve $\frac{d^2y}{dx^2} + y = \csc x$ *5*

8- a) Find the Laplace Transform of $t e^{-3t} \sin at$ *5*

b) Compute $\mathcal{L}^{-1} \frac{1}{(s^2 + a^2)(s^2 + b^2)}$ *5*