Solution Manual

Matrix:
A rectangular array or a formation of a collection of real numbers, say 0, 1, 2, 3, 4 and 7 etc. and then enclosed by brackets [ ] are known as **Matrix**. For example $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
In a matrix, the entries presented in horizontal way are called **rows** & all the entries presented in vertical way are called **columns**. For example in above matrix there are two rows one which contain 1, 2 and other 3, 4. There are two columns containing element 1,3 and 2,4.
Order of a Matrix:
The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns, then M is said to be of order m-by-n. For example in above matrix order of matrix is 2-by-2.
Equal Matrices:
Two matrices are said to be equal if and only if

- They have same order
- Their corresponding entries are equal

For example:

A = $\begin{bmatrix} 7 & 0 \\ 3 & 2 \end{bmatrix}$ & B= $\begin{bmatrix} 4+3 & 0 \\ 3 & 2 \end{bmatrix}$ are equal.

EX#1.1

Q.1: Find the order of the matrix.

A = $\begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$.
Order of A = 2-by-2

B = $\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$
Order of B = 2-by-2

C=$\begin{bmatrix} 2 & 4 \end{bmatrix}$
Order of C = 1-by-2

D = $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$
Order of D = 3-by-1

E=$\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$
Order of E = 3-by-2

F = [2]
Order of F = 1-by-1

Q.2: Which of the following matrices are equal?

A = [3]; B= [3 5]; C= [5-2]; D=[5 3]; E=$\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$;

F=$\begin{bmatrix} 2 \\ 6 \end{bmatrix}$; G=$\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$; H=$\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$; I=[3 3+2];

J=$\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Answer:
In above matrices following are equal: **A = C , B = I , E = H , F = G**
H = J , E = J

Q.3: Find the value of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

From the definition of equality of matrices:

a + c = 0 .... (1) ; a + 2b = -7 .... (3)
c – 1 = 3 ....(2) ; 4d – 6 = 2d ....(4)

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$\Rightarrow c = 3 + 1$

c = 4

So,

a + 4 = 0 ; from (4): 4d – 2d = 6

a = -4 ; 2d = 6

Put value of 'a' in (3) ; d = 6/2 = 3

-4 + 2b = -7

2b = -7 + 4

b = $-\frac{3}{2}$

Result: The values of a, b, c and d are -4, -3/2, 4 and 3 respectively.

(END OF EX # 1.1)

Types of matrices:

1. **Row Matrix:**
   A matrix is called a row matrix, if it has only one row. For example: M = [5 3].
2. **Column Matrix:**
   A matrix is called a row matrix, if it has only one column. For example: M = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
3. **Rectangular Matrix:**
   A matrix is called rectangular, if the number of rows of is not equal to the number of its columns. For example: M = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$
4. **Square Matrix:**
   A matrix is called a square matrix, if its number of rows is equal to its number of columns. For example: H=$\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$
5. **Null or Zero Matrix:**
   A matrix is called a null or zero matrix, if each of its entries is 0. For example: N =$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
6. **Negative Matrix:**
   Let A be a matrix. Then its negative, -A is obtained by changing the signs of all the entries of A For example negative of Z =$\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$ is -Z=$\begin{bmatrix} -4 & 0 \\ -6 & -2 \end{bmatrix}$
7. **Symmetric Matrix:**
   A square matrix A is symmetric, if A^t = A. For example: Z =$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$
8. **Skew-symmetric Matrix:**
   A square matrix A is said to be skew-symmetric, if A^t = –A. For example: A = $\begin{bmatrix} 0 & -6 \\ 6 & 0 \end{bmatrix}$
9. **Diagonal Matrix:**
   A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not zero and non-diagonal entries are zero. For example: M = $\begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}$
10. **Scalar Matrix:**
    A diagonal matrix is called a scalar matrix, if all the diagonal entries are same. For example: M =$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

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11. Identity Matrix:

A scalar matrix is called identity (unit) matrix, if all diagonal entries are 1. It is denoted by I. For example: **I** = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

EX # 1.2

Q.1: From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

A = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Null Matrix

B = $[2 \;\; 3 \;\; 4]$ Row Matrix

C = $\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$ Column Matrix

D = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Identity Matrix

E = $[\,0\,]$ Null Matrix

F = $\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$ Column Matrix

Q.2: From the following Matrices, identify

a) Square Matrix **b) Rectangular Matrix** **c) Row Matrix** **d) Column Matrix**

e) Identity Matrix **f) Null Matrix**

i. $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ Ans: Rectangular Matrix

ii. $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ Ans: Column Matrix (Also comes under the category of Rectangular Matrix)

iii. $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$ Ans: Square Matrix

iv. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Ans: Identity Matrix

v. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ Ans: Rectangular Matrix

vi. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ Ans: Column & Rectangular Matrix

vii. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Ans: Square Matrix

viii. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ Ans: Null Matrix

Q.3: From the following matrices, identify diagonal, scalar and unit (identity) matrices.

A = $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ Ans: Scalar Matrix

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B = $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ Ans: Diagonal Matrix

C = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Ans: Identity Matrix

D = $\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$ Ans: Diagonal Matrix

E = $\begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$ Ans: Scalar Matrix

EX#1.3

Q.1: Which of the following matrices are conformable for addition?

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix},\ B = \begin{bmatrix} 3 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix},\ D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix},$$

$$E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix},\ F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

Solution:

Since matrices ‘A & E’ and matrices ‘B & D’ and matrices ‘C & F’ have same order. So they are conformable for addition.

Q.2: Find additive inverse of the following matrices:

$$A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix},\ C = \begin{bmatrix} 4 \\ -2 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix},\ E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

Solution:

Additive inverse of A = - **A** = $-\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$

Additive inverse of B = - **B** = $-\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$

Additive inverse of C = - **C** = $-\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$

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Additive inverse of D = - **D** = $-\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$

Additive inverse of E = - **E** = $-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Additive inverse of F = - **F** = $-\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix} = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$

Q.3: If A = $\begin{bmatrix} \mathbf{-1} & \mathbf{2} \\ \mathbf{2} & \mathbf{1} \end{bmatrix}$ **, B =** $\begin{bmatrix} \mathbf{1} \\ \mathbf{-1} \end{bmatrix}$, $C = [\mathbf{1}\ \ -\mathbf{1}\ \ \mathbf{2}]$ **and D =** $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{0} & \mathbf{2} \end{bmatrix}$ **then find:**

(i). A + $\begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

(ii). **B +** $\begin{bmatrix} \mathbf{-2} \\ \mathbf{3} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1-2 \\ -1+3 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{-1} \\ \mathbf{2} \end{bmatrix}$

(iii). **C + [-2 1 3] = [1+(-2) -1+1 2+3]**

= [-1 0 5]

(iv). **D +** $\begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{2} & \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$

$= \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{3} \\ \mathbf{1} & \mathbf{0} & \mathbf{3} \end{bmatrix}$

(v). **2A = 2**$\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi). (-1)B = (-1) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii). (-2)C = (-2) $[\mathbf{1}\ \ -\mathbf{1}\ \ \mathbf{2}] = [\mathbf{-2}\ \ \mathbf{2}\ \ -\mathbf{4}]$

(viii). 3D = 3 $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 6 \end{bmatrix}$

(ix). 3C = 3 $[\mathbf{1}\ \ -\mathbf{1}\ \ \mathbf{2}] = [\mathbf{3}\ \ -\mathbf{3}\ \ \mathbf{6}]$

Q.4: Perform the indicated operations and simplify the following:

(i) $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$

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Solution:

(i): $= \left(\begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \left(\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 2+1 \\ 3+1 & 1+0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

(ii):

$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 0-1 & 2-1 \\ 3-1 & 0-0 \end{bmatrix}\right)$

$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \left(\begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}\right)$

$= \begin{bmatrix} 1+(-1) & 1+1 \\ 1+2 & 0+0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$

(iii): [2 3 1] + ([1 0 2] - [2 2 2])

$= [2\ 3\ 1] + ([1-2\ 0-2\ 2-2])$

$= [2\ 3\ 1] + [-1\ -2\ 0]$

$= [2+(-1)\ \ 3+(-2)\ \ 1+0]$

$= [1\ \ 1\ \ 1]$

(iv): $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{2} & \mathbf{2} \\ \mathbf{3} & \mathbf{3} & \mathbf{3} \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 2+1 & 3+1 \\ -1+2 & -1+2 & -1+2 \\ 0+3 & 1+3 & 2+3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$

(v): $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} \\ \mathbf{3} & \mathbf{1} & \mathbf{2} \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} & \mathbf{-1} \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 2+0 & 3+(-2) \\ 2+(-2) & 3+1 & 1+0 \\ 3+0 & 1+2 & 2+(-1) \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 & 1 \\ 0 & 4 & 1 \\ 3 & 3 & 1 \end{bmatrix}$

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(vi): $\left(\begin{bmatrix}\mathbf{1} & \mathbf{2}\\ \mathbf{0} & \mathbf{1}\end{bmatrix}+\begin{bmatrix}\mathbf{2} & \mathbf{1}\\ \mathbf{1} & \mathbf{0}\end{bmatrix}\right)+\begin{bmatrix}\mathbf{1} & \mathbf{1}\\ \mathbf{1} & \mathbf{1}\end{bmatrix}$

$$=\left(\begin{bmatrix}1+2 & 2+1\\ 0+1 & 1+0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

$$=\left(\begin{bmatrix}3 & 3\\ 1 & 1\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3+1 & 3+1\\ 1+1 & 1+0\end{bmatrix}$$

$$=\begin{bmatrix}4 & 4\\ 2 & 1\end{bmatrix}$$

Q.5: For the Matrices A = $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\ \mathbf{2} & \mathbf{3} & \mathbf{1}\\ \mathbf{1} & \mathbf{-1} & \mathbf{0}\end{bmatrix}, \boldsymbol{B}=\begin{bmatrix}\mathbf{1} & \mathbf{-1} & \mathbf{1}\\ \mathbf{2} & \mathbf{-2} & \mathbf{2}\\ \mathbf{3} & \mathbf{1} & \mathbf{3}\end{bmatrix}$ ***and C*** $=\begin{bmatrix}\mathbf{-1} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{-2} & \mathbf{3}\\ \mathbf{1} & \mathbf{1} & \mathbf{2}\end{bmatrix}$ **verify the following rules:**

(i). A + C = C + A

$$\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$\begin{bmatrix}1+(-1) & 2+0 & 3+0\\ 2+0 & 3+(-2) & 1+3\\ 1+1 & -1+1 & 0+2\end{bmatrix}=\begin{bmatrix}-1+1 & 0+2 & 0+3\\ 0+2 & -2+3 & 3+1\\ 1+1 & 1+(-1) & 2+0\end{bmatrix}$$

$$\begin{bmatrix}0 & 2 & 3\\ 2 & 1 & 4\\ 2 & 0 & 2\end{bmatrix}=\begin{bmatrix}0 & 2 & 3\\ 2 & 1 & 4\\ 2 & 0 & 2\end{bmatrix}$$

$L.H.S = R.H.S$

(ii). A + B = B + A

$$\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}=\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$$

$$\begin{bmatrix}1+1 & 2+(-1) & 3+1\\ 2+2 & 3+(-2) & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}=\begin{bmatrix}1+1 & -1+2 & 1+3\\ 2+2 & -2+3 & 2+1\\ 3+1 & 1+(-1) & 3+0\end{bmatrix}$$

$$\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$L.H.S = R.H.S$

(iii). **B + C = C + B (do yourself)**

(iv). **A + (B + A) = 2A + B**

$$\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}\right)=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

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$$\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+1 & 1+(-1) & 3+0\end{bmatrix}\right)=\begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}\right)=\begin{bmatrix}2+1 & 4+(-1) & 6+1\\4+2 & 6+(-2) & 2+2\\2+3 & -2+1 & 0+3\end{bmatrix}$$

$$\begin{bmatrix}1+2 & 2+1 & 3+4\\2+4 & 3+1 & 1+3\\1+4 & -1+0 & 0+3\end{bmatrix}=\begin{bmatrix}2+1 & 4+(-1) & 6+1\\4+2 & 6+(-2) & 2+2\\2+3 & -2+1 & 0+3\end{bmatrix}$$

$$\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}=\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

$L.H.S = R.H.S$

(v). **(C - B) + A = C + (A - B)**

$$\left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$\left(\begin{bmatrix}-1-1 & 0-(-1) & 0-1\\0-2 & -2-(-2) & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1-1 & 2-(-1) & 3-1\\2-2 & 3-(-2) & 1-2\\1-3 & -1-1 & 0-3\end{bmatrix}\right)$$

$$\left(\begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}0 & 3 & 2\\0 & 5 & -1\\-2 & -2 & -3\end{bmatrix}\right)$$

$$\left(\begin{bmatrix}-2+1 & 1+2 & -1+3\\-2+2 & 0+3 & 1+1\\-2+1 & 0+(-1) & -1+0\end{bmatrix}\right)=\begin{bmatrix}-1+0 & 0+3 & 0+2\\0+0 & -2+5 & 3+(-1)\\1+(-2) & 1+(-2) & 2+(-3)\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & 2\\0 & 3 & 2\\-1 & -1 & -1\end{bmatrix}=\begin{bmatrix}1 & 3 & 2\\0 & 3 & 2\\-1 & -1 & -1\end{bmatrix}$$

$L.H.S = R.H.S$

(vi). **2A + B = A + (A + B)** **(Do yourself)**
(vii).(C-B) A = (C - A) – B **(Do yourself)**
(viii). **(A + B) + C = A + (B + C)** **(Do yourself)**
(ix). **A + (B - C) = (A - C) + B** **(Do yourself)**
(x). **2A + 2B = 2(A + B)**

$$2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+2\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}=2\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

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$$\begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix}+\begin{bmatrix}2 & -2 & 2\\4 & -4 & 4\\6 & 2 & 6\end{bmatrix}=2\left(\begin{bmatrix}1+1 & 2+(-1) & 3+1\\2+2 & 3+(-2) & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}\right)$$
$$\begin{bmatrix}2+2 & 4+(-2) & 6+2\\4+4 & 6+(-4) & 2+4\\2+6 & -2+2 & 0+6\end{bmatrix}=2\left(\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}\right)$$
$$\begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix}=\begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix}$$
$$L.H.S = R.H.S$$

Q.6: If A = $\begin{bmatrix}\mathbf{1} & \mathbf{-2}\\\mathbf{3} & \mathbf{4}\end{bmatrix}$ $\boldsymbol{and\ B} = \begin{bmatrix}\mathbf{0} & \mathbf{7}\\\mathbf{-3} & \mathbf{8}\end{bmatrix}$ **find:**

(i). **3A - 2B**

$= 3\begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}-2\begin{bmatrix}0 & 7\\-3 & 8\end{bmatrix}$

$= \begin{bmatrix}3 & -6\\9 & 12\end{bmatrix}-\begin{bmatrix}0 & 14\\-6 & 16\end{bmatrix}$

$= \begin{bmatrix}3-0 & -6-14\\9-(-6) & 12-16\end{bmatrix}$

$= \begin{bmatrix}3 & -20\\15 & -4\end{bmatrix}$

(ii). $2A^t - 3B^t$

$A^t = \begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}$

$B^t = \begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$

$= 2\begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}-3\begin{bmatrix}0 & -3\\7 & 8\end{bmatrix}$

$= \begin{bmatrix}2 & 6\\-4 & 8\end{bmatrix}-\begin{bmatrix}0 & -9\\21 & 24\end{bmatrix}$

$= \begin{bmatrix}2-0 & 6-(-9)\\-4-21 & 8-24\end{bmatrix}$

$= \begin{bmatrix}2 & 15\\-25 & -16\end{bmatrix}$

Q.7: If $2\begin{bmatrix}\mathbf{2} & \mathbf{4}\\\mathbf{-3} & \boldsymbol{a}\end{bmatrix}-\mathbf{3}\begin{bmatrix}\mathbf{1} & \boldsymbol{b}\\\mathbf{8} & \mathbf{-4}\end{bmatrix}=\begin{bmatrix}\mathbf{7} & \mathbf{10}\\\mathbf{18} & \mathbf{1}\end{bmatrix}$ **then find the value of a & b.**

$$\begin{bmatrix}4 & 8\\-6 & 2a\end{bmatrix}-\begin{bmatrix}3 & 3b\\24 & -12\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

$$\begin{bmatrix}4-3 & 8-3b\\-6-24 & 2a-(-12)\end{bmatrix}=\begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$$

From equality of matrices:

8 - 3b = 10 ; 2a + 12 = 1

-3b = 10 - 8 ; 2a = 1-12

-3b = 2 ; 2a = -11

b = 2/-3 = -2/3 ; a= -11/2

Q.8: **If** $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$ $\boldsymbol{and\ B} = \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{0} \end{bmatrix}$ **then verify that:**

(i). $(A + B)^t = A^t + B^t$

L.H.S:

$$A + B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+1 \\ 0+2 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

$$(A + B)^t = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

R.H.S:

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 0+2 \\ 2+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

Hence Proved: **L.H.S = R.H.S**

(ii). $(A - B)^t = A^t - B^t$

L.H.S:

$$A + B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-1 \\ 0-2 & 1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$$

$$(A + B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

R.H.S:

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

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$$A^t + B^t = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}1-1 & 0-2\\2-1 & 1-0\end{bmatrix}$$

$$= \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$

Hence Proved: **L.H.S = R.H.S**

(iii). $A + A^t$ is symmetric.

$$A^t = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$A + A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} = \begin{bmatrix}1+1 & 2+0\\0+2 & 1+1\end{bmatrix}$$

$$= \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

Now,

$$\mathbf{(A + A^t)^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} = (A + A^t)}$$

Hence proved that $A + A^t$ is symmetric.

(iv). $A - A^t$ is skew-symmetric.

$$A^t = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$A + A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} = \begin{bmatrix}1-1 & 2-0\\0-2 & 1-1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

Now,

$$\mathbf{(A + A^t)^t = \begin{bmatrix}0 & -2\\2 & 0\end{bmatrix} = -(A + A^t)}$$

Hence proved that $A + A^t$ is skew- symmetric.

(v). $B + B^t$ is symmetric.

$$B^t = \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$B + B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix} = \begin{bmatrix}1+1 & 1+2\\2+1 & 0+0\end{bmatrix}$$

$$= \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}$$

Now,

$$\mathbf{(B + B^t)^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix} = (B + B^t)^t}$$

Hence proved that $B + B^t$ is symmetric.

(vi). $B - B^t$ is skew-symmetric.

$$B^t = \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$

$$B - B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix} = \begin{bmatrix}1-1 & 1-2\\2-1 & 0-0\end{bmatrix}$$

$$= \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

Now,

$$\mathbf{(B + B^t)^t = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix} = -(B + B^t)^t}$$

Hence proved that $B + B^t$ is skew-symmetric.

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Multiplication of Matrices: Two matrices A & B can be multiplied if number of column of A are equal to number of rows of B.

Ex# 1.4

Q.1: Which of the following product of matrices is conformable for multiplication?

(i). $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Ans: Yes. Since number of columns of first is two that is equal to numbers of rows of second. Therefore they can be multiplied.

(ii). $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Ans: yes.

(iii). $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Ans: No

(iv). $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Ans: yes

(v). $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Ans: Yes

Q.2: If $\boldsymbol{A} = \begin{bmatrix} \mathbf{3} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \mathbf{6} \\ \mathbf{5} \end{bmatrix}$, find (i) AB and (ii) BA (if possible).

(i). AB = $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

AB = $\begin{bmatrix} (3 \times 6) + (0 \times 5) \\ (-1 \times 6) + (2 \times 5) \end{bmatrix}$

AB = $\begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix}$

AB = $\begin{bmatrix} 18 \\ 4 \end{bmatrix}$

(ii). BA is not possible.

Q.3: Find the product of following matrices:

(i). $[1 \;\; 2]\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

= $[(1 \times 4) \quad (2 \times 0)]$

= $[4 \quad 0]$

(ii). $[1 \;\; 2]\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

= $[(1 \times 5) \quad (2 \times -4)]$

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$= [5 \quad -8]$

(iii). $[3 \;\; 0]\begin{bmatrix}4\\0\end{bmatrix}$

$= [(3\times 4) \quad (0\times 0)]$

$= [12 \quad 0]$

(iv). $[6 \;\; -0]\begin{bmatrix}4\\0\end{bmatrix}$

$= [(6\times 4) \quad (-0\times 0)]$

$= [24 \quad 0]$

(v). $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

$= \begin{bmatrix}(1\times 4)+(2\times 0) & (1\times 5)+(2\times -4)\\(-3\times 4)+(0\times 0) & (-3\times 5)+(0\times -4)\\(6\times 4)+(-1\times 0) & (6\times 5)+(-1\times -4)\end{bmatrix}$

$= \begin{bmatrix}4+0 & 5-8\\-12+0 & -15+0\\24+0 & 30+4\end{bmatrix}$

$= \begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$

Q.4: Multiply the following matrices.

a) $\begin{bmatrix}\mathbf{2} & \mathbf{3}\\\mathbf{1} & \mathbf{1}\\\mathbf{0} & \mathbf{-2}\end{bmatrix}\begin{bmatrix}\mathbf{2} & \mathbf{-1}\\\mathbf{3} & \mathbf{0}\end{bmatrix}$

$= \begin{bmatrix}(2\times 2)+(3\times 3) & (2\times -1)+(3\times 0)\\(1\times 2)+(1\times 3) & (1\times -1)+(1\times 0)\\(0\times 2)+(-2\times 3) & (0\times -1)+(-2\times 0)\end{bmatrix}$

$= \begin{bmatrix}4+9 & -2+0\\2+3 & -1+0\\0-6 & 0+0\end{bmatrix}$

$= \begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$

b) $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\\mathbf{4} & \mathbf{5} & \mathbf{6}\end{bmatrix}\begin{bmatrix}\mathbf{1} & \mathbf{2}\\\mathbf{3} & \mathbf{4}\\\mathbf{-1} & \mathbf{1}\end{bmatrix}$

$= \begin{bmatrix}(1\times 1)+(2\times 3)+(3\times -1) & (1\times 2)+(2\times 4)+(3\times 1)\\(4\times 1)+(5\times 3)+(6\times -1) & (4\times 2)+(5\times 4)+(6\times 1)\end{bmatrix}$

$= \begin{bmatrix}1+6-3 & 2+8+3\\4+15-6 & 8+20+6\end{bmatrix}$

$= \begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$

c) $\begin{bmatrix}\mathbf{1} & \mathbf{2}\\\mathbf{3} & \mathbf{4}\\\mathbf{-1} & \mathbf{-1}\end{bmatrix}\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\\mathbf{4} & \mathbf{5} & \mathbf{6}\end{bmatrix}$

$= \begin{bmatrix}(1\times 1)+(2\times 4) & (1\times 2)+(2\times 5) & (1\times 3)+(2\times 6)\\(3\times 1)+(4\times 4) & (3\times 2)+(4\times 5) & (3\times 3)+(4\times 6)\\(-1\times 1)+(-1\times 4) & (-1\times 2)+(-1\times 5) & (-1\times 3)+(-1\times 6)\end{bmatrix}$

$$= \begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1-4 & -2-5 & -3-6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ -5 & -7 & -9 \end{bmatrix}$$

d) $\begin{bmatrix} \mathbf{8} & \mathbf{5} \\ \mathbf{6} & \mathbf{4} \end{bmatrix}\begin{bmatrix} \mathbf{2} & -\frac{5}{2} \\ \mathbf{-4} & \mathbf{4} \end{bmatrix}$

$$= \begin{bmatrix} (8\times 2)+(5\times -4) & \left(8\times -\frac{5}{2}\right)+(5\times 4) \\ (6\times 2)+(4\times -4) & \left(6\times -\frac{5}{2}\right)+(4\times 4) \end{bmatrix}$$

$$= \begin{bmatrix} 16-20 & -40+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 20 \\ -4 & 1 \end{bmatrix}$$

e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} (-1\times 0)+(2\times 0) & (-1\times 0)+(2\times 0) \\ (1\times 0)+(3\times 0) & (1\times 0)+(3\times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q.5: Let $A = \begin{bmatrix} \mathbf{-1} & \mathbf{3} \\ \mathbf{2} & \mathbf{0} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{-3} & \mathbf{-5} \end{bmatrix}$ $\boldsymbol{and\ C} = \begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} \end{bmatrix}, \boldsymbol{verify\ whether}$:

(i). AB =BA

L.H.S

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times 1)+(3\times -3) & (-1\times 2)+(3\times -5) \\ (2\times 1)+(0\times -3) & (2\times 2)+(0\times -5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

R.H.S

$$BA = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times -1)+(2\times 2) & (1\times 3)+(2\times 0) \\ (-3\times -1)+(-5\times 2) & (-3\times 3)+(-5\times 0) \end{bmatrix}$$

$$= \begin{bmatrix} -1+4 & 3+0 \\ 3-10 & -9+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 \\ -7 & -9 \end{bmatrix}$$

S0, AB ≠ BA

(ii). A(BC) = (AB)C

L.H.S

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

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$$= \begin{bmatrix} (1\times 2)+(2\times 1) & (1\times 1)+(2\times 3) \\ (-3\times 2)+(-5\times 1) & (-3\times 1)+(-5\times 3) \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$$

$$BC = \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

Now,

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times 4)+(3\times -11) & (-1\times 7)+(3\times -18) \\ (2\times 4)+(0\times -11) & (2\times 7)+(0\times -18) \end{bmatrix}$$

$$= \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix}$$

$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$$

R.H.S

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times 1)+(3\times -3) & (-1\times 2)+(3\times -5) \\ (2\times 1)+(0\times -3) & (2\times 2)+(0\times -5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-10\times 2)+(-17\times 1) & (-10\times 1)+(-17\times 3) \\ (2\times 2)+(4\times 1) & (2\times 1)+(4\times 3) \end{bmatrix}$$

$$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$$

Hence, A(BC) = (AB)C

(iii). A(B + C) = AB + AC

L.H.S

$$B + C = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

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$$= \begin{bmatrix} (-1\times 3)+(3\times -2) & (-1\times 3)+(3\times -2) \\ (2\times 3)+(0\times -2) & (2\times 3)+(0\times -2) \end{bmatrix}$$

$$= \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

R.H.S

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times 1)+(3\times -3) & (-1\times 2)+(3\times -5) \\ (2\times 1)+(0\times -3) & (2\times 2)+(0\times -5) \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times 2)+(3\times 1) & (-1\times 1)+(3\times 3) \\ (2\times 2)+(0\times 1) & (2\times 1)+(0\times 3) \end{bmatrix}$$

$$= \begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10+1 & -17+8 \\ 2+4 & 4+2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

Hence, $A(B+C) = AB + AC$

(iv). A(B – C) = AB – AC

$$B - C = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ -4 & -8 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -4 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times -1)+(3\times -4) & (-1\times 1)+(3\times -8) \\ (2\times -1)+(0\times -4) & (2\times 1)+(0\times -8) \end{bmatrix}$$

$$= \begin{bmatrix} 1-12 & -1-24 \\ -2+0 & 2+0 \end{bmatrix}$$

A(B - C) = $\begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix}$

R.H.S

AB = $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

= $\begin{bmatrix} (-1\times 1)+(3\times -3) & (-1\times 2)+(3\times -5) \\ (2\times 1)+(0\times -3) & (2\times 2)+(0\times -5) \end{bmatrix}$

= $\begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$

= $\begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$

AC = $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

= $\begin{bmatrix} (-1\times 2)+(3\times 1) & (-1\times 1)+(3\times 3) \\ (2\times 2)+(0\times 1) & (2\times 1)+(0\times 3) \end{bmatrix}$

= $\begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$

= $\begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$

AB - AC = $\begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$

= $\begin{bmatrix} -10-1 & -17-8 \\ 2-4 & 4-2 \end{bmatrix}$

= $\begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix}$

Hence, A(B - C) = AB – AC

Q.6: For the matrices

$$\boldsymbol{A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \; and \; C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}}$$

Verify that:

(i). $(AB)^t = B^tA^t$

L.H.S

AB = $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

= $\begin{bmatrix} (-1\times 1)+(3\times -3) & (-1\times 2)+(3\times -5) \\ (2\times 1)+(0\times -3) & (2\times 2)+(0\times -5) \end{bmatrix}$

= $\begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$

= $\begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$

$(AB)^t$ = $\begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$

R.H.S

$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$

$A^t = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$

$B^t\ A^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} (1 \times -1) + (-3 \times 3) & (1 \times 2) + (-3 \times 0) \\ (2 \times -1) + (-5 \times 3) & (2 \times 2) + (-5 \times 0) \end{bmatrix}$

$= \begin{bmatrix} -1 - 9 & 2 + 0 \\ -2 - 15 & 4 + 0 \end{bmatrix}$

$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$

Hence, $(AB)^t = B^tA^t$

(ii). Do yourself

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Singular Matrices: A square matrix A is called singular, if the determinant of A is equal to zero. i.e., $|A| = 0$.

Non-Singular Matrices: A square matrix A is called singular, if the determinant of A is not equal to zero. i.e., $|A| \neq 0$.

Ex# 1.5

Q.1: Find the determinant of following matrices:

(i). $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} = (-1 \times 0) - (2 \times 1) = 0 - 2 = -2$

(ii). $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} = (1 \times -2) - (2 \times 3) = -2 - 6 = -8$

(iii). $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

$|C| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} = (3 \times 2) - (3 \times 2) = 6 - 6 = 0$

(iv). $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$|D| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (1 \times 2) = 12 - 2 = 10$

Q.2: Find which of the following matrices are singular or non-singular?

(i). $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} = (3 \times 4) - (2 \times 6) = 12 - 12 = 0$

So, A is a singular matrix.

(ii). $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = (4 \times 2) - (3 \times 1) = 8 - 3 = 5$

So, B is a non-singular matrix.

(iii). $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix} = (7 \times 5) - (3 \times -9) = 35 + 27 = 62$

So, C is a non-singular matrix.

(iv). $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix} = (5 \times 4) - (-2 \times -10) = 20 - 20 = 0$

So, D is a singular matrix.

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Q.3: Find the multiplicative inverse of following matrices (if exist):

(i). $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$|A| = (-1 \times 0) - (3 \times 2)$

$= -6$

$Adj\ (A) = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$

$A^{-1} = \frac{1}{-6} \times \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 \times \frac{1}{-6} & -3 \times \frac{1}{-6} \\ -2 \times \frac{1}{-6} & 1 \times \frac{1}{-6} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{6} \end{bmatrix}$

(ii). $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$|B| = (1 \times (-5)) - (-3 \times 2)$

$= 1$

$Adj\ (B) = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} \times Adj\ (B)$

$B^{-1} = \frac{1}{1} \times \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

(iii). Same as above. Do yourself.

(iv). $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

$|A| = (5 \times 4) - (-10 \times -2)$

$= 0$

$Since\ |A| = 0, so\ its\ multiplicative\ Inverse\ does\ not\ exists.$

Q.4: If $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{6} \end{bmatrix}$ & $\boldsymbol{B} = \begin{bmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-2} \end{bmatrix}$ **Then,**

(i). $\boldsymbol{A(Adj\ A) = (Adj\ A)A = (det\ A)I}$

$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{6} \end{bmatrix}$

L.H.S=

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$Adj\ (A) = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

$A(Adj\ A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

$A(Adj\ A) = \begin{bmatrix} 6-8 & -2+2 \\ 24-24 & -8+6 \end{bmatrix}$

$A(Adj\ A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

Middle Side=M.S.

$(Adj\ A)A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

$(Adj\ A)A = \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$

$(Adj\ A)A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

R.H.S=

$(det\ A) = |A| \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = 6 - 8 = -2$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(det\ A)I = -2 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

Hence prove that

$\boldsymbol{A(Adj\ A) = (Adj\ A)A = (det\ A)I}$

(ii). $\boldsymbol{BB^{-1} = I = B^{-1}B}$

$\boldsymbol{B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}}$

$|B| = (3 \times (-2)) - (2 \times (-1))$

$= -4$

$Adj\ (B) = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$B^{-1} = \dfrac{1}{|B|} \times Adj\ (B)$

$B^{-1} = \dfrac{1}{-4} \times \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$B^{-1} = -\dfrac{1}{4} \times \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

L.H.S=

$\boldsymbol{BB^{-1}} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \times (-\frac{1}{4}) \times \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$BB^{-1} = (-\frac{1}{4}) \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

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$$BB^{-1} = (-\frac{1}{4})\begin{bmatrix}-6+2 & 3-3\\-4+4 & 2-6\end{bmatrix}$$

$$BB^{-1} = \left(-\frac{1}{4}\right)\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix} = \begin{bmatrix}-4\times-\frac{1}{4} & 0\times-\frac{1}{4}\\0\times-\frac{1}{4} & -4\times-\frac{1}{4}\end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I \dots\dots\dots\dots\dots\dots\dots\dots..(i)$$

R.H.S=

$$B^{-1}B = (-\frac{1}{4})\times\begin{bmatrix}-2 & 1\\-2 & 3\end{bmatrix}\begin{bmatrix}3 & -1\\2 & -2\end{bmatrix}$$

$$B^{-1}B = (-\frac{1}{4})\begin{bmatrix}-6+2 & 2-2\\-6+6 & 2-6\end{bmatrix}$$

$$B^{-1}B = \left(-\frac{1}{4}\right)\begin{bmatrix}-4 & 0\\0 & -4\end{bmatrix} = \begin{bmatrix}-4\times-\frac{1}{4} & 0\times-\frac{1}{4}\\0\times-\frac{1}{4} & -4\times-\frac{1}{4}\end{bmatrix}$$

$$\boldsymbol{B^{-1}B = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I \dots\dots\dots\dots\dots\dots\dots\dots..(ii)}$$

From (i) & (ii) it is proved that:

$$\boldsymbol{BB^{-1} = I = B^{-1}B}$$

Q.5: Determine whether the given matrices are multiplicative inverses of each other

(i). $\begin{bmatrix}3 & 5\\4 & 7\end{bmatrix}$ and $\begin{bmatrix}7 & -5\\-4 & 3\end{bmatrix}$

$let\ A = \begin{bmatrix}3 & 5\\4 & 7\end{bmatrix}$

Than

$A^{-1} = \begin{bmatrix}7 & -5\\-4 & 3\end{bmatrix}$

As we know that

$AA^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$

$AA^{-1} = \begin{bmatrix}3 & 5\\4 & 7\end{bmatrix}\begin{bmatrix}7 & -5\\-4 & 3\end{bmatrix}$

$= \begin{bmatrix}21-20 & -15+15\\28-28 & -20+21\end{bmatrix}$

$= \boldsymbol{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I}$

Hence verified that $\begin{bmatrix}3 & 5\\4 & 7\end{bmatrix}$ and $\boldsymbol{\begin{bmatrix}7 & -5\\-4 & 3\end{bmatrix}}$ **are multiplicative inverses of each other.**

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(ii). $\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$ and $\begin{bmatrix}-3 & 2\\2 & -1\end{bmatrix}$

$let\ A = \begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$

Than

$\boldsymbol{A^{-1}} = \begin{bmatrix}\mathbf{-3} & \mathbf{2}\\\mathbf{2} & \mathbf{-1}\end{bmatrix}$

As we know that

$\boldsymbol{AA^{-1}} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$

$AA^{-1} = \begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}\begin{bmatrix}-3 & 2\\2 & -1\end{bmatrix}$

$= \begin{bmatrix}-3+4 & 2-2\\-6+6 & 4-3\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$

(iii).Hence verified that $\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$ and $\begin{bmatrix}\mathbf{-3} & \mathbf{2}\\\mathbf{2} & \mathbf{-1}\end{bmatrix}$ **are multiplicative inverses of each other.**

Q.6: If

$$\boldsymbol{A} = \begin{bmatrix}\mathbf{4} & \mathbf{0}\\\mathbf{-1} & \mathbf{2}\end{bmatrix}, \boldsymbol{B} = \begin{bmatrix}\mathbf{-4} & \mathbf{-2}\\\mathbf{1} & \mathbf{-1}\end{bmatrix}\ \boldsymbol{and\ D} = \begin{bmatrix}\mathbf{3} & \mathbf{2}\\\mathbf{-2} & \mathbf{1}\end{bmatrix}$$

Verify that:

(i). $(\boldsymbol{AB})^{-1} = \boldsymbol{B^{-1}A^{-1}}$

L.H.S

$\mathrm{AB} = \begin{bmatrix}\mathbf{4} & \mathbf{0}\\\mathbf{-1} & \mathbf{2}\end{bmatrix}\begin{bmatrix}\mathbf{-4} & \mathbf{-2}\\\mathbf{1} & \mathbf{-1}\end{bmatrix}$

$\mathrm{AB} = \begin{bmatrix}-16+0 & -8+0\\4+2 & 2-2\end{bmatrix}$

$\mathrm{AB} = \begin{bmatrix}-16 & -8\\6 & 0\end{bmatrix}$

$|AB| = 0 + 48$

$= 48$

$Adj\ (AB) = \begin{bmatrix}0 & 8\\-6 & -16\end{bmatrix}$

$(AB)^{-1} = \dfrac{1}{|AB|} \times Adj\ (AB)$

$(AB)^{-1} = \dfrac{1}{48} \times \begin{bmatrix}0 & 8\\-6 & -16\end{bmatrix}$

$(AB)^{-1} = \begin{bmatrix}0 \times \dfrac{1}{48} & 8 \times \dfrac{1}{48}\\-6 \times \dfrac{1}{48} & -16 \times \dfrac{1}{48}\end{bmatrix}$

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$$(AB)^{-1} = \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

R.H.S

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{4} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} \end{bmatrix}$$

$|A| = 8 + 0$

$= 8$

$$Adj\ (A) = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$$

$$A^{-1} = \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{-4} & \mathbf{-2} \\ \mathbf{1} & \mathbf{-1} \end{bmatrix}$$

$|B| = 4 + 2$

$= 6$

$$Adj\ (B) = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times Adj\ (B)$$

$$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$\boldsymbol{B^{-1}A^{-1}} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\boldsymbol{B^{-1}A^{-1}} = \frac{1}{48} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\boldsymbol{B^{-1}A^{-1}} = \frac{1}{48} \times \begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$$

$$\boldsymbol{B^{-1}A^{-1}} = \begin{bmatrix} 0 \times \frac{1}{48} & 8 \times \frac{1}{48} \\ -6 \times \frac{1}{48} & -16 \times \frac{1}{48} \end{bmatrix}$$

$$\boldsymbol{B^{-1}A^{-1}} = \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

hence prove that

L.H.S = R.H.S

(ii). Do yourself

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Ex# 1.6

Q.1: Use matrices, if possible, to solve the following systems of linear equations by:

i. **Matrix inversion method**
ii. **Cramer's rule**

(i).

$$2x - 2y = 4$$

$$3x + 2y = 6$$

Writing in matrices form:

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Matrix inversion method

If we let:

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Than we can write above equation as:

$$AX = B$$

$$X = A^{-1}B \text{ ....(1)}$$

So,

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = (2 \times 2) - \left(3 \times (-2)\right) = 4 - (-6) = 10$$

$$Adj\ (A) = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$$

$$A^{-1} = \frac{1}{10} \times \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

Putting values in equation (1):

$$X = \frac{1}{10} \times \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{10} \times \begin{bmatrix} 8 + 12 \\ -12 + 12 \end{bmatrix}$$

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$$X = \frac{1}{10} \times \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 20 \times \frac{1}{10} \\ 0 \times \frac{1}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

So, x= 2 ; y=0

Solution set: (2,0)

(i).

$$2x - 2y = 4$$

$$3x + 2y = 6$$

Cramer's rule

Writing in matrices form:

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

According to cramer rule:

$$\boldsymbol{x} = \frac{|\boldsymbol{A_x}|}{|\boldsymbol{A}|} \text{..................}\textbf{(1)}$$

$$\boldsymbol{y} = \frac{|\boldsymbol{A_y}|}{|\boldsymbol{A}|} \text{..................}\textbf{(2)}$$

Here

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} b_1 & -2 \\ b_2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & b_1 \\ 3 & b_2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = (2 \times 2) - \big(3 \times (-2)\big) = 4 - (-6) = 10$$

$$|A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix} = (4 \times 2) - \big(6 \times (-2)\big) = 8 - (-12) = 20$$

$$|A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = (2 \times 6) - (3 \times 4) = 12 - 12 = 0$$

So , from (1) and (2)

$$x = \frac{|A_x|}{|A|} = \frac{20}{10} = 2$$

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$$y = \frac{|A_y|}{|A|} = \frac{0}{10} = 0$$

So, x= 2 ; y=0
Solution set: (2,0)

(ii).

$$\mathbf{2x + y \ = 3}$$

$$\mathbf{6x + 5y \ = 1}$$

Writing in matrices form:

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Matrix inversion method

If we let:

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Than we can write above equation as:

$$AX = B$$

$$X = A^{-1}B \text{ ....(1)}$$

So,

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = (2 \times 5) - (6 \times 1) = 10 - 6 = 4$$

$$Adj\ (A) = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$$

$$A^{-1} = \frac{1}{4} \times \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

Putting values in equation (1):

$$X = \frac{1}{4} \times \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$X = \frac{1}{4} \times \begin{bmatrix} 15 - 1 \\ -18 + 2 \end{bmatrix}$$

$$X = \frac{1}{10} \times \begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$X = \begin{bmatrix} 14 \times \frac{1}{4} \\ -16 \times \frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7/2 \\ -4 \end{bmatrix}$$

So, $x = 7/2$; $y = -4$
Solution set: $(7/2, -4)$

Cramer rule

$$2x + y = 3$$

$$6x + 5y = 1$$

Writing in matrices form:

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

According to cramer rule:

$$\boldsymbol{x} = \frac{|A_x|}{|A|} \text{.................}\textbf{(1)}$$

$$\boldsymbol{y} = \frac{|A_y|}{|A|} \text{.................}\textbf{(2)}$$

Here

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$A_x = \begin{bmatrix} b_1 & 1 \\ b_2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & b_1 \\ 6 & b_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = (2 \times 5) - (6 \times 1) = 10 - 6 = 4$$

$$|A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = (5 \times 3) - (1 \times 1) = 15 - 1 = 14$$

$$|A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix} = (2 \times 1) - (3 \times 6) = 2 - 18 = -16$$

So , from (1) and (2)

$$x = \frac{|A_x|}{|A|} = \frac{14}{4} = \frac{7}{2}$$

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$$y = \frac{|A_y|}{|A|} = \frac{-16}{4} = -4$$

So, $x = {}^{7}/_{2}$; $y = 4$

Solution set: $({}^{7}/_{2}, -4)$

(iii). Do by yourself

(iv).

$$\mathbf{3x - 2y = 4}$$

$$\mathbf{-6x + 4y = 7}$$

Writing in matrices form:

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Than we can write above equation as:

$$AX = B$$

$X = A^{-1}B$ …………………………..(1)

So,

$$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} = (3 \times 4) - (-6 \times (-2)) = 12 - 12 = 0$$

$Since\ |A| = 0, therefore\ the\ solution\ for\ these\ equations\ does\ not\ exist.$

(v). Same as part (i) do yourself.

(vi). Same as part (i) do yourself.

(vii). Same as part (i) do yourself.

Solve the following word problems by using

1. **Matrix Inversion Method**
2. **Cramer's Rule**

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Q.2: The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle.

Solution:

Let

Length of rectangle = x

Width of rectangle = y

By condition of question:

$x = 4y$

$2(x + y) = 150$ $\quad\quad\quad\quad$ $\therefore perimeter\ of\ rectangle = 2\ (length + width)$

Simplifying above equations:

$$x - 4y = 0$$

$$2x + 2y = 150$$

1. **Matrix Inversion method**

Writing in matrices form:

$$\begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

Than we can write above equation as:

$AX = B$

$X = A^{-1}B$ ....(1)

So,

$$|A| = \begin{vmatrix} 1 & -4 \\ 2 & 2 \end{vmatrix} = (2 \times 1) - (-4 \times 2) = 2 - (-8) = 10$$

$$Adj\ (A) = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$$

$$A^{-1} = \frac{1}{4} \times \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$$

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Putting values in equation (1):

$$X = \frac{1}{10} \times \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

$$X = \frac{1}{10} \times \begin{bmatrix} 0 + 600 \\ 0 + 150 \end{bmatrix}$$

$$X = \frac{1}{10} \times \begin{bmatrix} 600 \\ 150 \end{bmatrix}$$

$$X = \begin{bmatrix} 600 \times \frac{1}{10} \\ 150 \times \frac{1}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 60 \\ 15 \end{bmatrix}$$

So, $\mathrm{x} = 60$; $\mathrm{y} = 15$

Solution set: (60 , 15)

Length of rectangle = 60 cm

Width of rectangle= 15 cm

Cramer rule

$$x - 4y = 0$$

$$2x + 2y = 150$$

Writing in matrices form:

$$\begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

According to Cramer rule:

$$\boldsymbol{x} = \frac{|A_x|}{|A|} \text{..................(1)}$$

$$\boldsymbol{y} = \frac{|A_y|}{|A|} \text{..................(2)}$$

Here

$$A = \begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} b_1 & -4 \\ b_2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 150 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & b_1 \\ 2 & b_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 150 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -4 \\ 2 & 2 \end{vmatrix} = (2 \times 1) - (-4 \times 2) = 2 - (-8) = 10$$

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$|A_x| = \begin{vmatrix} 0 & -4 \\ 150 & 2 \end{vmatrix} = (0 \times 2) - (-4 \times 150) = 0 - (-600) = 600$

$|A_y| = \begin{vmatrix} 1 & 0 \\ 2 & 150 \end{vmatrix} = (1 \times 150) - (2 \times 0) = 150 - 0 = 150$

So , from (1) and (2)

$$x = \frac{|A_x|}{|A|} = \frac{600}{10} = 60$$

$$y = \frac{|A_y|}{|A|} = \frac{150}{10} = 15$$

So, $x = 60$; $y = 15$

Solution set: (60 , 15)

Length of rectangle = 60 cm

Width of rectangle= 15 cm

Q.3: **Two sides of a rectangle differ by 3.5cm. Find the dimensions of the rectangle if its perimeter is 67cm.**

Solution:

Let

Length of rectangle = x

Width of rectangle = y

By condition of question:

$x - y = 3.5$

$2(x + y) = 67$ $\quad\quad \because perimeter\ of\ rectangle = 2\ (length + width)$

Simplifying above equations:

$$x - y = 3.5$$

$$2x + 2y = 67$$

1. **<u>Matrix Inversion method</u>**

Writing in matrices form:

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

If we let:

$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$

$X = \begin{bmatrix} x \\ y \end{bmatrix}$

$B = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$

Than we can write above equation as:

$$AX = B$$

$$X = A^{-1}B \text{ ....(1)}$$

So,

$$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = (2 \times 1) - (-1 \times 2) = 2 - (-2) = 4$$

$$Adj\ (A) = \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$$

$$A^{-1} = \frac{1}{4} \times \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$$

Putting values in equation (1):

$$X = \frac{1}{4} \times \begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

$$X = \frac{1}{4} \times \begin{bmatrix} 7 + 67 \\ -7 + 67 \end{bmatrix}$$

$$X = \frac{1}{4} \times \begin{bmatrix} 74 \\ 60 \end{bmatrix}$$

$$X = \begin{bmatrix} 74 \times \frac{1}{4} \\ 60 \times \frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18.5 \\ 15 \end{bmatrix}$$

So, $x = 18.5$; $y = 15$

Solution set: (18.5 , 15)

Length of rectangle = 18.5 cm

Width of rectangle= 15 cm

Cramer rule

$$x - y = 3.5$$

$$2x + 2y = 67$$

Writing in matrices form:

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

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According to Cramer rule:

$$\boldsymbol{x} = \frac{|\boldsymbol{A_x}|}{|\boldsymbol{A}|} \ldots\ldots\ldots\ldots\ldots\ldots\mathbf{(1)}$$

$$\boldsymbol{y} = \frac{|\boldsymbol{A_y}|}{|\boldsymbol{A}|} \ldots\ldots\ldots\ldots\ldots\ldots\mathbf{(2)}$$

Here

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} b_1 & -4 \\ b_2 & 2 \end{bmatrix} = \begin{bmatrix} 3.5 & -1 \\ 67 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & b_1 \\ 2 & b_2 \end{bmatrix} = \begin{bmatrix} 1 & 3.5 \\ 2 & 67 \end{bmatrix}$$

$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = (2 \times 1) - (-1 \times 2) = 2 - (-2) = 4$

$|A_x| = \begin{vmatrix} 3.5 & -1 \\ 67 & 2 \end{vmatrix} = (3.5 \times 2) - (-1 \times 67) = 7 - (-67) = 74$

$|A_y| = \begin{vmatrix} 1 & 3.5 \\ 2 & 67 \end{vmatrix} = (1 \times 67) - (2 \times 3.5) = 67 - 7 = 60$

So , from (1) and (2)

$$x = \frac{|A_x|}{|A|} = \frac{74}{4} = 18.5$$

$$y = \frac{|A_y|}{|A|} = \frac{60}{4} = 15$$

So, $x = 18.5$; $y = 15$

Solution set: (18.5 , 15)

Length of rectangle = 18.5 cm

Width of rectangle= 15 cm

Q.4: The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Solution:

Let

One of the equal angle = x

Third angle=y

By condition of question:

$x + x - 16 = y$

$2x - y = 16$

As we know that

Sum of three angles of triangles = 180 degrees

$$x + x + y = 180$$

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Simplifying above equations:

$$2x - y = 16$$

$$2x + y = 180$$

1. **Matrix Inversion method**

Writing in matrices form:

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

Than we can write above equation as:

$$AX = B$$

$X = A^{-1}B$ ....(1)

So,

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = (2 \times 1) - (-1 \times 2) = 2 - (-2) = 4$$

$$Adj\ (A) = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$$

$$A^{-1} = \frac{1}{4} \times \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$$

Putting values in equation (1):

$$X = \frac{1}{4} \times \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

$$X = \frac{1}{4} \times \begin{bmatrix} 16 + 180 \\ -32 + 360 \end{bmatrix}$$

$$X = \frac{1}{4} \times \begin{bmatrix} 196 \\ 328 \end{bmatrix}$$

$$X = \begin{bmatrix} 196 \times \frac{1}{4} \\ 328 \times \frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

So, $x = 49$; $y = 82$

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Solution set: (49 , 82)
First of two equal angle = 49°
Second of two equal angle = 49°
Third angle = 82^0

Cramer rule

$$2x - y = 16$$

$$2x + y = 180$$

Writing in matrices form:

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

According to Cramer rule:

$$\boldsymbol{x} = \frac{|A_x|}{|A|} \text{..................(1)}$$

$$\boldsymbol{y} = \frac{|A_y|}{|A|} \text{..................(2)}$$

Here

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} b_1 & -1 \\ b_2 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -1 \\ 180 & 1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & b_1 \\ 2 & b_2 \end{bmatrix} = \begin{bmatrix} 2 & 16 \\ 2 & 180 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} = (2 \times 1) - (-1 \times 2) = 2 - (-2) = 4$$

$$|A_x| = \begin{vmatrix} 16 & -1 \\ 180 & 1 \end{vmatrix} = (16 \times 1) - (-1 \times 180) = 16 - (-180) = 196$$

$$|A_y| = \begin{vmatrix} 2 & 16 \\ 2 & 180 \end{vmatrix} = (2 \times 180) - (2 \times 16) = 360 - 32 = 328$$

So , from (1) and (2)

$$x = \frac{|A_x|}{|A|} = \frac{196}{4} = 49$$

$$y = \frac{|A_y|}{|A|} = \frac{328}{4} = 82$$

So, $x = 49$; $y = 82$
Solution set: (49 , 82)
First of two equal angle = 49°
Second of two equal angle = 49°

Third angle = 82^0

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Q.5: One acute angle of a right triangle is 12°more than twice the other acute angle. Find the acute angles of the right triangle.

Let

One acute angle = x

Second cute angle = y

As we know that

Third angle = 90^0

By condition:

$$x - 12 = 2y$$

$$x - 2y = 12 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

As we know that

Sum of three angles of triangles = 180 degrees

$$x + y + 90 = 180$$

$$x + y = 90 \ldots\ldots\ldots\ldots..(2)$$

Now do yourself.

Q.6: Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Let

Speed of one car = x

Speed of other car = y

By condition of question:

$x - y = 6$………..(1)

After $4\frac{1}{2}$ hours:

$$4.5x + 4.5y = 477$$

$$4.5(x + y) = 477$$

$$(x + y) = 477/4.5$$

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$$x + y = 106 \ldots\ldots\ldots\ldots (2)$$

Simplifying above equations:

$$x - y = 6$$

$$x + y = 106$$

1. **Matrix Inversion method**

Writing in matrices form:

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

Than we can write above equation as:

$$AX = B$$

$$X = A^{-1}B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

So,

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = (1 \times 1) - (-1 \times 1) = 1 - (-1) = 2$$

$$Adj\ (A) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\ (A)$$

$$A^{-1} = \frac{1}{2} \times \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Putting values in equation (1):

$$X = \frac{1}{2} \times \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$X = \frac{1}{2} \times \begin{bmatrix} 6 + 106 \\ -6 + 106 \end{bmatrix}$$

$$X = \frac{1}{2} \times \begin{bmatrix} 112 \\ 100 \end{bmatrix}$$

$$X = \begin{bmatrix} 112 \times \frac{1}{2} \\ 100 \times \frac{1}{2} \end{bmatrix}$$

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$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 56 \\ 50 \end{bmatrix}$$

So, $x = 56$; $y = 50$
Solution set: (56 ,50)
Speed of a car =56 kmh^{-1}
Speed of other car =50 kmh^{-1}

Cramer rule

$$x - y = 6$$
$$x + y = 106$$

Writing in matrices form:

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

If we let:

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

According to Cramer rule:

$$\boldsymbol{x = \frac{|A_x|}{|A|}} \text{..................}\textbf{(1)}$$
$$\boldsymbol{y = \frac{|A_y|}{|A|}} \text{..................}\textbf{(2)}$$

Here

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$A_x = \begin{bmatrix} b_1 & -1 \\ b_2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 106 & 1 \end{bmatrix}$$
$$A_y = \begin{bmatrix} 1 & b_1 \\ 2 & b_2 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 1 & 106 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = (1 \times 1) - (-1 \times 1) = 1 - (-1) = 2$$
$$|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix} = (6 \times 1) - (-1 \times 106) = 6 - (-106) = 112$$
$$|A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix} = (1 \times 106) - (1 \times 6) = 106 - 6 = 100$$

So , from (1) and (2)

$$x = \frac{|A_x|}{|A|} = \frac{112}{2} = 56$$
$$y = \frac{|A_y|}{|A|} = \frac{100}{2} = 50$$

So, $x = 56$; $y = 50$
Solution set: (56 ,50)
Speed of a car =56 kmh^{-1}
Speed of other car =50 kmh^{-1}