Question # 1

If α, β *are the roots of the equations*
$x^2 + px + q = 0$ *then evaluate*

Solution : $x^2 + px + q = 0$
$a = 1,\ b = p,\ c = q$

Sum of roots,

$$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p$$

$$\alpha + \beta = -p$$

Product of roots,

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

$$\alpha\beta = q$$

(i) $\alpha^2 + \beta^2$

Solution :

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-p)^2 - 2(q)$$

$$= p^2 - 2q$$

(ii) $\alpha^3\beta + \alpha\beta^3$

Solution : $\alpha^3\beta + \alpha\beta^3$

$$= \alpha\beta\left(\alpha^2 + \beta^2\right)$$

$$= \alpha\beta\left[(\alpha + \beta)^2 - 2\alpha\beta\right]$$

$$= q\left[(-p)^2 - 2q\right]$$

$$= q\left(p^2 - 2q\right)$$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Solution : $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{1}{\alpha\beta}\left(\alpha^2 + \beta^2\right)$$

$$= \frac{1}{q}\left[(-p)^2 - 2 \times q\right]$$

$$= \frac{1}{q}\left(p^2 - 2q\right)$$

Q.2 *If* α, β *are the roots of the equation*
$4x^2 - 5x + 6 = 0$, *then find the value*

Solution : $4x^2 - 5x + 6 = 0$

$$ax^2 + bx + c = 0$$

$$a = 4,\ b = -5,\ c = 6$$

Sum of roots,

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{4} = \frac{5}{4}$$

Product of roots,

$$\alpha\beta = \frac{c}{a} = \frac{6}{4}$$

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$

Solution : $\frac{1}{\alpha} + \frac{1}{\beta}$

$$= \frac{\alpha + \beta}{\alpha\beta} = \frac{5/4}{6/4} = \frac{5}{6}$$

(ii) $\alpha^2\beta^2$

Solution: $\alpha^2\beta^2$

$$=(\alpha\beta)^2=\left(\frac{6}{4}\right)^2=\left(\frac{3}{2}\right)^2=\frac{9}{4}$$

(iii) $\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2}$

Solution: $\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2}$

$$=\frac{\beta+\alpha}{\alpha^2\beta^2}=\frac{\alpha+\beta}{(\alpha\beta)^2}=\frac{5/4}{(6/4)^2}$$

$$=\frac{5/4}{36/16}=\frac{5}{4}\times\frac{16}{36}=\frac{5}{9}$$

(iv) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Solution: $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$

Using formula:

$$(\alpha+\beta)^3=\alpha^3+\beta^3+3(\alpha\beta)(\alpha+\beta)$$

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)$$

$$=\frac{(\alpha+\beta)^3-3(\alpha\beta)(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{(5/4)^3-3(6/4)(5/4)}{(6/4)}$$

$$=\left(\frac{125}{64}-\frac{90}{16}\right)\frac{4}{6}=\left(\frac{125-360}{64}\right)\frac{4}{6}$$

$$=\frac{-235}{96}$$

Q.3 If α,β are the roots of the equation $lx^2+mx+n=0$ then find the value

Solution: $lx^2+mx+n=0$

$$ax^2+bx+c=0$$

$$a=l,\ b=m,\ c=n$$

$f\ \alpha,\beta$ are the roots of given equation

Sum of roots $=\frac{-b}{a}$

$$\alpha+\beta=\frac{-m}{l}$$

Product of roots= $\frac{c}{a}$

$$\alpha\beta=\frac{n}{l}$$

(i) $\alpha^3\beta^2+\alpha^2\beta^3$

Solution: $\alpha^3\beta^2+\alpha^2\beta^3$

$$=\alpha^2\beta^2(\alpha+\beta)$$

$$=(\alpha\beta)^2(\alpha\beta)^2=\left(\frac{n}{l}\right)^2\left(\frac{-m}{l}\right)$$

$$=\left(\frac{n^2}{l^2}\right)\left(\frac{-m}{l}\right)=\frac{-mn^2}{l^3}$$

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Solution: $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$=(\alpha+\beta)^2-2\alpha\beta\div(\alpha\beta)^2$$

$$=\left(\frac{-m}{l}\right)^2-2\left(\frac{n}{l}\right)\div\left(\frac{n}{l}\right)^2$$

$$=\frac{m^2}{l^2}-\frac{2n}{l}\div\frac{n^2}{l^2}=\left(\frac{m^2-2nl}{l^2}\right)\times\frac{l^2}{n^2}$$

$$=\frac{1}{n^2}\left(m^2-2nl\right)$$