

Chapter No.1**Matrix**

A rectangular array of number arranged into rows and columns is called matrix

OR

The combination of rows and columns in square or rectangular form is called matrix

Row Matrix:

A matrix is called a row matrix, if it has only one row

e.g $M = [5 \quad 3]$

Column Matrix:

A matrix is called column matrix, if it has only one column

e.g $M = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Rectangular matrix:

A matrix is called rectangular. If the number of rows not equal to number of columns

e.g $M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

Square matrix:

A matrix is called square matrix, if the number of rows are equal to number of columns

e.g $H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$

Order of Matrix

The number of rows and columns in a matrix specifies its order. If a matrix M has a rows and a columns them M is said to be of order, m-b-n

e.g

$M = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ is order of 2-by-3

Equal matrix

Two numbers are said to be equal if and only

They have same order

They corresponding entries are equal

e.g

$$A = \begin{bmatrix} 7 & 0 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 4+3 & 0 \\ 3 & 2 \end{bmatrix} \text{ are equal}$$

Null or Zero matrix:

A matrix is called a Null or Zero matrix, if each of its entries is Zero (0)

e.g $N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Symmetric matrix:

A matrix is called symmetric matrix if $A^t=A$

e.g $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Skew Symmetric matrix:

A square matrix is called skew symmetric $A^t=-A$

e.g $A = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$

Scalar matrix:

A matrix is called scalar matrix if all the diagonal entries are same and non diagonal entries is zero

e.g $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Diagonal matrix:

A square matrix is called diagonal matrix if at least any one of the entries of its diagonal is not zero and non diagonal entries must all be zero

e.g $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Identity matrix:

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1

e.g $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Singular matrix:

A square matrix is called singular matrix if the determinant of M is equal to zero i.e
 $[M]=0$

$$M = \begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix}$$

i.g

$$|M| = \begin{vmatrix} 4 & 4 \\ 3 & 3 \end{vmatrix} = 12 - 12 = 0$$

Non singular matrix:

A square matrix is called non singular matrix, if the determinant M is not equal to zero
 i.e $|M| \neq 0$

e.g $M = \begin{bmatrix} 6 & 2 \\ 3 & 4 \end{bmatrix}$

Transpose of matrix:

A matrix obtained by changing the rows into columns or to columns in to rows of a matrix is called Transpose of matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \end{bmatrix} \quad A' = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 2 & 3 \end{bmatrix}$$

Negative of matrix:

Let A be a matrix then its negative $-A$ is obtained by changing the signs of all the entries.

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \quad \text{Then } -A = \begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$$

Adjoint of matrix:

Adjoint of square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of others entries. Adjoint of matrix A is denoted as Adj A.

For example $Adj A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Chapter # 02

REAL AND COMPLEX NUMBER

Set of real number:

Union of two disjoint sets, the set of rational number Q and the set of irrational number Q`.

$$R = Q \cup Q'$$

Complex number:

A number of the form $Z=a+ bi$ where a and b are the real numbers and $i=\sqrt{-1}$ is called complex number. e.g. $Z=2+6i$

Complex conjugate:

The number $a+ bi$ and $a-bi$ are conjugate of each other.

Rational number:

The number of the form $\frac{p}{q}$ where p, q integers and $q \neq 0$ are called rational numbers.

e.g. $\frac{2}{3}, \sqrt{\frac{16}{25}}$

Irrational number:

The number which cannot be express in the form of $\frac{p}{q}$ where p, q integers and $q \neq 0$ are called irrational numbers. e.g. $\sqrt{2}, 3.14$

Natural numbers:

The numbers $1, 2, 3, \dots$ which we use for counting objects are called natural numbers.

e.g. $N = \{1, 2, 3, \dots\}$

Whole numbers:

If we include 0 in the set of natural numbers then it is called whole number and it is denoted by W .

e.g. $W = \{0, 1, 2, 3, \dots\}$

Integers:

Set of integers consists of positive integers, 0 and is denoted by Z .

e.g. $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Radicals and Radicands:

If a real number then any real number x such that $X=n\sqrt[n]{a}$ is called the n th root of a , in the radical $n\sqrt[n]{a}$, n is radical and a is called radicand.

Chapter#3

LOGARITHM

Scientific Notation:

Number written in the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer called scientific notation.

Common Logarithm:

If the base of logarithm is taken as 10 then logarithm is called common logarithm.

Natural Logarithm:

Logarithm having base e is called Napier logarithm or Natural logarithm.

Characteristic:

The integral part of the logarithm of any number is called the characteristic.

Mantissa:

The decimal part of the logarithm of a number is called the mantissa and is always positive.

CHAPTER#4

ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS

Algebraic expression:

An algebraic expression is that in which constants and variables or both are combined by addition and subtraction.

e.g. $5x^2 - 4x$

Rational expression:

The quotient $p(x)/q(x)$ of two polynomial $p(x)$ and $q(x)$ where $q(x) \neq 0$ is called a rational expression.

$$\frac{3x + 1}{2x + 5}$$

e.g.

Surd:

An irrational radical with radicand is called a surd. e.g. $\sqrt{3}$, $3\sqrt{7}$

Monomial Surd:

A surd which contain a single term is called monomial surd.

e.g. $\sqrt{2}, \sqrt{3}$

Binomial Surd:

A surd which contain sum of two monomial surds is called binomial surd.

e.g. $\sqrt{3}+\sqrt{7}$

Chapter#5

Factorization

Factorization:

The process of expressing an algebraic expression in term of its factor is called factorization.

Remainder theorem:

If a polynomial $f(x)$ is divided by a linear divisor $(x-a)$, then the remainder is $f(a)$

Factor theorem:

The polynomial $(x-a)$ is a factor of the polynomial $f(x)$ if and only if $f(a)=0$

Zero of polynomial:

If a specific number $x=a$ is substituted for the variable x in a polynomial $p(x)$ so that the value.

$P(a)$ is zero then $x=a$ is called zero of polynomial $p(x)$

Chapter#6

Algebraic Manipulation

HCF(Highest common factor)

If two or more algebraic expression are given then their common factor of highest power is called HCF of the expression.

LCM(Least common factor)

The product of common factor together with non common factors of the given expression is called LCM.

Relation between HCF and LCM

$LCM.HCF=p(x)q(x)$ where $p(x)$ and $q(x)$ are given expression

Chapter No.7

Linear Equation & Inequalities

Equation:

An equation is a statement that says the two given algebraic expressions are

e.g $x+2=3$

Linear Equation:

A linear equation in one unknown variable x is an equation of the form $ax+b=0$

$A, b \in \mathbb{R}$ and $a \neq 0$

Types of equations

Identity Equations:

An identity is an equation that is satisfied by every number for which both sides are defined

e.g $x+3=3+x$

Conditional equation:

A conditional equation is an equation that is satisfied by at least one number but is not an identity Eg $2x+1=9$

Inconsistent Equation:

An inconsistent equation is an equation whose solution set is the empty set

Eg. $x=x+5$ because no value of x satisfied it

Rational equation:

When the variable in an equation occurs under a radical sign the equation is called rational equation e.g $\sqrt{2x-3}-7=0$

Extraneous Solution:

A solution that does not satisfy the original equation is called extraneous solution

Chapter No.8

Linear Graphs And Their Applications

Cartesian Plane

The plane formed by two straight lines perpendicular to each other is called Cartesian plane.

Co Ordinate Axes

The line intersecting each other perpendicularly are called Co-Ordinate Axes.

Order Pair of element

An order pair is a pair of elements in which elements are written in specific order.

Origin

The point of intersection of two co-ordinate axes is called origin.

Abscissa and Ordinate

The x co ordinate of point is called Abscissa and y co ordinate is called ordinate
e.g (3,6) have 3 is Abscissa and 6 is ordinate.

Collinear point

Two or more than two points lie on the same line are called Collinear point.

Chapter No.9

Introduction to Co-Ordinate Geometry

Plane geometry and co-ordinate geometry

The study of geometrical shapes in a plane is called plane geometry. Co- ordinate geometry is the study of geometrical shapes in the Cartesian plane.

Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is define as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

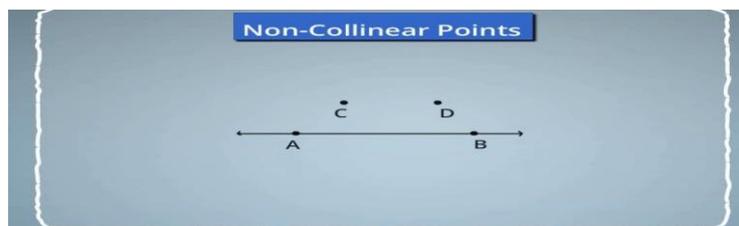
Collinear Point:

Two or more than two points which lie on the same straight line are called collinear point



Non Collinear point:

Points are non collinear is they do not lie on the same straight line are called non collinear Point.



Rectangle:

A figure formed in the plane by four non collinear points is called a rectangle if

- i. Its opposite sides are of equal measure

- ii. Its opposite sides are parallel to each other

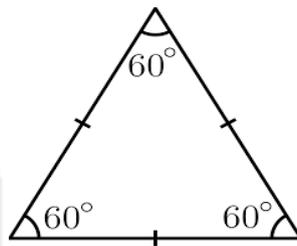
Parallelogram:

A figure formed by four non collinear point in the plane is called parallelogram if

- i. Its opposite sides are of equal measure
- ii. Its opposite sides are parallel
- iii. Measure of none of the angle is 90°

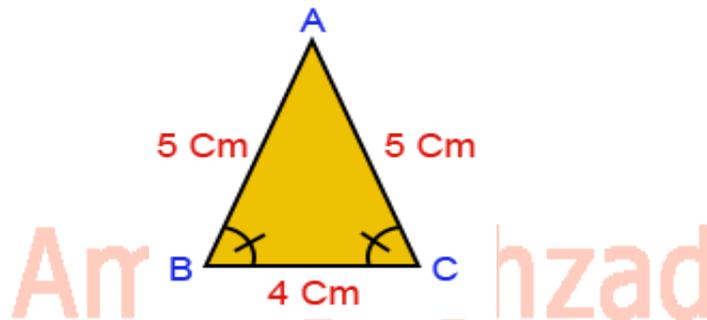
Equilateral triangle:

If the length of all the three sides of a triangle are same then the triangle is called an equilateral triangle.



Isosceles triangle:

If length of two sides of the triangle is equal and length of the 3rd side is different then that triangle is called isosceles triangle



Right triangle:

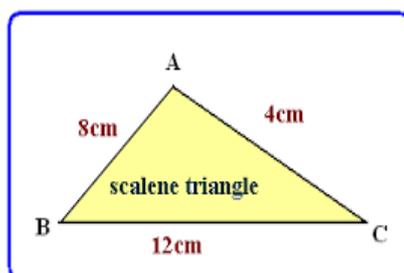
A right triangle is that in which one of the angles has measure equal to 90°

Square:

A square is a closed figure in the plane formed by four noncollinear points such that length of all sides equal and measure of each angle is 90°

Scalene triangle:

Its measure of three sides of the triangle are different then the triangle is called scalene triangle



Chapter No.10

Congruent Triangles

Congruent triangle:

Two triangles are said to be congruent if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

ASA postulate:

In any correspondence of two triangles if one side and any angles of the triangle are congruent to the corresponding sides and angles of the other then the triangles are congruent. This statement is called ASA postulate.

SSS postulate:

In any correspondence of two triangles if three sides of a triangle are congruent to the corresponding three sides of the other then the triangles are congruent. This statement is called SSS postulate.

H.S postulate

In any correspondence of two right-angled triangles if the hypotenuse and one side of a triangle are congruent to the corresponding hypotenuse and side of the other then the triangles are congruent.

Chapter No.11

Parallelogram and Triangles

Trapezium:

A trapezium is a quadrilateral with two parallel sides and two non-parallel sides.

Rectangle:

A rectangle is a parallelogram with all the angles at the vertices equal to 90° .

Area of rectangle = length \times width

Q. If the line segment joining the midpoints of the sides of a triangle is parallel to the third side, what is the relation with its size?

Ans. If the line segment joining the midpoint of the sides of a triangle is parallel to the 3rd sides and is equal one half of its length here

Chapter No.12

Line Bisector and Angle Bisector

Right bisector of a line segment:

A line l is called a right bisector of a line segment if l is perpendicular to the line segment and passes through its midpoint

Angle bisector:

Angle bisector is the ray which divide and angle into two equal parts

Chapter No.13

Sides and Angles of a Triangle

Scalene triangle:

A triangle is called scalene triangle if measure of all the sides is different

Q. Which side of a right angle triangle is longer then each of other two sides ?

Ans. The hypotenuses of right angle triangle is longer than each of other two sides

Chapter No.14

Ratio and Proportion

Ratio:

A relation between two quantities of the same kind is called Ratio for example $a : b = \frac{a}{b}$

Proportion:

The equality of two ratios is called proportional four number a, b, c, d are in proportion if A:
B=C:D

Similar triangles:

Triangles are called similar if they are equiangular and measure of their corresponding sides are proportional

Practical application of similar triangle:

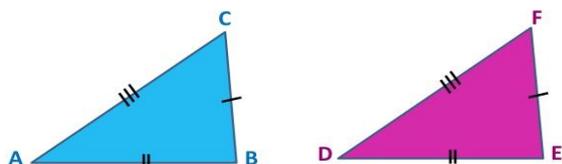
Photographer can develop prints of different sizes from the same negative. In spite of the difference in sizes. There picfures look like each other. One photograph is simply on enlargement of another

Congruent triangles:

Two triangles are said to be congruent symbol (\cong) if there exists a correspondence between them. Such that all the corresponding sides are angles are congruent.

Congruency Symbols

Triangles are labelled with Letters and special symbols used.



$$AB = DE \quad \text{and} \quad BC = EF \quad \text{and} \quad AC = DF$$

$$\triangle ABC \cong \triangle DEF \quad (\text{by SSS Rule})$$

Chapter No.15

Pythagoras Theorem

Pythagoras theorem:

In a right angle triangle the square of the length of hypotenuse is equal to the sum of the square of the length of the other two sides

$$(\text{Hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

Right triangle:

A right triangle is that in which one of the angles as measure equal to 90° .

Acute angle triangle :

If sum of square of two sides is greater than the square of 3rd side then the angle is called acute angle triangle $a^2 + b^2 > c^2$

Obtuse angle triangle:

The triangle in which of sum of squares are two sides is less then the square of 3rd side is called obtuse angle triangle

Chapter No.16

Theorems Related With Area

Area of the figure:

The region enclosed by bounding lines of a closed figure is called the area of the figure

Triangular region:

A triangular region is the union of a triangle and its interior i.e The three line segments forming the triangle and its interior

Rectangular region:

A Rectangular region is the union of a rectangle and its interior

Parallel lines:

Two lines which on extending in both the directions infinitely never intersect at a point are called parallel lines

Altitude or height of a parallelogram:

If one side of parallelogram is taken as its base the perpendicular distance between that sides parallel to it is called altitude or height parallelogram

Chapter No.17

Practical Geometry Triangle

Centroid of a triangle:

The point where the three medians of a triangle meet is called centriod of the triangle

Circumcentre of a triangle:

The point of concurrency of three right bisectors of sides of triangle is called its circumcentre.

Orthocentre of a triangle:

The point of concurrency of the three altitude of a triangle is called orthocenter of triangle

Incentre of a triangle:

The point of concurrency of the three bisectors of a interior angles of triangle is called in-center of triangle

Concurrent lines:

The line passing through the same point are called concurrent lines

Median of a triangle:

A line segment joining a vertex of a triangle to the midpoint of the opposite side is called a median of triangle

Altitude of a triangle:

If one side of a triangle is taken as base the perpendicular distance between the sides and the side parallel to it is called altitude or height of the triangle

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