**ON STOLARSKY AND RELATED MEANS**

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ABSTRACT. We give a simple proof of the Stolarsky means inequality as well as some related inequalities for similar means of Stolarsky type.

1. **Introduction and Preliminaries**

Let us consider the following means

where and are positive real numbers and are any real numbers but 0 .

These means, known in literature, are called Stolarsky means. Namely Stolarsky[1] in 1975 (see also [2, p.120]) introduced these means. Stolarsky proved that the function is increasing in both and i.e. for and , we have

In this paper, first we shall give a simple proof of inequality (1). Further we shall introduce two new classes of means of Stolarsky type.

2. **A Simple Proof of Stolarsky Means Inequality**

Note that is continuous, this means it is enough to prove (1) in the case where and .

We consider the following function

where and , and

Now

This implies is monotonically increasing. So for

i.e.

Let

then

i.e.

This implies is log-convex in Jensen sense.

Also , which implies is continuous for all . And therefore -convex.

We need following lemma which proof can be found in [2].

Lemma 2.1. Let be log-convex function and if, , then the following inequality is valid:

Applying Lemma 2.1 for , (let we get an inequality

Since is continuous, we have (1).

**CONCLUSION**

In the literature, many researchers have published so many results on different major generalizations of convex function. Many authors today focus on interval-valued functions, which is known as the -convex interval-valued function. Additionally, we give the rigorous proof of the famous Hermite-Hadamard type inequality for -convex in intervalvalued.

**REFERENCES**

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