MathCity.org	Exercise 2.5 (Solutions)
Merging man and maths	Mathematics (Science Group): 10^{th} Written by Amir Shehzad, Version: 1.0

Q.1 Write the quadratic equation having following roots.

(a) 1,5

Solution: Since 1 and 5 are the roots of the required quadratic equation, therefore

Sum of roots=S=1+5=6

Product of roots=P=1 $\times$ 5=5

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - 6x + 5 = 0$

(b) 4,9

Solution: Since 4 and 9 are the roots of the required quadratic equation, therefore

Sum of roots=S=4+9=13

Product of roots=P=4 $\times$ 9=36

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - 13x + 36 = 0$

(c) -2,3

Solution: Since -2 and 3 are the roots of the required quadratic equation, therefore

Sum of roots=S=-2+3=1

Product of roots=P=-2 $\times$ 3=-6

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - x + 6 = 0$

(d) 0, -3

Solution: Since 0 and -3 are the roots of the required quadratic equation, therefore

Sum of roots=S=0+(-3) =-3

Product of roots=P=0 $\times$ -3=0

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - (-3)x + 0 = 0 \qquad \Rightarrow x^2 + 3x = 0$

(e) 2, -6

Solution: Since 2 and -6 are the roots of the required quadratic equation, therefore

Sum of roots=S=2+(-6) =2-6=-4

Product of roots=P=2 $\times$ -6=-12

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - (-4)x + (-12) = 0 \quad \Rightarrow x^2 + 4x - 12 = 0$

(f) -1, -7

Solution: Since -1 and -7 are the roots of the required quadratic equation, therefore

Sum of roots=S= (-1) +(-7) =-1-7=-8

Product of roots=P= (-1) $\times$ (-7) =7

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - (-8)x + 7 = 0 \quad \Rightarrow x^2 + 8x + 7 = 0$

(g) 1+i , 1-i

Solution: Since 1+i and 1-i are the roots of the required quadratic equation, therefore

Sum of roots=S= 1+i +1-i =2

Product of roots=P= (1+i) $\times$ (1-i)

$$P = (1)^2 - (i)^2 = 1 - (-1)$$

$=1+1=2$

As $x^2-Sx+P=0$ so the required equation is $x^2-2x+2=0$

(h) $3+\sqrt{2}$, $3-\sqrt{2}$

Solution: Since $3+\sqrt{2}$ and $3-\sqrt{2}$ are the roots of the required quadratic equation, therefore

Sum of roots=S= $3+\sqrt{2}+3-\sqrt{2}=6$

Product of roots=P= $(3+\sqrt{2})\times(3-\sqrt{2})$

$$P=\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$$

$$=(3)^2-\left(\sqrt{2}\right)^2=9-2=7$$

As $x^2-Sx+P=0$ so the required equation is $x^2-6x+7=0$

Q.2 If α,β are the roots of the equation $x^2-3x+6=0$. From equation whose roots are:

Solution: As α,β are the roots of the equation $x^2-3x+6=0$.

$$a=1,\ b=-3,\ c=6$$

Therefore,

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-3)}{1}$$

$$=3\ \Rightarrow \boxed{\alpha+\beta=3}$$

$$\alpha\beta=\frac{c}{a}=\frac{6}{1}=6\ \Rightarrow \boxed{\alpha\beta=6}$$

(a) $2\alpha+1, 2\beta+1$

Solution: Sum of roots

S= $2\alpha+1, 2\beta+1$

S= $2\alpha+2\beta+2$

S= $2(\alpha+\beta)+2$

S=2(3) +2=6+2=8 $\Rightarrow \boxed{S=8}$

Product of roots

$$P=(2\alpha+1)(2\beta+1)$$
$$P=4\alpha\beta+2\alpha+2\beta+1$$
$$P=4\alpha\beta+2(\alpha+\beta)+1$$
$$P=4(6)+2(3)+1$$
$$P=24+6+1=31$$
$$\boxed{P=31}$$

Using $x^2-Sx+P=0$, we have

$x^2-8x+31=0$

(b) α^2,β^2

Solution: Sum of roots $=S=\alpha^2,\beta^2$

$$S=(\alpha+\beta)^2-2\alpha\beta$$
$$S=(3)^2-2(6)=9-12$$
$$S=-3\ \Rightarrow \boxed{S=-3}$$

Product of roots

$$P=\alpha^2,\beta^2=(\alpha\beta)^2$$
$$P=(6)^2=36\ \Rightarrow \boxed{P=36}$$

Using $x^2-Sx+P=0$, we have

$x^2-(-3)x+36=0$

$x^2+3x+36=0$

(c) $\frac{1}{\alpha},\frac{1}{\beta}$

Solution: Sum of roots $=S=\frac{1}{\alpha},\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$$S=(\alpha+\beta).\frac{1}{\alpha\beta}$$

$$S = 3\times\frac{1}{6} = \frac{3}{6} = \frac{1}{2} \Rightarrow \boxed{S = \frac{1}{2}}$$

Product of roots

$$P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{\alpha\beta}$$

$$P = \frac{1}{6} \Rightarrow \boxed{P = \frac{1}{6}}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

Multiplying by '6' on both sides

$$6x^2 - 3x + 1 = 0$$

(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution: Sum of roots $= S = \frac{\alpha}{\beta}, \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\beta\alpha}$

$$S = .\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(3)^2 - 2(6)}{6}$$

$$S = \frac{9-12}{6} = \frac{-3}{6} \Rightarrow \boxed{S = \frac{-1}{2}}$$

Product of roots

$$P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = \frac{\alpha\beta}{\beta\alpha}$$

$$P = \frac{6}{6} \Rightarrow \boxed{P = 1}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 + \frac{1}{2}x + 1 = 0$$

Multiplying by '2' on both sides

$$2x^2 + x + 2 = 0$$

(e) $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

Solution: Sum of roots $= S = (\alpha+\beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$S = (\alpha+\beta)\frac{\beta+\alpha}{\alpha\beta} = (\alpha+\beta) + \frac{(\alpha+\beta)}{\alpha\beta}$$

$$S = 3 + \frac{3}{6} = 3 + \frac{1}{2} = \frac{6+1}{2} \Rightarrow \boxed{S = \frac{7}{2}}$$

Product of roots

$$P = (\alpha+\beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = (\alpha+\beta)\left(\frac{\beta+\alpha}{\beta\alpha}\right)$$

$$P = (\alpha+\beta)\left(\frac{\alpha+\beta}{\beta\alpha}\right) = 3\left(\frac{3}{6}\right) \Rightarrow \boxed{P = \frac{3}{2}}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 + \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying by '2' on both sides

$$2x^2 - 7x + 3 = 0$$

Q.3 If α, β are the roots of the equation $x^2 + px + q = 0$. From equation whose roots are

Solution: Since α, β are the roots of the equation $x^2 + px + q = 0$.

$$ax^2 + bx + c = 0$$

By compering the coefficients of these equations, we have

$$a = 1,\ b = p,\ c = q$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1}$$

$$= -p \Rightarrow \boxed{\alpha + \beta = -p}$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q \Rightarrow \boxed{\alpha\beta = q}$$

(a) α^2, β^2

Sum of roots $= S = \alpha^2 + \beta^2$

$$= (\alpha+\beta)^2 - 2\alpha\beta$$

$$=(-p)^2-2q \quad \Rightarrow \boxed{p^2-2q}$$

$$\text{Product of roots} = \text{P}=\alpha^2\beta^2=(\alpha\beta)^2$$

$$=(q)^2 \qquad \Rightarrow \boxed{q^2}$$

Using $x^2-Sx+P=0$, we have

$x^2-(p^2-2q)x+q^2=0$.

(b)

$\dfrac{\alpha}{\beta},\dfrac{\beta}{\alpha}$

$$\text{Sum of roots } = \text{S}=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{(-p)^2-2q}{q}$$

$$=\frac{p^2-2q}{q} \qquad \Rightarrow \boxed{\frac{p^2-2q}{q}}$$

$$\text{Product of roots} = \text{P}=\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)=\left(\frac{\alpha\beta}{\beta\alpha}\right)$$

$$=\left(\frac{q}{q}\right)=1 \quad \Rightarrow \boxed{1}$$

Using $x^2-Sx+P=0$, we have

$$x^2-\left(\frac{p^2-2q}{q}\right)x+1=0$$

Multiplying by q

$$qx^2-(p^2-2q)x+q=0$$

For updates and news visit

http://www.mathcity.org

these notes are written by Amir shehzad

http://www.mathcity.org/people/amir