Chapter 7

Linear equation and inequalities

Linear Equation:

The equation of the from $ax+b=0, a\neq 0$ is called linear equation.

Example 1: i). $2x+3=0$

Example 1: ii). $\frac{5}{2}y-4=0$

Example 1: iii). $5x-15=2x+3$

Example 1: iv). $\frac{1}{3}x-\frac{2}{3}=4$

Example 2: $2x+3=1-6(x-1)$

Sol: Given $2x+3=1-6(x-1)$

$$2x+3=1-6(x-1)$$
$$2x+3=1-6x+6$$
$$2x+6x=1+6-3$$
$$8x=4$$
$$x=\frac{4}{8}=\frac{1}{2}$$

Verification, putting $x=\frac{1}{2}$ in given equation

$$2\left(\frac{1}{2}\right)+3=1-6\left(\frac{1}{2}-1\right)$$
$$1+3=1-3+6$$
$$4=4\ True$$
$$S.S=\left\{\frac{1}{2}\right\}$$

Example 3: $3x+\frac{x}{5}-5=\frac{1}{5}+5x$

Sol: Given $3x+\frac{x}{5}-5=\frac{1}{5}+5x$

$$3x+\frac{x}{5}-5=\frac{1}{5}+5x$$
$$3x+\frac{x}{5}-5x=\frac{1}{5}+5$$
$$\frac{15x+x-25x}{5}=\frac{1+25}{5}$$
$$16x-25x=26$$
$$-9x=26$$
$$x=\frac{26}{-9} \qquad \therefore S.S=\left\{\frac{26}{-9}\right\}$$

Example 4: Age of mother is 13 times age of daughter. It will be only five times after four year. Find their present ages.

Sol: Let age of daughter = x

Age of mother = $13x$ From the set of fact

$$13x+4=5(x+4)$$
$$13x+4=5x+20$$
$$13x-5x=20-4$$
$$8x=16$$
$$x=2$$

Therefore age of daughter = 2 year

Age of mother = 13(2) = 26 year

Example 5: A number consists of two digits. The sum of digits is 8. If the digits are interchanged the new number is 36 less then original number. Find the number.

Solution: Suppose the digit at unit place = x

Digit at ten place = y

So the original number = 10 y + x

After interchanging the place of digits

New number = 10x + y

According to the given conditions

Sum of two digits is 8

i.e., $x+y=8 \quad \Rightarrow y=8-x$ ……(1)

New number + 36 = original number

$$10x+y+36=10y+x$$
$$10x-x+y-10y=-36$$
$$9x-9y=-36$$
$$x-y=-4$$

Putting the value of y = 8 - x

$$x-(8-x)=-4$$
$$x-8+x=-4$$
$$2x=-4+8$$
$$2x=4$$
$$x=\frac{4}{2}=2$$

So y = 8 - 2 = 6

Original number = 10 y + x
= 10 (6) + 2
= 60 + 2
= 62

Exercise 7.1

Q1. Find the solution sets of the following equations and verify the answer

i). $5x+8=23$

Sol: Given $5x+8=23$

$$5x=23-8$$
$$5x=15$$
$$x=\frac{15}{5}=3$$

Verification put x = 3 in given linear equation

$$5(3)+8=23$$
$$15+8=23$$
$$23=23$$

Verified so, $S.S=\{3\}$

ii). $\frac{3}{5}x-\frac{2}{3}=2$

Sol: Given $\frac{3}{5}x-\frac{2}{3}=2$

Multiply each term by 15

$$15\times\frac{3}{5}x-15\times\frac{2}{3}=15\times 2$$
$$3(3x)-5(2)=30$$
$$9x-10=30$$
$$9x=30+10$$
$$x=\frac{40}{9}$$

Verification put $x=\frac{40}{9}$ in given equation

$$\frac{3}{5}\left(\frac{40}{9}\right)-\frac{2}{3}=2$$
$$\frac{8}{3}-\frac{2}{3}=2$$
$$\frac{8-2}{3}=2$$
$$\frac{6}{3}=2$$

2 = 2 verified

$$S.S=\left\{\frac{40}{9}\right\}$$

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iii). $6x-5=2x+9$

Sol: Given $6x-5=2x+9$

$$6x-2x=9+5$$
$$4x=14$$
$$x=\frac{14}{4}=\frac{7}{2}$$

Verification put $x=\frac{7}{2}$ In given equation

$$6\left(\frac{7}{2}\right)-5=2\left(\frac{7}{2}\right)+9$$
$$3(7)-5=7+9$$
$$21-5=16$$

16=16 verified

$$S.S=\left\{\frac{7}{2}\right\}$$

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iv). $\frac{2}{x-1}=\frac{1}{x-2}$

Sol: Given $\frac{2}{x-1}=\frac{1}{x-2}$

By cross multiplication

$$2(x-2)=1(x-1)$$
$$2x-4=x-1$$
$$2x-x=-1+4$$
$$x=3$$

Verification put x = 3 in given equation

$$\frac{2}{3-1}=\frac{1}{3-2}$$
$$\frac{2}{2}=\frac{1}{1}$$

1 = 1 verified

$$S.S=\{3\}$$

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v). $\frac{1}{7x+13}=\frac{2}{9}$

Solution: we have

$$\frac{1}{7x+13}=\frac{2}{9}$$
$$1(9)=2(7x+13) \quad \text{(Cross Multiplying)}$$
$$9=14x+26$$
$$9-26=14x+26-26 \quad \text{(Subtract 26)}$$
$$-17=14x$$
$$\frac{-17}{14}=\frac{14x}{14} \quad \text{(Dividing by 14)}$$
$$\frac{-17}{14}=x$$

Put $x=\frac{-17}{14}$ in the original equation

$$\frac{1}{7\left(\frac{-17}{14}\right)+13}=\frac{2}{9}$$
$$\frac{1}{\frac{-17}{2}+\frac{13}{1}}=\frac{2}{9}$$
$$\frac{1}{\frac{-17+26}{2}}=\frac{2}{9}$$
$$\frac{1}{\frac{9}{2}}=\frac{2}{9}$$
$$1\div\frac{9}{2}=\frac{2}{9}$$
$$\frac{2}{9}=\frac{2}{9}$$

LHS = RHS (Satisfied)

$$S.S=\left\{\frac{-17}{14}\right\}$$

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vi). $10(x-4)=4(2x-1)+5$

Sol: Given $10(x-4)=4(2x-1)+5$

$$10x-40=8x-4+5$$
$$10x-8x=-4+5+40$$
$$2x=41$$
$$x=\frac{41}{2}$$

Verification put $x=\frac{41}{2}$ in given equation

$$10\left(\frac{41}{2}-4\right)=4\left(2\times\frac{41}{2}-1\right)+5$$
$$5\times 41-40=4(41-1)+5$$
$$205-40=4(40)+5$$
$$165=160+5$$

165 = 165 verified

$$S.S=\left\{\frac{41}{2}\right\}$$

Q2. Awais thought of a number, add 3 with it. Then he Double the sum. He got 40. What was the original number

Solution: Suppose the number = x
Add 3 = x + 3
Double the sum = 2 (x + 3)
According to question

$2(x+3)=40$

$2x+6=40$

$2x=40-6$

$2x=34$

$x=\frac{34}{2}$

$x=17$

So the number = 17

Q3. The sum of two numbers is -4 and their difference is 6. What are the numbers.
Sol: Let the first number = x
And the second number = y
From the first set of fact $x+y=-4$ .....(1)
From the second fact of fact $x-y=6$ ....(2)
Adding eq (1) and (2) we get

$x+y=-4$

$x-y=6$

$2x=2$

Or $x=1$ put in (1)

$1+y=-4$

$y=-4-1$

$y=-5$

Therefore first number x =1
Second number y = - 5

Q4. The sum of three consecutive odd integers is 81. Find the numbers.
Solution: consider three consecutive odd no.s
First odd number = x
Second odd number = x + 2
Third odd number = x + 4
According to the given condition

$x+x+2+x+4=81$

$3x+6=81$

$3x=81-6$

$3x=75$

$x=25$

So the first number = 25
Second number = 25 + 2 = 27
Third number = 25 + 4 = 29

Q5. A man is 41 year old and his son is 9 year old. In how many years will the father be three times as old as the son?
Sol: Father's Age = 41 year
Son's Age = 9 year
From the set of fact $3(9+x)=41+x$

$27+3x=41+x$

$3x-x=41-27$

$2x=14$

$x=7$

After 7 year father's age will be 3 time age of son.

Q6. The tens digit of a certain two-digit number exceeds the unit digit by 4 and is 1 less than twice the ones digit. Find the number.
Sol: Digit at unit place = x
Digit at ten place = y
Then two-digit number = $10y+x$
From the first set of fact
$y=x+4$ ....(1)
From the second set of fact
$y=2x-1$ ....(2)
By comparing eq (1) and (2) we get

$2x-1=x+4$

$2x-x=4+1$

$x=5$ put in (1)

$y=5+4$

$y=9$

Therefore two-digit number = $10(9)+5$
= 95

Q7. sum of two digits is 10. If the place of digits are changed then the new number is decreased by 18. Find the numbers
Solution: Suppose the digit at unit place = x
Digit at ten place = y
So the original number = 10 y + x
After interchanging the place of digits
New number = 10x + y
According to the given conditions
Sum of two digits is 10
i.e., $x+y=10 \Rightarrow y=10-x$ ......(1)
New number + 18 = original number

$10x+y+18=10y+x$

$10x+x+y-10y=-18$

$9x-9y=-18$

$x-y=-2$

Putting the value of y = 10 – x

$x-(10-x)=-2$

$x-10+x=-2$

$x+x=-2+10$

$2x=8$

$x=\frac{8}{2}$

$x=4$

So y = 10 – 4 = 6
Original number = 10 y + x
= 10 (6) + 4
= 60 + 4
= 64

Q8. If the breadth of the room is one fourth of its length and the perimeter of the room is 20m. Find length and breadth of the room.
Solution: Suppose the length of the room = x
Breadth of the room = $\frac{x}{4}$

According to the condition
Perimeter = 20 m

$2(L+B)=20m$

$2\left(x+\frac{x}{4}\right)=20m$

$\frac{4}{4}.\frac{x}{1}+\frac{x}{4}=\frac{20m}{2}$

$\frac{5x}{4}=10m$

$x=10m\times\frac{4}{5}$

$x=8m$

So the length of the room = 8 m
Than breadth of the room= $\frac{8m}{4}=2m$

Radical Equation; The equation of the form $\sqrt{ax+b}=c$ is called radical equation;
Note that Variables must have rational exponent Or variable must be radicand.

Example 6: Solve $\sqrt{2x}+5=9$

Sol: Given $\sqrt{2x}+5=9$

$\sqrt{2x}=9-5$

$\sqrt{2x}=4$

Squaring both sides

$\left(\sqrt{2x}\right)^2=(4)^2$

$2x=16$

$x=\frac{16}{2}=8$

Verification put $x=8$ in given radical eq

$\sqrt{2(8)}+5=9$

$\sqrt{16}+5=9$

$4+5=9$

9 = 9 verification

$S.S=\{8\}$

Example 7: Solve $\sqrt{3x-2}=\sqrt{5x+4}$

Sol: Given $\sqrt{3x-2}=\sqrt{5x+4}$

Squaring both sides

$\left(\sqrt{3x-2}\right)^2=\left(\sqrt{5x+4}\right)^2$

$3x-2=5x+4$

$3x-5x=4+2$

$-2x=6$

$x=\frac{6}{-2}=-3$

Verification Put x = - 3 in radical equation

$\sqrt{3(-3)-2}=\sqrt{5(-3)+4}$

$\sqrt{-9-2}=\sqrt{-15+4}$

$\sqrt{-11}=\sqrt{-11}$ verified

$S.S=\{-3\}$

Example 8: Find the solution set of the equation $\sqrt{3x+2}+6=2$

Sol: Given $\sqrt{3x+2}+6=2$

$\sqrt{3x+2}=2-6$

$\sqrt{3x+2}=-4$

Squaring both sides

$\left(\sqrt{3x+2}\right)^2=(-4)^2$

$3x+2=16$

$3x=16-2$

$x=\frac{14}{3}$

Verification put $x=\frac{14}{3}$ in given radical eq

$\sqrt{3\left(\frac{14}{3}\right)+2}+6=2$

$\sqrt{14+2}+6=2$

$\sqrt{16}+6=2$

$4+6=2$

$10\neq 2$ false

Therefore $S.S=\{\ \}$

Exercise 7.2

Solve the following radical equation;

Q1: $2\sqrt{a}-3=7$

Sol: Given $2\sqrt{a}-3=7$

$2\sqrt{a}=7+3$

$2\sqrt{a}=10$

$\sqrt{a}=\frac{10}{2}=5$

Taking square on both sides

$\left(\sqrt{a}\right)^2=(5)^2$

$a=25$

Verification put $a=25$ in the given radical eq

$2\sqrt{25}-3=7$

$2(5)-3=7$

$10-3=7$

$7=7$ True

$S.S=\{25\}$

Q2: $8+3\sqrt{b}=20$

Sol: Given $8+3\sqrt{b}=20$

$3\sqrt{b}=20-8$

$3\sqrt{b}=12$

$\sqrt{b}=4$

Taking square on both sides

$$\left(\sqrt{b}\right)^2=(4)^2$$
$$b=16$$

Verification put $b=16$ in the given radical eq

$$8+3\sqrt{16}=20$$
$$8+3(4)=20$$
$$8+12=20$$
$$20=20 \text{ True}$$
$$S.S=\{16\}$$

Q3: $7-\sqrt{2b}=3$

Sol: Given $7-\sqrt{2b}=3$

$$7-3=\sqrt{2b}$$
$$\sqrt{2b}=4$$

Taking square on both sides

$$\left(\sqrt{2b}\right)^2=(4)^2$$
$$2b=16$$
$$b=\frac{16}{2}=8$$

Verification put $b=8$ in the given radical eq

$$7-\sqrt{2(8)}=3$$
$$7-\sqrt{16}=3$$
$$7-4=3$$
$$3=3 \text{ Ture}$$
$$S.S=\{8\}$$

Q4: $8\sqrt{r}-5=\sqrt{r}+9$

Sol: Given $8\sqrt{r}-5=\sqrt{r}+9$

$$8\sqrt{r}-\sqrt{r}=+9+5$$
$$7\sqrt{r}=14$$
$$\sqrt{r}=\frac{14}{7}=2$$

Taking square on both sides

$$\left(\sqrt{r}\right)^2=(2)^2$$
$$r=4$$

Verification put $r=4$ in the given radical eq

$$8\sqrt{4}-5=\sqrt{4}+9$$
$$8(2)-5=2+9$$
$$16-5=11$$
$$11=11 \text{ True}$$
$$S.S=\{4\}$$

Q5. $20-3\sqrt{t}=\sqrt{t}-4$

Sol: Given $20-3\sqrt{t}=\sqrt{t}-4$

$$20+4=\sqrt{t}+3\sqrt{t}$$
$$24=4\sqrt{t}$$
$$4\sqrt{t}=24$$
$$\sqrt{t}=\tfrac{24}{4}=6$$

Taking square on both sides

$$\left(\sqrt{t}\right)^2=(6)^2$$
$$t=36$$

Verification put $t=36$ in the given radical eq

$$20-3\sqrt{36}=\sqrt{36}-4$$
$$20-3(6)=6-4$$
$$20-18=2$$
$$2=2 \text{ True}$$
$$S.S=\{36\}$$

Q6: $2\sqrt{5x}-3=7$

Sol: Given $2\sqrt{5x}-3=7$

$$2\sqrt{5x}=7+3$$
$$2\sqrt{5x}=10$$
$$\sqrt{5x}=\frac{10}{2}=5$$

Taking square on both sides

$$\left(\sqrt{5x}\right)^2=(5)^2$$
$$5x=25$$
$$x=\frac{25}{5}=5$$

Verification put $x=5$ in the given radical eq

$$2\sqrt{5x}-3=7$$
$$2\left(\sqrt{5(5)}\right)-3=7$$
$$2\sqrt{25}-3=7$$
$$2(5)-3=7$$
$$10-3=7$$
$$7=7 \text{ True}$$
$$S.S=\{5\}$$

Q7: $\sqrt{2x-7}+8=11$

Sol: Given $\sqrt{2x-7}+8=11$

$$\sqrt{2x-7}=11-8$$
$$\sqrt{2x-7}=3$$

Taking square on both sides

$$\left(\sqrt{2x-7}\right)^2=(3)^2$$
$$2x-7=9$$
$$2x=9+7$$
$$x=\frac{16}{2}=8$$

Verification put $x=8$ in the given radical eq

$$\sqrt{2x-7}+8=11$$
$$\sqrt{2(8)-7}+8=11$$
$$\sqrt{16-7}+8=11$$
$$\sqrt{9}+8=11$$
$$3+8=11$$

11 = 11 True

$S.S = \{8\}$

Q8: $22 = 17 + \sqrt{40 - 3y}$

Sol: Given $22 = 17 + \sqrt{40 - 3y}$

$22 - 17 = \sqrt{40 - 3y}$

$5 = \sqrt{40 - 3y}$

Taking square on both sides

$(5)^2 = \left(\sqrt{40 - 3y}\right)^2$

$25 = 40 - 3y$

$3y = 40 - 25$

$y = \frac{15}{3} = 5$

Verification put $y = 5$ in the given radical eq

$22 = 17 + \sqrt{40 - 3(5)}$

$22 = 17 + \sqrt{40 - 15}$

$22 = 17 + \sqrt{25}$

$22 = 17 + 5$

22 = 22 True

$S.S = \{5\}$

Absolute Value:

The absolute of a real number x, is defined as

$$|x| = \begin{cases} x & if \quad x \geq 0 \\ -x & if \quad x < 0 \end{cases}$$

Example 9: solve $|x - 1| = 7$

Sol: Given $|x - 1| = 7$,

Then there are two possibilities

Either

$x - 1 = -7$

$x = -7 + 1$

$x = -6$

or

$x - 1 = 7$

$x = 7 + 1$

$x = 8$

$S.S = \{-6, 8\}$

Example 8: Find the solution set of

$|3x - 5| + 7 = 11$

Sol: Given $|3x - 5| + 7 = 11$

Or $|3x - 5| = 11 - 7$

$|3x - 5| = 4$

There are two possibilities

Either

$3x - 5 = -4$

$3x = -4 + 5$

$3x = 1$

$x = \frac{1}{3}$

or

$3x - 5 = 4$

$3x = 4 + 5$

$3x = 9$

$x = \frac{9}{3} = 3$

$S.S = \left\{\frac{1}{3}, 3\right\}$

Exercise 7.3

Solve for x

Q1. $|x + 3| = 5$

Sol: Given

Then there are two possibilities

Either

$x + 3 = -5$

$x = -5 - 3$

$x = -8$

or

$x + 3 = 5$

$x = 5 - 3$

$x = 2$

$S.S = \{-8, 2\}$

Q2. $|-5x + 1| = 6$

Sol: Given $|-5x + 1| = 6$

Then there are two possibilities

Either

$-5x + 1 = -6$

$-5x = -6 - 1$

$-5x = -7$

$x = \frac{7}{5}$

or

$-5x + 1 = 6$

$-5x = 6 - 1$

$-5x = 5$

$x = -1$

$S.S = \left\{\frac{7}{5}, -1\right\}$

Q3. $\left|\frac{3}{4}x - 8\right| = 1$

Sol: Given $\left|\frac{3}{4}x - 8\right| = 1$

Then there are two possibilities

Either

$\frac{3}{4}x - 8 = -1$

$\frac{3}{4}x = -1 + 8$

$x = 7 \times \frac{4}{3}$

$x = \frac{28}{3}$

or

$\frac{3}{4}x - 8 = 1$

$\frac{3}{4}x = 1 + 8$

$x = 9 \times \frac{4}{3}$

$x = 12$

$S.S = \left\{\frac{28}{3}, 12\right\}$

Q4. $|x - 4| = 3$

Sol: Given $|x - 4| = 3$

Then there are two possibilities

Either

$x - 4 = -3$

$x = -3 + 4$

$x = 1$

or

$x - 4 = 3$

$x = 3 + 4$

$x = 7$

$S.S = \{1, 7\}$

Q5. $|3x + 4| = -2$

Sol: Given $|3x + 4| = -2$ There is no such a number whose absolute value is negative

Therefore $S.S = \{\ \}$

Q6. $|2x-9| = 0$

Sol: Given $|2x-9| = 0$

$2x-9=0$

$2x=9$

$x=\frac{9}{2}$

$S.S=\left\{\frac{9}{2}\right\}$

Q7. $\left|\frac{3x-2}{5}\right| = 7$

Sol: Given $\left|\frac{3x-2}{5}\right| = 7$

Then there are two possibilities

Either	or
$\frac{3x-2}{5}=-7$	$\frac{3x-2}{5}=7$
$3x-2=-7\times 5$	$3x-2=7\times 5$
$3x-2=-35$	$3x-2=35$
$3x=-35+2$	$3x=35+2$
$3x=-33$	$3x=33$
$x=\frac{-33}{3}=-11$	$x=\frac{37}{3}$

$S.S=\left\{-11, \frac{37}{3}\right\}$

Q8. $4|5x-2|+3=11$

Sol: Given $4|5x-2|+3=11$

$4|5x-2|=11-3$

$|5x-2|=\frac{8}{4}=2$

Then there are two possibilities

Either	or
$5x-2=-2$	$5x-2=2$
$5x=-2+2$	$5x=2+2$
$5x=0$	$5x=4$
$x=\frac{0}{5}$	$x=\frac{4}{5}$
$x=0$	

$S.S=\left\{0, \frac{4}{5}\right\}$

Q9. $\frac{2}{5}|4x-3|-9=-1$

Sol: Given $\frac{2}{5}|4x-3|-9=-1$

$\frac{2}{5}|4x-3|=-1+9$

$\frac{2}{5}|4x-3|=8$

$|4x-3|=8\times\frac{5}{2}$

$|4x-3|=20$

Then there are two possibilities

Either	or
$4x-3=-20$	$4x-3=20$
$4x=-20+3$	$4x=20+3$
$4x=-17$	$4x=23$
$x=\frac{-17}{4}$	$x=\frac{23}{4}$

$S.S=\left\{\frac{-17}{4}, \frac{23}{4}\right\}$

Linear inequality in one variable

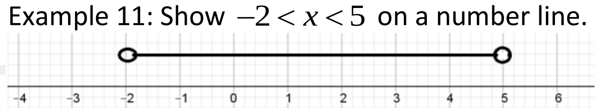
A linear inequality in one variable say x is an inequality in one of the following forms can be converted into the following forms:

$ax+b>c$ $\quad ax+b<c$

$ax+b\geq c$ $\quad ax+b\leq c$

Where a, b, c are any real numbers with $a\neq 0$ and x is also a real variable.

Example 11: Show $-2<x<5$ on a number line.



Properties of inequality of Real numbers:

1.Trichotomy Property

For any $a,b\in R$ either $a<b$ or $a=b$ or $b<a$

Either $a<b$ is also written as $b>a$

2.Transitive property

For all $a,b,c\in R$, $a<b$ and $b<c \Rightarrow a<c$

For all $a,b,c\in R$, $a>b$ and $b>c \Rightarrow a>c$

4.Additive property

(1). For all $a,b,c\in R$, $a<b \Rightarrow a+c<b+c$

For all $a,b,c\in R$, $a<b \Rightarrow c+a<c+b$

(2).For all $a,b,c\in R$, $a>b \Rightarrow a+c>b+c$

$\forall\ a,b,c\in R$, $a>b$ and $\Rightarrow c+a>c+b$

5.Multiplicative Property

(1). $\forall\ a,b,c\in R, c>0$, $a<b \Rightarrow a.c<b.c$

For all $a,b,c\in R, c>0$, $a>b \Rightarrow a.c>b.c$

(2) $\forall\ a,b,c\in R, c<0$, $a<b \Rightarrow a.c>b.c$

For all $a,b,c\in R, c<0$, $a>b \Rightarrow a.c<b.c$

Example 12: write the name of properties used in following statements.

i). $21<31 \Rightarrow 31<41$

Additive property

ii). $15>8 \Rightarrow 22>15$

Additive property

iii). 10 < 20 $\Rightarrow$ 30 < 60

multiplicative property

iv). - 12 > - 15 $\Rightarrow$ 24 < 30

multiplicative property

v). if x > 4 & 4 > z then x > z

multiplicative property

Example 13: you are checking a bag at an airport. Bags can weight no more than 50 kg. your bag weight 16.8 kg. Find the possible weight w that you can add to the bag.

Sol; Form the set of fact

Weight of bag + weight you can add $\leq$ weight limit

16.8 + w $\leq$ 50

W $\leq$ 50 - 16.8

W $\leq$ 33.2

You can add no more than 33.2 kg

Example 14:i). solve the inequality

$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$ where x is a natural number

Sol: We have $2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

$\frac{x}{2}+2<\frac{3}{2}$ Multiply by 2

$x+4<3$

$x<-1$

There is no natural number which is less than – 1

$S.S=\{\ \}$

Example 14:ii). solve the inequality

$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$ where x is a real number

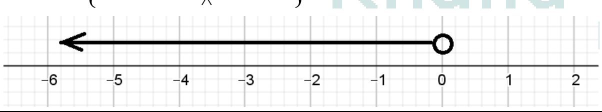
Sol: We have $2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

$\frac{x}{2}+2<\frac{3}{2}$ Multiply by 2

$x+4<3$

$x<-1$

$S.S=\{x:x\in R_{\wedge}x<-1\}$



Example 15i): Solve the inequality

$x-\frac{5}{7}\leq\frac{15+2x}{7}$ where x is an natural number

Sol: Given $x-\frac{5}{7}\leq\frac{15+2x}{7}$ Multiply by 7

$7x-5\leq 15+2x$

$7x-2x\leq 15+5$

$5x\leq 20$

$x\leq\frac{20}{5}$

$x\leq 4$

$S.S=\{1,2,3,4\}$

Example 15ii): Solve the inequality

$x-\frac{5}{7}\leq\frac{15+2x}{7}$ where x is an real number

Sol: Given $x-\frac{5}{7}\leq\frac{15+2x}{7}$ Multiply by 7

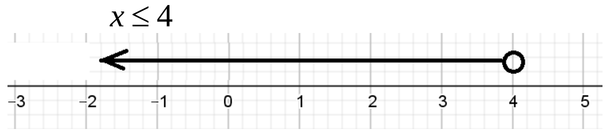
$7x-5\leq 15+2x$

$7x-2x\leq 15+5$

$5x\leq 20$

$x\leq\frac{20}{5}$

$x\leq 4$



$S.S=\{x:x\in R_{\wedge}x\leq 4\}$

Example 16: Solve the inequality $\frac{x+3}{2}\leq\frac{x-5}{3}$ where $x\in R$

Sol: Given $\frac{x+3}{2}\leq\frac{x-5}{3}$ multiply by 6

$6\left(\frac{x+3}{2}\right)\leq 6\left(\frac{x-5}{3}\right)$

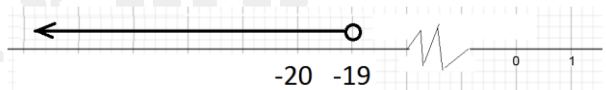
$3(x+3)\leq 2(x-5)$

$3x+9\leq 2x-10$

$3x-2x\leq -10-9$

$x\leq -19$

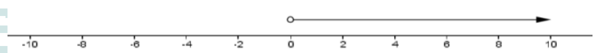
$S.S=\{x:x\in R_{\wedge}x\leq -19\}$



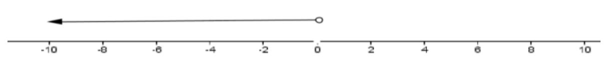
Exercise 7.4

Q1: Show the following inequalities on number line.

i). $x>0$



ii). $x<0$



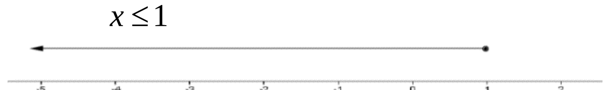
iii). $\frac{x-3}{2}\leq -1$

Sol: given $\frac{x-3}{2}\leq -1$

$x-3\leq -2$

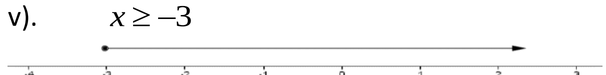
$x\leq -2+3$

$x\leq 1$



iv). $x\leq -5$

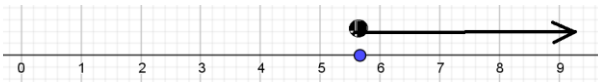
v). $x\geq -3$



vi). $\frac{3x-2}{6}>\frac{5}{2}$

Sol: given $\frac{3x-2}{6}>\frac{5}{2}$

$$3x-2>\frac{5}{2}\times 6$$
$$3x-2>15$$
$$3x>15+2$$
$$3x>17$$
$$x>\frac{17}{3}$$
$$x>5.67$$



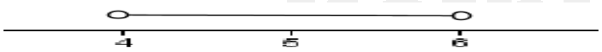
vii). $-5\le x\le 6$

viii). $3\ge x\ge -2$

ix). $0<\frac{x}{4}-1<\frac{1}{2}$

Sol: given $0<\frac{x}{4}-1<\frac{1}{2}$ multiply by 4

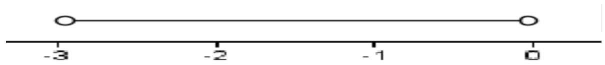
$$0<x-4<2$$
$$0+4<x<2+4$$
$$4<x<6$$



x). $0<\frac{x+3}{2}<\frac{3}{2}$

Sol: Given $0<\frac{x+3}{2}<\frac{3}{2}$ multiply by 2

$$0<x+3<3$$
$$0-3<x<3-3$$
$$-3<x<0$$



Q2. Find the solution set of the following inequalities.

i). $7-2x\ge 1, x\in N$

Sol: Given $7-2x\ge 1, x\in N$

$$7-1\ge 2x$$
$$6\ge 2x$$
$$2x\le 6$$
$$x\le 3$$
$$S.S=\{1,2,3\}$$

ii). $5x+4<34, x\in N$

Sol: Given $5x+4<34, x\in N$

$$5x<34-4$$
$$5x<30$$
$$x<6$$
$$S.S=\{1,2,3,4,5\}$$

iii). $\frac{8x+1}{2}<2x-1.5, x\in R$

Sol: Given $\frac{8x+1}{2}<2x-1.5, x\in R$

Multiply by 2

$$8x+1<2(2x-1.5)$$
$$8x+1<4x-3$$
$$8x-4x<-3-1$$
$$4x<-4$$
$$x<-1$$
$$S.S=\{x: x\in R_\wedge x<-1\}$$

iv). $(4x+3)\ge 23, x\in\{1,2,3,4,5,6\}$

Sol: Given $(4x+3)\ge 23, x\in\{1,2,3,4,5,6\}$

$$4x\ge 23-3$$
$$4x\ge 20$$
$$x\ge 5$$
$$S.S=\{5,6\}$$

v). $5x+1\ge 13-x, x\in\{-2,-1,0,1,2,3,4,5\}$

Sol: Given $5x+1\ge 13-x, x\in\{-2,-1,0,1,2,3,4,5\}$

$$5x+1\ge 13-x$$
$$5x+x\ge 13-1$$
$$6x\ge 12$$
$$x\ge 2$$
$$S.S=\{2,3,4,5\}$$

vi). $\frac{2x+6}{2}\le\frac{x-9}{3}, x\in R$

Sol: Given $\frac{2x+6}{2}\le\frac{x-9}{3}, x\in R$ Multiply by 6

$$6\left(\frac{2x+6}{2}\right)\le 6\left(\frac{x-9}{3}\right)$$
$$3(2x+6)\le 2(x-9)$$
$$6x+18\le 2x-18$$
$$6x-2x\le -18-18$$
$$4x\le -36$$
$$x\le -9$$
$$S.S=\{x: x\in R_\wedge x\le -9\}$$

vii). $\frac{x-1}{3}\le\frac{1-x}{2}, x\in z$

Sol: Given $\frac{x-1}{3}\le\frac{1-x}{2}, x\in z$ Multiply by 6

$$6\left(\frac{x-1}{3}\right)\le 6\left(\frac{1-x}{2}\right)$$
$$2(x-1)\le 3(1-x)$$
$$2x-2\le 3-3x$$
$$2x+3x\le 3+2$$
$$5x\le 5$$
$$x\le 1$$
$$S.S=\{1,0,-1,-2,\ldots\}$$

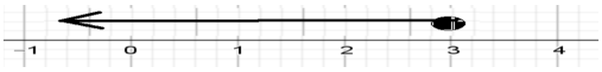
Q3. Solve the following inequalities and plot the solution on the number line.

i). $\frac{x}{12}\le\frac{1}{4}$

Sol: Given $\frac{x}{12}\le\frac{1}{4}$ multiply by 12

$$12\left(\frac{x}{12}\right)\le 12\left(\frac{1}{4}\right)$$

$$x\le 3$$



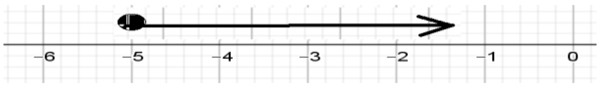
$S.S=\{x: x\in R_{\wedge} x\le 3\}$

ii). $x+7\ge 2$

Sol: Given $x+7\ge 2$

$$x\ge 2-7$$

$$x\ge -5$$



$S.S=\{x: x\in R_{\wedge} x\ge -5\}$

iii). $3(x-2)>15$

Sol: Given $3(x-2)>15$

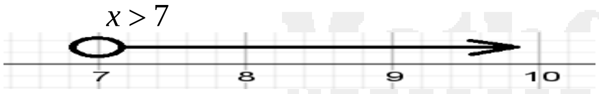
$$3x-6>15$$

$$3x>15+6$$

$$3x>21$$

$$x>\frac{21}{3}$$

$$x>7$$



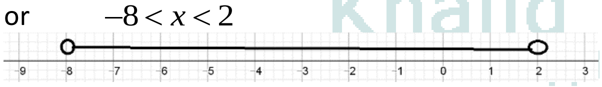
$S.S=\{x: x\in R_{\wedge} x>7\}$

iv). $\frac{1}{2}>\frac{x}{4}>-2$

Sol: Given $\frac{1}{2}>\frac{x}{4}>-2$ Multiply by 4

$$2>x>-8$$

or $-8<x<2$



$S.S=\{x: x\in R_{\wedge} -8<x<2\}$

v). $2.5\le \frac{x}{2}+1\le 4.5$

Sol: Given $2.5\le \frac{x}{2}+1\le 4.5$ Multiply by 2

Sol: Given $2.5\le \frac{x}{2}+1\le 4.5$

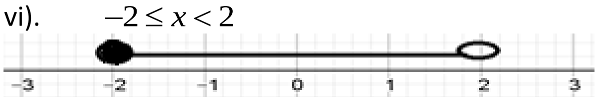
$5\le x+2\le 9$

$5-2\le x\le 9-2$

Or $3\le x\le 7$

$S.S=\{x: x\in R_{\wedge} 3\le x\le 7\}$

vi). $-2\le x<2$



$S.S=\{x: x\in R_{\wedge} -2\le x<2\}$

Review Exercise 7

Q1. Select the correct answer.

i). solve for x: $\frac{1}{2}|x-6|-4=-1$

a). $x=12$ b). $x=8$ & $x=4$

c). $x=12$ & $x=0$ d). there is no solution

ii). Solve for x: $|3x-1|=2$

a). $x=1$ & $x=\frac{1}{3}$ b). $x=1$ & $x=\frac{-1}{3}$

c). $x=1$ & $x=-1$ d). $x=1$

iii). The solution set of $\sqrt{5x+3}+2=4$ is

a). $\left\{\frac{1}{5}\right\}$ b). $\left\{\frac{-1}{5}\right\}$

c). $\{2\}$ d). $\{1\}$

iv). Solve for x, $\sqrt{x}=-10$

a). $\{-10\}$ b). $\{\ \}$

c). $\{100\}$ d). $\{10\}$

v). $\sqrt{2x+1}-5=4$ is a

a). linear equation b). Radical equation

c). Cubic equation d). Quadratic equation

vi). What is the solution for $|x-7|=1$?

a). $x=8$ b). $x=6$ & $x=8$

c). $x=8$ & $x=$-8 d). $x=6$

vii). The solution set of $\sqrt{5x+3}+2=4$ is

a). $\left\{\frac{1}{5}\right\}$ b). $\left\{\frac{-1}{5}\right\}$

c). $\{2\}$ d). $\{1\}$

viii). The solution set $\left|\frac{5x}{3}\right|=5$ is

a). $\{3\}$ b). $\{5,-5\}$

c). $\{4,-4\}$ d). $\{3,-3\}$

ix). Which one is the solution set of $|-x|=0$

a). $\{-1\}$ b). $\{1\}$

c). $\{\ \}$ d). $\{0\}$

x). solve for x; $\frac{x+2}{x-2}>0$

a). $(-2,\infty)$ b). $(-2,2)$

c). $(-2,\infty)\cup(2,\infty)$ d). $(-\infty,-2)\cup(2,\infty)$

Q2. Solve the following equation for x

i). $5(3x+1)=2(x-4)$

Sol: Given $5(3x+1)=2(x-4)$

$$15x+5=2x-8$$

$$15x-2x=-8-5$$

$$13x=-13$$

$$x=-1$$

$S.S=\{-1\}$

ii). $\frac{x-8}{3}+\frac{x-3}{2}=0$

Sol: Given $\frac{x-8}{3}+\frac{x-3}{2}=0$ Multiply by 6

$$6\left(\frac{x-8}{3}\right)+6\left(\frac{x-3}{2}\right)=6\times 0$$
$$2(x-8)+3(x-3)=0$$
$$2x-16+3x-9=0$$
$$2x+3x=16+9$$
$$5x=25$$
$$x=5$$
$$S.S=\{5\}$$

iii). $\sqrt{2(5x-1)}=\sqrt{2x+14}$

Sol: Given $\sqrt{2(5x-1)}=\sqrt{2x+14}$

Squaring both sides

$$\left(\sqrt{2(5x-1)}\right)^2=\left(\sqrt{2x+14}\right)^2$$
$$2(5x-1)=2x+14$$
$$10x-2=2x+14$$
$$10x-2x=14+2$$
$$8x=16$$
$$x=2$$

For radical equation verification

$$\sqrt{2(5(2)-1)}=\sqrt{2(2)+14}$$
$$\sqrt{2(10-1)}=\sqrt{4+14}$$
$$\sqrt{2\times 9}=\sqrt{18}$$
$$3\sqrt{2}=\sqrt{9\times 2}$$
$$3\sqrt{2}=3\sqrt{2}$$

Therefore $S.S=\{2\}$

iv). $|2x+7|=9$

Sol: Given $|2x+7|=9$

There are two possibilities

Either	Or
$2x+7=-9$	$2x+7=9$
$2x=-9-7$	$2x=9-7$
$2x=-16$	$2x=2$
$x=-8$	$x=1$

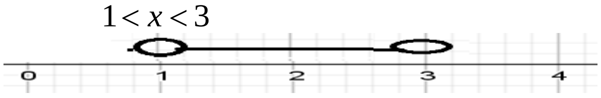
$$S.S=\{-8,1\}$$

Q3. Solve the following inequalities and graph the solution on the number line.

i). $-1<\frac{x-3}{2}<0$

Sol: Given $-1<\frac{x-3}{2}<0$ Multiply by 2

$$2(-1)<2\left(\frac{x-3}{2}\right)<2(0)$$
$$-2<x-3<0$$
$$-2+3<x<3$$
$$1<x<3$$

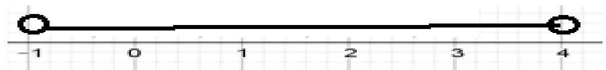


$$S.S=\{x : x\in R_{\wedge} 1<x<3\}$$

ii). $-1<\frac{x-4}{5}<0$

Sol: Given $-1<\frac{x-4}{5}<0$ Multiply by 5

$$5(-1)<5\left(\frac{x-4}{5}\right)<5\times 0$$
$$-5<x-4<0$$
$$-5+4<x<0+4$$
$$-1<x<4$$



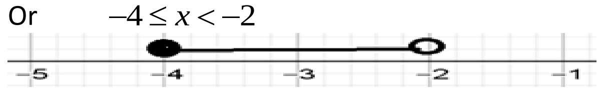
$$S.S=\{x : x\in R_{\wedge} -1<x<4\}$$

iii). $7<-3x+1\le 13$

Sol: Given $7<-3x+1\le 13$

$$7-1<-3x\le 13-1$$
$6<-3x\le 12$ divided by -3
$$-2>x\ge -4$$

Or $-4\le x<-2$



$$S.S=\{x : x\in R_{\wedge} -4\le x<-2\}$$

Q4. A father is 4 times older than his son. In 20 year he will be twice as old as his son. What ages have they now?

Sol: Let age of son = x

So, Age of father = $4x$

From the set of fact in question

$$4x+20=2(x+20)$$
$$4x+20=2x+40$$
$$4x-2x=40-20$$
$$2x=20$$
$$x=10$$

Therefore age of son = 10 year

And age of father = $4(10)=40$ year