Chapter 2

System of Real numbers

Terminating Decimal fraction:
The decimal fraction in which Given **finite numbers of digits** in its decimal part is called a **terminating decimal fraction.**

Recurring (non-terminating)Decimal fraction:
The decimal fraction (Non terminating) in which **some digits are repeated** again and again in the **same order** in its **decimal part**, is called a recurring decimal fraction.

Non-recurring Non-terminating Decimal fraction:
The non-terminating decimal fraction in which the **digits are not repeated in the same order** in its **decimal part** is called a non-terminating and non-recurring fraction.

Rational numbers: A number which can be expressed in the form of $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$ are called rational number.

Irrational numbers: A number which can not be expressed in the form of $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$ are called irrational number.

Real numbers:
Union of set of rational and irrational numbers is called the set of real numbers.

Exercise # 2.1

Consider the numbers

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Q1. From the following which are whole numbers?

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Solution: whole numbers are
2, 5, 3, 96, $\sqrt{36}$, 1,

Q2. From the following which are integers?

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Solution: integers are
3, $\sqrt{36}$, , -9, 1,

Q3. From following which are irrational numbers?

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Solution: irrational numbers are
$\sqrt{3}$, $\sqrt{7}$, $-\sqrt{14}$, π

Q4. From following which are natural number?

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Solution: natural numbers are
3, $\sqrt{36}$, 1,

Q5. From the following which are rational number.

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Solution: rational number are 2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, -9, 1, $4\frac{2}{3}$, 0.333...

Q6.From the following Which are real numbers

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Solution: real number 2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Q7. From the following which are rational numbers but not an integers?

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Solution: rational numbers but not an integers
$\frac{5}{7}$, $\frac{-7}{6}$, $4\frac{2}{3}$, 0.333...

Q8. From the following which are integer but not whole numbers

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Sol: Integer but not whole numbers -9

Q9. Form the following which are integers but not a natural numbers?

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Sol: Integer but not natural numbers -9

Q10. Which are real numbers but not a integers?

2.5, 3, $\frac{5}{7}$, - 1.96, $\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, -9, 1, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Sol: Real numbers but not an integers 2.5, $\frac{5}{7}$, -1.96

$\sqrt{36}$, $\frac{-7}{6}$, $\sqrt{3}$, $\sqrt{7}$, $-\sqrt{14}$, π, $4\frac{2}{3}$, 0.333...

Q11. Write the decimal representation of each of the following numbers. $\frac{1}{6},\frac{6}{7},\frac{2}{9},\frac{1}{8}$

Solution Given $\frac{1}{6},\frac{6}{7},\frac{2}{9},\frac{1}{8}$

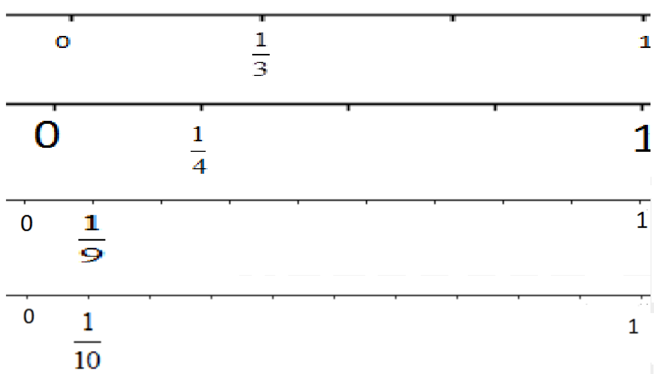
$\frac{1}{6}=0.1666\cdots=0.1\overline{6}$

$\frac{6}{7}=0.857142857142\cdots=0.\overline{857142}$

$\frac{2}{9}=0.222\cdots=0.\overline{2}$

$\frac{1}{8}=0.125$

Q4. Depict each number on a number line.



Properties of Real Numbers:-

Closure Property:-

(1). W.r.t. Addition

For each $a,b\in R$, $a+b\in R$

(2). W.r.t. Multiplication

For each $a,b\in R$, $a.b\in R$

Commutative property:-

(1). W.r.t. Addition

For each $a,b\in R$, $a+b=b+a\in R$

(2). W.r.t. Multiplication

For each $a,b\in R$, $a.b=b.a\in R$

Associative property:

(1). W.r.t. Addition

For each $a,b,c\in R$, $(a+b)+c=a+(b+c)$

(2). W.r.t. Multiplication

For each $a,b,c\in R$, $(a.b).c=a.(b.c)$

Identity:

(1). W.r.t. Addition

There exists $0\in R$ such that

$a+0=0+a=a$

(2). W.r.t. Multiplication

There exists $1\in R$ such that $a.1=1.a=a$

Inverse:

(1). W.r.t. Addition

For each $a\in R$ then there exists an element m $(-a)\in R$ such that

$a+(-a)=(-a)+a=0$

(2). W.r.t. Multiplication

For each $a\in R$ then there exists an element $\frac{1}{a}\in R$ such that

$a.(\frac{1}{a})=(\frac{1}{a}).a=1$

Distributive property:

(1). Multiplication over Addition

For all $a,b,c\in R$, $a(b+c)=a.b+a.c$

(2). Addition over Multiplication

For all $a,b,c\in R$, $(a+b)c=a.c+b.c$

Properties of equality of Real numbers:

Reflexive Property

For all $a\in R$, $a=a$

Symmetric Property

For all $a,b\in R$, $a=b\Leftrightarrow b=a$

Transitive property

For all $a,b,c\in R$, $a=b$ and $b=c\Rightarrow a=c$

Additive Property

For all $a,b,c\in R$, $a=b$ and $\Rightarrow a+c=b+c$

For all $a,b,c\in R$, $a=b$ and $\Rightarrow c+a=c+b$

Multiplicative Property

For all $a,b,c\in R$, $a=b$ and $\Rightarrow a.c=b.c$

For all $a,b,c\in R$, $a=b$ and $\Rightarrow c.a=c.b$

Cancellation Property

(1). W.r.t. Addition For all $a,b,c\in R$, $a+c=b+c$ and $\Rightarrow a=b$

For all $a,b,c\in R$, $c+a=c+b$ and $\Rightarrow a=b$

(2). W.r.t. Multiplication For all $a,b,c\in R$, $a.c=b.c$ and $\Rightarrow a=b$

For all $a,b,c\in R$, $c.a=c.b$ and $\Rightarrow a=b$

Properties of inequality of real numbers

Trichotomy Property

For any $a,b\in R$ either $a<b$ or $a=b$ or $b<a$

Either $a<b$ is also written as $b>a$

Archimidean property

For all $a,b\in R$ $n>0$ then there exists a natural number n such that $na>b$

Transitive property

For all $a,b,c\in R$, $a<b$ and $b<c\Rightarrow a<c$

For all $a,b,c\in R$, $a>b$ and $b>c\Rightarrow a>c$

Additive property

(1). For all $a,b,c\in R$, $a<b$ a$\Rightarrow a+c<b+c$

For all $a,b,c\in R$, $a<b$ an$\Rightarrow c+a<c+b$

(2).For all $a,b,c\in R$, $a>b$ a$\Rightarrow a+c>b+c$

For all $a,b,c \in R$, $a>b$ and $\Rightarrow c+a>c+b$

Multiplicative Property

(1).For all $a,b,c \in R, c>0$, $a<b$ and $\Rightarrow a.c<b.c$

For all $a,b,c \in R, c>0$, $a>b$ and $\Rightarrow a.c>b.c$

(2) For all $a,b,c \in R, c<0$, $a<b$ and $\Rightarrow a.c>b.c$

For all $a,b,c \in R, c<0$, $a>b$ and $\Rightarrow a.c<b.c$

Inequality multiplicative inverse

For all If $a \neq 0, b \neq 0$

(1) $a<b \Rightarrow \frac{1}{a}>\frac{1}{b}$

(2). $a>b \Rightarrow \frac{1}{a}<\frac{1}{b}$

Exercise 2.2

Q1. Write name of properties used in following equations. Latter a,b represents real numbers.

i). 1+(4+3)=(1+4)+3

Associative law of addition

ii). 5(a+b)=5a+5b

Distributive Law of multiplication over addition

iii). a+0=0+a=a

Existence of additive identity

iv). $5\times\frac{1}{5}=\frac{1}{5}\times5=1$

Existence of multiplicative inverse

Q2. Write the missing number.

i). $2+(\_\_+4)=(2+6)+4$

Answer missing number is 6

ii). $7+(4+2)=13$, so $(7+4)+2=$ ....

Answer missing number is 13

iii). $9\times(3\times4)=108$, so $(9\times3)\times4=$.......

Answer missing number is 108

iv). $5\times(8\times9)=(5\times\_\_)\times9$

Answer missing number is 8

Q3. Choose the correct option.

i). $8\times(6\times7)=$................

a). $8\times6-7$ b). $8-(6-7)$

c). 8×12 d). $(8\times6)\times7$

ii). In which of the following illustrates the associative law of addition?

a). $3+(2+4)=(4+4)+1$

b). $3+(2+4)=(3+2)+4$

c). $3+(2+4)=(5+2)+2$

d). $3+(2+4)=(2+6)+1$

iii). Which of the following illustrates the associative law of multiplication?

a). $4\times(3\times6)=(6\times6)\times2$

b). $4\times(3\times6)=(3\times12)\times2$

c). $4\times(3\times6)=(4\times3)\times6$

d). $4\times(3\times6)=(3\times8)\times3$

Q4. Do this without using distributive property.

i). $39\times63+39\times37$

Solution: Given $39\times63+39\times37$
$=2457+1443$
$=3900$

ii) $81\times450+81\times550$

Solution: Given $81\times450+81\times550$
$=36450+44550$
$=81000$

iii). $50\times161-50\times81$

Solution: Given $50\times161-50\times81$
$=8050-4050$
$=4000$

iv). $827\times60-327\times60$

Solution: Given $827\times60-327\times60$
$=49620-19620$
$=30000$

Exponent or index: If a real number x is multiplied with itself 4 times, then the product will be x^5. Similarly, the product of a real number x with itself n times will be written as $x^{n+1}=x.x.x.....x$
(n times multiplication of x with itself)

Radicand: In $\sqrt{x}$, x is called the radicand.

Or Index:

In $\sqrt[q]{x}$, q is the index.

Or Index Or Exponent:

In x^n, n is called exponent or index.

Base: In x^n, x is called Base & n is exponent

Exercise # 2.3

Q1. Write down the index and radicand for each of the following expressions.

i). $\sqrt{\frac{11}{y}}$

Answer: Here index 2 Radicand $\frac{11}{y}$

ii). $\sqrt[3]{\frac{13}{3x}}$

Answer: Here index 3 Radicand $\frac{13}{3x}$

iii). $\sqrt[5]{ab^2}$

Answer: Here index 5 Radicand ab^2

Q2. Transform the following radical forms into exponential forms. Do not simplify

i). $\sqrt{36}$

Exponential form = $(36)^{\frac{1}{2}}$

ii). $\sqrt{1000}$

Exponential form = $(1000)^{\frac{1}{2}}$

iii). $\sqrt[3]{8}$

Exponential form = $(8)^{\frac{1}{3}}$

iv). $\sqrt[n]{q}$

Exponential form = $(q)^{\frac{1}{n}}$

v). $\sqrt{(5-6a^2)^3}$

Exponential form = $(5-6a^2)^{\frac{3}{2}}$

vi). $\sqrt[3]{-64}$

Exponential form = $(-64)^{\frac{1}{3}}$

Q3. Transform the following exponential form of an expression into radical form

i). $-7^{\frac{1}{3}}$

radical form = $-\sqrt[3]{7}$

ii). $x^{\frac{-3}{2}}$

radical form = $\sqrt{x^{-3}}$

iii). $(-8)^{\frac{1}{5}}$

radical form = $\sqrt[5]{-8}$

iv). $y^{\frac{3}{4}}$

radical form = $\sqrt[4]{y^3}$

v). $b^{\frac{4}{5}}$

radical form = $\sqrt[5]{b^4}$

vi). $(3x)^{\frac{1}{q}}$

radical form = $\sqrt[q]{3x}$

Q4. Simplify

i). $\sqrt[3]{125x}$

solution Given $\sqrt[3]{125x}$

$$=\sqrt[3]{125}\times\sqrt[3]{x}$$
$$=5^{\frac{3}{3}}\times\sqrt[3]{x}$$
$$=5\sqrt[3]{x}$$

ii). $\sqrt[3]{\frac{8}{27}}$

solution Given $\sqrt[3]{\frac{8}{27}}$

$$=\sqrt[3]{\frac{2^3}{3^3}}$$
$$=\frac{2^{\frac{3}{3}}}{3^{\frac{3}{3}}}$$
$$=\frac{2}{3}$$

iii). $\sqrt{\frac{625x^3y^4}{25xy^2}}$

solution Given $\sqrt{\frac{625x^3y^4}{25xy^2}}$

$$\sqrt{\frac{625x^3y^4}{25xy^2}}$$
$$=\sqrt{25x^2y^2}$$
$$=\sqrt{5^2x^2y^2}$$
$$=5^{\frac{2}{2}}x^{\frac{2}{2}}y^{\frac{2}{2}}=5xy$$

iv). $\sqrt{(3y-5)^2}$

solution Given $\sqrt{(3y-5)^2}$

$$=(3y-5)^{\frac{2}{2}}$$
$$=(3y-5)$$

v). $6\sqrt{18}$

solution Given $6\sqrt{18}$

$$=6\sqrt{9\times 2}$$
$$=6\sqrt{9}\sqrt{2}$$
$$=6\times 3\sqrt{2}$$
$$=18\sqrt{2}$$

vi). $\sqrt[3]{54x^3y^3z^2}$

solution Given $\sqrt[3]{54x^3y^3z^2}$

$$=\sqrt[3]{27\times 2x^3y^3z^2}$$
$$=\sqrt[3]{3^3x^3y^3\times 2z^2}$$
$$=3^{\frac{3}{3}}x^{\frac{3}{3}}y^{\frac{3}{3}}\sqrt[3]{2z^2}$$
$$=3xy\sqrt[3]{2z^2}$$

Laws of Exponents:

1. Sum of powers:
   $x^m\times x^n=x^{m+n}$
2. power of a product
   $(x.y)^n=x^n.y^n$
3. power of power
   $(x^m)^n=x^{m\times n}=x^{mn}$
4. quotient of powers with same base:
   $\frac{x^m}{x^n}=x^m.x^{-n}=x^{m-n}$ where $m>n$
   $\frac{x^m}{x^n}=\frac{1}{x^n.x^{-m}}=\frac{1}{x^{n-m}}$ where $m<n$
   If $m=n$, $\frac{x^m}{x^m}=x^m.x^{-m}=x^{m-m}=x^0=1$
5. Power of fraction
   $\left(\frac{x}{y}\right)^n=\frac{x^n}{y^n}$

6. Rational exponents

If a is any positive natural number $m,n\in N$ and $n>1, m>1$, then $a^{\frac{m}{n}}$ means nth root of a^m and is defined as $a^{\frac{m}{n}}=\left(a^m\right)^{\frac{1}{n}}=\sqrt[n]{a^m}$ or

$$a^{\frac{m}{n}}=\left(a^{\frac{1}{n}}\right)^m=\left(\sqrt[n]{a}\right)^m$$

1. $x^{\frac{m}{n}}.x^{\frac{p}{q}}=x^{\frac{m}{n}+\frac{p}{q}}$

2. $\left(x^{\frac{m}{n}}\right)^{\frac{p}{q}}=x^{\frac{m}{n}\times\frac{p}{q}}=x^{\frac{mp}{nq}}$

3. $(x.y)^{\frac{m}{n}}=x^{\frac{m}{n}}.y^{\frac{m}{n}}$

4. $\left(\frac{x}{y}\right)^{\frac{m}{n}}=\frac{x^{\frac{m}{n}}}{y^{\frac{m}{n}}}$, $y\neq 0$

5. $\frac{x^{\frac{m}{n}}}{x^{\frac{p}{q}}}=x^{\frac{m}{n}-\frac{p}{q}}$, $x\neq 0$

6. $x^{-\frac{p}{q}}=\frac{1}{x^{\frac{p}{q}}}$, $x\neq 0$

Exercise # 2.4

Q1. Write base, exponent and the value of the following.

i). $(2)^{-9}=\frac{1}{1024}$

Base = 2 exponent = - 9 value = $\frac{1}{1024}$

ii). $\left(\frac{a}{b}\right)^p=\frac{a^p}{b^p}$

Base = $\frac{a}{b}$ exponent = p value = $\frac{a^p}{b^p}$

iii). $(-4)^2=16$

Base = - 4 exponent = 2 value = 16

Q2. If a,b denote the real numbers then simplify the following.

i). $a^3\times a^5$

Solution: Given $a^3\times a^5$

$=a^{3+5}$

$=a^8$

ii). $\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{\frac{-2}{3}}$

Solution: Given $\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{\frac{-2}{3}}$

$=\left(\frac{b}{a}\right)^{\frac{3}{2}-\frac{2}{3}}$

$=\left(\frac{b}{a}\right)^{\frac{9-4}{6}}$

$=\left(\frac{b}{a}\right)^{\frac{5}{6}}$

iii). $(-a)^4\times(-a)^3$

Solution: Given $(-a)^4\times(-a)^3$

$=(-a)^{4+3}=(-a)^7$

$=-a^7$

iv). $\left(-2a^2b^3\right)^3$

Solution: Given $\left(-2a^2b^3\right)^3$

$=(-2)^3a^{2\times3}b^{3\times3}$

$=-8a^6b^9$

v) $a^3(-2b)^2$

Solution: Given $a^3(-2b)^2$

$=a^3\times 4b^2$

$=4a^3b^2$

vi). $\left(a^2b\right)\left(a^2b\right)$

Solution: Given $\left(a^2b\right)\left(a^2b\right)$

$=a^{2+2}b^{1+1}$

$=a^4b^2$

vii). $\frac{a^0b^0}{2}$

Solution: Given $\frac{a^0b^0}{2}$

$=\frac{1}{2}$ $\therefore x^0=0$ $\frac{a^0b^0}{2}$

viii). $\left(-3a^2b^2\right)^2$

Solution: Given $\left(-3a^2b^2\right)^2$

$=(-3)^2a^{2\times2}b^{2\times2}$

$=9a^4b^4$

xi). $\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

Solution: Given $\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

$=\frac{a^{2\times\frac{3}{2}}}{b^{4\times\frac{3}{2}}}$

$=\frac{a^3}{b^6}$

Q3. Simplify the following

i). $\frac{7^6}{7^4}$

Solution: Given $\frac{7^6}{7^4}$

$$= 7^{6-4}$$
$$= 7^{2}$$

ii). $\dfrac{2^4.5^3}{10^2}$

Solution: Given $\dfrac{2^4.5^3}{10^2}$

$$= \frac{2^4.5^3}{(2\times 5)^2}$$
$$= \frac{2^4.5^3}{2^2.5^2}$$
$$= 2^{4-2}.5^{3-2}$$
$$= 2^2.5$$
$$= 4.5$$
$$= 20$$

iii). $\left\{\dfrac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

Solution: Given $\left\{\dfrac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

$$= \left\{(a+b)^{2-1}.(c+d)^{3-2}\right\}^3$$
$$= (a+b)^3.(c+d)^3$$

iv). $\left(\sqrt[3]{a}\right)^{\frac{1}{2}}$

Solution: Given $\left(\sqrt[3]{a}\right)^{\frac{1}{2}}$

$$= \left(\sqrt[3]{a}\right)^{\frac{1}{2}} = a^{\frac{1}{3}\times\frac{1}{2}}$$
$$= a^{\frac{1}{6}}$$

v). $\sqrt[5]{x^5}.\sqrt[4]{x^4}$

Solution: Given $\sqrt[5]{x^5}.\sqrt[4]{x^4}$

$$= x^{\frac{5}{5}}.x^{\frac{4}{4}}$$
$$= x^1.x^1$$
$$= x^2$$

Q4. Simplify the following in such away that answer should not contain fractional or negative exponents.

i). $\left(\dfrac{25}{81}\right)^{\frac{1}{2}}$

Solution: Given $\left(\dfrac{25}{81}\right)^{\frac{1}{2}}$

$$= \left(\frac{5^2}{9^2}\right)^{\frac{1}{2}}$$
$$= \frac{5^{2\times\frac{1}{2}}}{9^{2\times\frac{1}{2}}}$$
$$= \frac{5}{9}$$

ii). $\dfrac{(ab)^{\frac{1}{a}}}{\left(\frac{1}{ab}\right)^{\frac{1}{b}}}$

Solution: Given $\dfrac{(ab)^{\frac{1}{a}}}{\left(\frac{1}{ab}\right)^{\frac{1}{b}}}$

$$= \frac{(ab)^{\frac{1}{a}}}{(ab)^{\frac{-1}{b}}} = (ab)^{\frac{1}{a}}(ab)^{\frac{1}{b}}$$
$$= (ab)^{\frac{1}{a}+\frac{1}{b}}$$
$$= (ab)^{\frac{b+a}{ab}}$$
$$= (ab)^{\frac{a+b}{ab}}$$
$$= a^{\frac{a+b}{ab}}.b^{\frac{a+b}{ab}}$$

iii). $\dfrac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$

Solution: Given $\dfrac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$

$$= \frac{2^{p+1}.3^{2p-q}.5^{p+q}.(2.3)^q}{(2.3)^p.(2.5)^{q+2}.(3.5)^p}$$
$$= \frac{2^{p+1}.3^{2p-q}.5^{p+q}.2^q.3^q}{2^p.3^p.2^{q+2}.5^{q+2}.3^p.5^p}$$
$$= \frac{2^{p+1+q}.3^{2p-q+q}.5^{p+q}}{2^{p+q+2}.3^{p+p}.5^{q+2+p}}$$
$$= \frac{2^{p+q+1}.3^{2p}.5^{p+q}}{2^{p+q+2}.3^{2p}.5^{q+p+2}}$$
$$= \frac{3^{2p-2p}}{2^{p+q+2-p-q-1}.5^{q+p+2-p-q}}$$
$$= \frac{3^0}{2^1\times 5^2} = \frac{1}{2\times 25} = \frac{1}{50}$$

iv). $\left(\frac{x^p}{x^q}\right)^{p+q}.\left(\frac{x^q}{x^r}\right)^{q+r}.\left(\frac{x^r}{x^p}\right)^{r+p}$

Solution: Given $\left(\frac{x^p}{x^q}\right)^{p+q}.\left(\frac{x^q}{x^r}\right)^{q+r}.\left(\frac{x^r}{x^p}\right)^{r+p}$

$$= \frac{x^{p(p+q)}}{x^{q(p+q)}}.\frac{x^{q(q+r)}}{x^{r(q+r)}}.\frac{x^{r(r+p)}}{x^{p(r+p)}}$$
$$= \frac{x^{p^2+pq}}{x^{pq+q^2}}.\frac{x^{q^2+qr}}{x^{qr+r^2}}.\frac{x^{r^2+rp}}{x^{rp+p^2}}$$
$$= x^{p^2+pq-pq-q^2}.x^{q^2+qr-qr-r^2}.x^{r^2+rp-rp-p^2}$$
$$= x^{p^2-q^2}.x^{q^2-r^2}.x^{r^2-p^2}$$
$$= x^{p^2-q^2+q^2-r^2+r^2-p^2}$$
$$= x^0 = 1$$

Q5. Prove that $\left(\dfrac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$

Solution: Taking LHS $\left(\dfrac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}}$

$$= \left(\frac{2^{2\times 5}.2^{6\times 3}.2^3}{2^{3\times 5}.2^{7\times 2}}\right)^{\frac{1}{2}}$$

$=\left(\frac{2^{10}.2^{18}.2^{3}}{2^{15}.2^{14}}\right)^{\frac{1}{2}}$

$=\left(2^{10+18+3-15-14}\right)^{\frac{1}{2}}$

$=\left(2^{31-29}\right)^{\frac{1}{2}}=2^{2\times\frac{1}{2}}$

$=2$ = RHS Hence proved.

Complex numbers:
In 1795, Gauss, a German Mathematician, gave the concept of Complex Number as $a+ib$ or $Z=a+\sqrt{-1}b$ where a is called the real part and b is called imaginary part.

Use of complex numbers:
Complex numbers play a very important role in Mathematics and science. The use of complex numbers is indispensable in physics, Aeronautical and Electrical Engineering especially in the analysis of Electric circuits.

Conjugate of Complex numbers:
If $z=a+ib$ then conjugate of Z is defined as $\bar{z}=\overline{a+ib}=a-ib$, is obtaining by interchanging the sign of imaginary part.

Equality of two complex numbers:
Let $z_1=a+ib$ and $z_2=c+id$ be two complex numbers if $z_1=z_2$ iff real parts $a=c$ and imaginary parts $b=d$

Addition of two complex numbers:
Let $z_1=a+ib$ and $z_2=c+id$ be two complex numbers. Addition of z_1 and z_2 denoted by z_1+z_2 and define as

$z_1+z_2=a+ib+c+id$

$=a+c+i(b+d)$

Subtraction of two complex numbers:
Let $z_1=a+ib$ and $z_2=c+id$ be two complex numbers. subtraction of z_1 and z_2 denoted by z_1-z_2 and define as

$z_1-z_2=(a+ib)-(c+id)$

$=a+ib-c-id$

$=a-c+i(b-d)$

Multiplication of two complex numbers:
Let $z_1=a+ib$ and $z_2=c+id$ be two complex numbers. Multiplication of z_1 and z_2 denoted by $z_1.z_2$ and define as

$z_1.z_2=(a+ib)(c+id)$

$=ac+iad+ibc+i^2bd$

$=ac+(-1)bd+i(ad+bc)$

$=ac-bd+i(ad+bc)$

Division of two complex numbers:
Let $z_1=a+ib$ and $z_2=c+id$ be two complex numbers. Division of z_1 and z_2 denoted by $\frac{z_1}{z_2}$ and define as

$\frac{z_1}{z_2}=\frac{a+ib}{c+id}$

$\frac{z_1}{z_2}=\frac{a+ib}{c+id}\times\frac{c-id}{c-id}$

$\frac{z_1}{z_2}=\frac{ac-iad+ibc-i^2bd}{c^2-(id)^2}$

$\frac{z_1}{z_2}=\frac{ac-(-1)bd+ibc-iad}{c^2-(-1)^2d^2}$

$\frac{z_1}{z_2}=\frac{ac+bd+i(bc-ad)}{c^2+d^2}$

Modulus or the Absolute value of Z:
The Modulus or the Absolute value of Z is defined as $|z|=|a+ib|=\sqrt{a^2+b^2}$

Example: Add complex number $5+2i, 3+i$
Solution: Given to add $5+2i, 3+i$, Now
$(5+2i)+(3+i)=5+3+2i+i$
$=8+3i$

Example: subtract $7+3i$ from $5-8i$
Sol: Given to subtract $7+3i$ from $5-8i$
$(5-8i)-(7+3i)=5-8i-7-3i$
$=5-7-8i-3i$
$=-2-11i$

Exp:Let $z_1=2-i$ and $z_2=3+i$ then find z_1z_2
Solution: Given $z_1=2-i$ and $z_2=3+i$
Now $z_1z_2=(2-i)(3+i)$
$=6+2i-3i-i^2$
$=6-i-(-1)$
$=6+1-i$
$=7-i$

Example: Let $z_1=3+4i$ and $z_2=3-2i$ find the quotient $\frac{z_1}{z_2}$
Solution: Given $z_1=3+4i$ and $z_2=3-2i$
Now $\frac{z_1}{z_2}=\frac{3+4i}{3-2i}$ to rationalize denominator

$\frac{z_1}{z_2}=\frac{3+4i}{3-2i}\times\frac{3+2i}{3+2i}$

$\frac{z_1}{z_2}=\frac{9+6i+12i+8i^2}{3^2-(2i)^2}$

$\frac{z_1}{z_2}=\frac{9+18i+8(-1)}{9-4i^2}$

$$\frac{z_1}{z_2}=\frac{9-8+18i}{9-4(-1)}$$

$$\frac{z_1}{z_2}=\frac{1+18i}{9+4}=\frac{1+18i}{13}$$

$$\frac{z_1}{z_2}=\frac{1}{13}+\frac{18}{13}i$$

Exercise 2.5

Q1. Add the following complex numbers.

i). $8+9i,\ \ 5+2i$

Solution; given that $8+9i,\ \ 5+2i$

Now $(8+9i)+(5+2i)=8+9i+5+2i$

$=8+5+9i+2i$

$=13+11i$

ii). $6+3i,\ \ 3-5i$

Solution; Given that $6+3i,\ \ 3-5i$

Now $(6+3i)+(3-5i)=6+3i+3-5i$

$=6+3+3i-5i$

$=9-2i$

iii). $2i+3, 8-5\sqrt{-1}$

Solution; since $i=\sqrt{-1}$ so given complex number becomes $2i+3, 8-5i$

Now $(2i+3)+(8-5i)=2i+3+8-5i$

$=3+8+2i-5i$

$=11-3i$

iv). $\sqrt{3}+\sqrt{2}i,\qquad 3\sqrt{3}-2\sqrt{2}i$

Solution; Given $\sqrt{3}+\sqrt{2}i,\qquad 3\sqrt{3}-2\sqrt{2}i$

Now

$(\sqrt{3}+\sqrt{2}i)+(3\sqrt{3}-2\sqrt{2}i)=\sqrt{3}+3\sqrt{3}+\sqrt{2}i-2\sqrt{2}i$

$=4\sqrt{3}-\sqrt{2}i$

Q2. Subtract.

i). $-2+3i$ from $6-3i$

Solution; given that $-2+3i$, $6-3i$

Now $(6-3i)-(-2+3i)=6-3i+2-3i$

$=6+2-3i-3i$

$=8-6i$

ii). $9+4i$ from $9-8i$

Solution; given that $9+4i$, $9-8i$

$(9-8i)-(9+4i)=9-8i-9-4i$

$=9-9-8i-4i$

$=0-12i$

iii). $1-3i$ from $8-i$

Solution; Given $1-3i$ from $8-i$

Now $(8-i)-(1-3i)=8-i-1+3i$

$=8-1-i+3i$

$=7+2i$

iv). $6-7i$ from $6+7i$

Solution; $6-7i$ from $6+7i$

Now $(6+7i)-(6-7i)=6+7i-6+7i$

$=6-6+7i+7i$

$=0+14i$

Q3. Multiply the following complex numbers.

i). $1+2i,\ \ 3-8i$

Solution: Given $1+2i,\ \ 3-8i$

Now $(1+2i)(3-8i)=3-8i+6i-16i^2$

$=3-2i-16(-1)$

$=3+16-2i$

$=19-2i$

ii). $2i,\qquad 4-7i$

Solution: Given $2i,\qquad 4-7i$

Now $2i(4-7i)=8i-14i^2$

$=8i-14(-1)$

$=14+8i$

iii). $5-3i,\ \ 2-4i$

Solution: Given $5-3i,\ \ 2-4i$

Now $(5-3i)(2-4i)=10-20i-6i+12i^2$

$=10-26i+12(-1)$

$=10-12-26i$

$=-2-26i$

iv). $\sqrt{2}+i, 1-\sqrt{2}i$

Solution: Given $\sqrt{2}+i, 1-\sqrt{2}i$

Now $(\sqrt{2}+i)(1-\sqrt{2}i)=\sqrt{2}-\sqrt{4}i+i-\sqrt{2}i^2$

$=\sqrt{2}-2i+i-\sqrt{2}(-1)$

$=\sqrt{2}+\sqrt{2}-i$

$=2\sqrt{2}-i$

Q4. Divide the first complex number by second.

i). $z_1=2+i, z_2=5-i$

Solution: given $z_1=2+i, z_2=5-i$

Now $\dfrac{z_1}{z_2}=\dfrac{2+i}{5-i}$

To Rationalize the denominator Multiply & dividing by conjugate of $5-i$ i.e., $5+i$

$$\frac{z_1}{z_2}=\frac{2+i}{5-i}\times\frac{5+i}{5+i}$$

$$\frac{z_1}{z_2}=\frac{10+2i+5i+i^2}{5^2-i^2}$$

$$\frac{z_1}{z_2}=\frac{10+7i+(-1)}{25-(-1)}$$

$$\frac{z_1}{z_2}=\frac{10-1+7i}{25+1}$$

$$\frac{z_1}{z_2}=\frac{9+7i}{26}$$

$$\frac{z_1}{z_2}=\frac{9}{26}+\frac{7}{26}i$$

ii). $z_1 = 3i + 4, z_2 = 1 - i$

Solution: Given $z_1 = 3i + 4, z_2 = 1 - i$

Now $\frac{z_1}{z_2} = \frac{3i+4}{1-i}$

To Rationalize the denominator Multiply and dividing by conjugate of $1-i$ i.e., $1+i$

$$\frac{z_1}{z_2} = \frac{3i+4}{1-i} \times \frac{1+i}{1+i}$$

$$\frac{z_1}{z_2} = \frac{3i+3i^2+4+4i}{1^2-i^2}$$

$$\frac{z_1}{z_2} = \frac{3(-1)+4+3i+4i}{1-(-1)}$$

$$\frac{z_1}{z_2} = \frac{-3+4+7i}{1+1}$$

$$\frac{z_1}{z_2} = \frac{1+7i}{2}$$

$$\frac{z_1}{z_2} = \frac{1}{2} + \frac{7}{2}i$$

Q5. Perform the indicated operations and reduce to the form $a + bi$

i). $(4-3i)+(2-3i)$

Solution: Given $(4-3i)+(2-3i)$

$$= 4 - 3i + 2 - 3i$$
$$= 4 + 2 - 3i - 3i$$
$$= 6 - 6i$$

ii). $(5-2i)-(4-7i)$

Solution: Given $(5-2i)-(4-7i)$

$$= 5 - 2i - 4 + 7i$$
$$= 5 - 4 - 2i + 7i$$
$$= 1 + 5i$$

iii). $2i(4-5i)$

Solution: Given $2i(4-5i)$

$$= 8i - 10i^2$$
$$= 8i - 10(-1)$$
$$= 10 + 8i$$

iv). $(2-3i) \div (4-5i)$

Solution: Given $(2-3i) \div (4-5i)$

$$= \frac{2-3i}{4-5i}$$

To Rationalize the denominator $\times$ & $\div$ by conjugate of $4-5i$ i.e., $4+5i$

$$= \frac{2-3i}{4-5i} \times \frac{4+5i}{4+5i}$$

$$= \frac{8+10i-12i-15i^2}{4^2-(5i)^2}$$

$$= \frac{8-2i-15(-1)}{16-25(-1)}$$

$$= \frac{8+15-2i}{16+25}$$

$$= \frac{23-2i}{41}$$

$$= \frac{23}{41} - \frac{2}{41}i$$

Q6. Find the complex conjugate of the following complex numbers.

i). $-8-3i$

Solution: Given $-8-3i$

$\overline{-8-3i} = -8+3i$

ii). $-4+9i$

Solution: Given $-4+9i$

$\overline{-4+9i} = -4-9i$

iii). $7+6i$

Solution: Given $7+6i$

$\overline{7+6i} = 7-6i$

iv). $\sqrt{5}-i$

Solution: Given $\sqrt{5}-i$

$\overline{\sqrt{5}-i} = \sqrt{5}+i$

Review Exercise 2

Q1. Tell whether the following are true or false

i). $3^{\frac{1}{3}} = \sqrt{3}$ False

ii). $2^{\frac{2}{3}} = \sqrt[3]{4}$ True

iii). $\sqrt{49} = \sqrt{7}$ False

iv). $\sqrt[3]{27} = x^3$ False

Q2. Select the correct answer.

i). The additive inverse of $\sqrt{5}$ is

a). $-\sqrt{5}$ b). $\frac{1}{\sqrt{5}}$

c). $\sqrt{-3}$ d). -5

ii). $2(3+4) = 2\times3 + 3\times4$ name the property

a). commutative b). Associative

c). Distributive d). closure

iii). $\sqrt{-1}\times\sqrt{-1} =$ ....

a). 1 b). i

c). - 1 d). 0

iv). Which of the following represents number greater than -3 but less then 6?

a). $\{x : -3 > x > 6\}$ b). $\{x : -3 \le x \le 6\}$

c). $\{x : -3 < x < 6\}$ d). $\{x : -3 \ge x \ge 6\}$

v). if $n = 8$ and $16 \times 2^m = 4^{n-8}$ then m = ?

a). - 4 b). -2

c). 0 c). 8

vi). $(i)(-i)$ = ...

a). 1 b). - 1

c). $-i$ d). i

vii). The multiplicative identity of real numbers is

a). 0 b). 1

c). - 1 d). R

viii). 0 is ……………………………………

a). a positive integer b). a negative integer
c). neither positive nor negative
d). Not an integer

ix). For $i=\sqrt{-1}$ if $3i(2+5i)=x+6i$ then $x=?$

a). 5 b). - 15
c). $5i$ d). $15i$

x). $\sqrt{0}=$ ……………………

a). 0 b). 1
c). - 1 d). not defined

xi). $\sqrt{-(-9)^2}=?$ note : $i=\sqrt{-1}$

a). 9 b). $9+i$
c). $9-i$ d). 9i

Q3. Simplify each of the following.

i). $\left(\frac{-2}{3}\right)^3$

Solution: Given $\left(\frac{-2}{3}\right)^3$

$$=\frac{(-2)^3}{3^3}=\frac{-8}{27}$$

ii). $(-2)^3(3)^2$

Solution: Given $(-2)^3(3)^2$

$$=-8\times 9$$
$$=-72$$

iii). $-3\sqrt{48}$

Solution: Given $-3\sqrt{48}$

$$=-3\sqrt{16\times 3}$$
$$=-3\sqrt{16}\sqrt{3}$$
$$=-3\times 4\sqrt{3}$$
$$=-12\sqrt{3}$$

iv). $\frac{5}{\sqrt[3]{9}}$

Solution: Given $\frac{5}{\sqrt[3]{9}}$

$$=\frac{5}{\sqrt[3]{9}}\times\frac{\sqrt[3]{3}}{\sqrt[3]{3}}$$
$$=\frac{5\sqrt[3]{3}}{\sqrt[3]{27}}=\frac{5\sqrt[3]{3}}{3^{\frac{3}{3}}}=\frac{5\sqrt[3]{3}}{3}$$

Q4. Multiply $8i,-8i$

Solution: Given to multiply $8i,-8i$

$$(8i)(-8i)$$
$$=-64i^2$$
$$=-64(-1)$$
$$=64$$

Q5. Divide $2-5i$ by $1-6i$

Solution: Given to divide $2-5i$ by $1-6i$

$$(2-5i)\div(1-6i)$$
$$=\frac{2-5i}{1-6i}$$

To Rationalize the denominator Multiply &dividing by conjugate of $1-6i$ i.e., $1+6i$

$$=\frac{2-5i}{1-6i}\times\frac{1+6i}{1+6i}$$
$$=\frac{2+12i-5i-30i^2}{1-36i^2}$$
$$=\frac{2+7i-30(-1)}{1-36(-1)}$$
$$=\frac{2+30+7i}{1+36}$$
$$=\frac{32}{37}+\frac{7}{37}i$$

Q6. Name the property used $7\times\frac{1}{7}=\frac{1}{7}\times 7=1$

Answer: Multiplicative inverse

Q7. Use laws of exponents to simplify

$$\frac{(81)^n.3^5+(3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

Solution: Given $\frac{(81)^n.3^5+(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

$$=\frac{(3^4)^n.3^5+(3)^{4n-1}(3^5)}{(3^{2\times 2n})(3^3)}$$
$$=\frac{3^{4n}.3^5+3^{4n-1}.3^5}{3^{4n}.3^3}$$
$$=\frac{3^{4n}.3^5(1+3^{-1})}{3^{4n}.3^3}$$
$$=3^2\left(1+\frac{1}{3}\right)=9\left(\frac{4}{3}\right)$$
$$=12$$