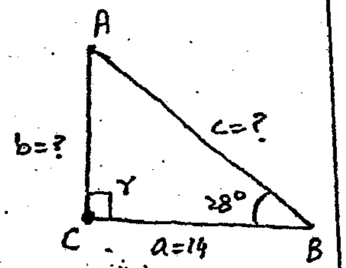
Exercise # 11.1

From Q 1 — Q 4 solve the right angled triangle in which $\gamma = 90^\circ$

① $a = 14$, $\beta = 28^\circ$



Sol
As $\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \alpha = 180^\circ - \beta - \gamma$

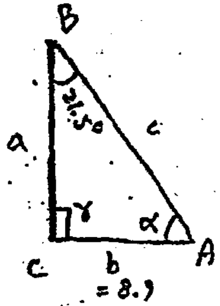
$\Rightarrow \alpha = 180^\circ - 28^\circ - 90^\circ$

$\Rightarrow \boxed{\alpha = 62^\circ}$ Ans

Now $\cos\beta = \dfrac{a}{c} \Rightarrow c = \dfrac{a}{\cos\beta} \Rightarrow c = \dfrac{14}{\cos 28^\circ} \Rightarrow \boxed{c = 15.85}$

and $\tan\beta = \dfrac{b}{a} \Rightarrow b = a\tan\beta \Rightarrow b = 14(\tan 28^\circ) \Rightarrow \boxed{b = 7.44}$

② $b = 8.9$, $\beta = 21.5^\circ$



As $\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \alpha = 180^\circ - \beta - \gamma$

$\Rightarrow \alpha = 180^\circ - 21.5^\circ - 90^\circ$

$\Rightarrow \boxed{\alpha = 68.5^\circ}$ Ans

Now $\tan\alpha = \dfrac{a}{b}$

$\Rightarrow a = b\tan\alpha$

$a = (8.9)(\tan 68.5^\circ) \Rightarrow \boxed{a = 22.6}$

Now by pathagorus theorem

$c^2 = a^2 + b^2$ OR $\cos\alpha = \dfrac{b}{c}$

$\Rightarrow c^2 = (22.6)^2 + (8.9)^2$

$\Rightarrow c^2 = 510.5 + 79.21$ $c = \dfrac{b}{\cos\alpha}$

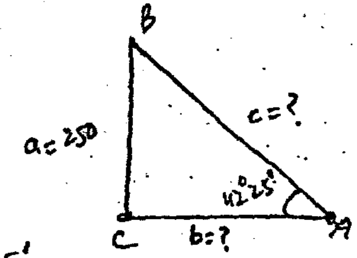
$c^2 = 589.71$

$\Rightarrow c = \sqrt{589.71}$ $c = \dfrac{8.9}{\cos 68.5^\circ} = \dfrac{8.9}{0.366}$

$\Rightarrow c = 24.28$ Ans

$\Rightarrow \boxed{c = 24.28}$ Ans

③ $a = 250$ $\alpha = 42^\circ 25'$



Sol
As $\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \beta = 180^\circ - \alpha - \gamma$

$\beta = 180^\circ - 42^\circ 25' - 90^\circ$

$\boxed{\beta = 47.58^\circ}$ or $\beta = 47^\circ 35'$ Ans

Now $\tan\alpha = \dfrac{a}{b}$ and $\sin\alpha = \dfrac{a}{c}$ Pathagorus theorem

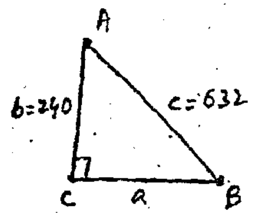
$\Rightarrow b = \dfrac{a}{\tan\alpha}$ $c = \dfrac{a}{\sin\alpha}$ OR $c^2 = a^2 + b^2$

$\Rightarrow b = \dfrac{250}{\tan 42^\circ 25'}$ $c = \dfrac{250}{\sin 47.58^\circ}$ $c = ?$

$\Rightarrow \boxed{b = 273.62}$ $\boxed{c = 338.85}$

(4) $c = 632$, $b = 240$

Sol By pathagorus theorem



$c^2 = a^2 + b^2$

$\Rightarrow a^2 = c^2 - b^2$

$\Rightarrow a^2 = (632)^2 - (240)^2$

$\Rightarrow a^2 = 341.824 \Rightarrow a = \sqrt{341.824} \Rightarrow \boxed{a = 584.65}$ Ans

Now $\cos\beta = \frac{a}{c}$ and $\sin\alpha = \frac{a}{c}$

$\Rightarrow \cos\beta = \frac{584.65}{632}$ $\quad$ $\sin\alpha = \frac{584.65}{632}$

$\Rightarrow \cos\beta = 0.9251$ $\quad$ $\Rightarrow \sin\alpha = 0.925$

$\Rightarrow \beta = \cos^{-1}(0.9251)$ $\quad$ $\Rightarrow \alpha = \sin^{-1}(0.925)$

$\Rightarrow \boxed{\beta = 22.31^\circ}$ $\quad$ $\Rightarrow \boxed{\alpha = 67.68^\circ}$

or $\alpha + \beta + \gamma = 180^\circ$

$\alpha = 180^\circ - \beta - \gamma$

$\alpha = 180^\circ - 22.31^\circ - 90^\circ$

$\boxed{\alpha = 67.69^\circ}$

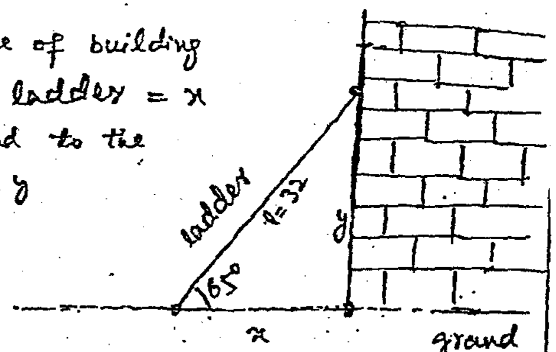
Q:5 A ladder 32 ft long leans against a building and makes an angle 65° with the ground. What is the distance from the base of the building to the foot of the ladder? What is the distance from ground to the top of the ladder?

Sol $l = 32$ ft

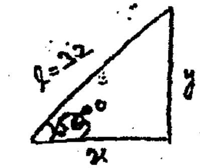
$\theta = 65^\circ$

Let Distance from base of building to the foot of ladder $= x$

& Distance from ground to the top of ladder $= y$



Sol The diagram is simplified to



$\sin 65^\circ = \frac{y}{32}$ & $\cos 65^\circ = \frac{x}{l} = \frac{x}{32}$

$\Rightarrow y = 32 \times \sin 65^\circ$ $\quad$ $\Rightarrow x = 32 \cos 65^\circ$

$\boxed{y = 29 \text{ ft}}$ & $\quad$ $\Rightarrow \boxed{x = 13.5 \text{ ft}}$

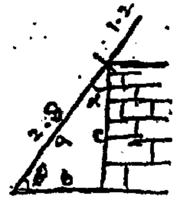
Q:6 A 4m plank rests against a wall 1.8m high so that 1.2m of it projects beyond the wall. Find the angle the plank makes with the wall & ground?

Sol Total length $l = 4m$

$a = l - 1.2$ $\quad$ $c = 1.8$

$a = 4 - 1.2$

$a = 2.8m$



Let Angle with ground $= \theta$

Angle with wall $= \alpha$

Now $\cos\alpha = \frac{c}{2.8}$ and $\sin\theta = \frac{c}{2.8}$

$\Rightarrow \cos\alpha = \frac{1.8}{2.8}$ $\quad\Rightarrow \sin\theta = \frac{1.8}{2.8}$

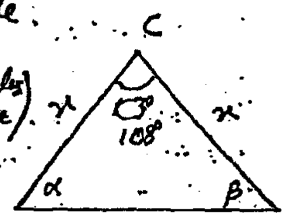
$\Rightarrow \cos\alpha = 0.64$ $\quad\Rightarrow \sin\theta = 0.64$

$\Rightarrow \alpha = \cos^{-1}(0.64)$ $\quad\Rightarrow \theta = \sin^{-1}(0.64)$

$\Rightarrow \boxed{\alpha = 50^\circ}$ $\quad\Rightarrow \boxed{\theta = 40^\circ}$

Angle with wall $\quad$ Angle with ground

Q:7 An isoceles triangle of 108° and a base 20cm long. Calculate its altitude.



Sol In the figure $\alpha = \beta$ ($\because$ Isoceles triangle)

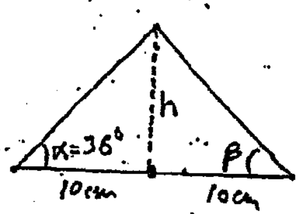
$\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \alpha + \beta + 108^\circ = 180^\circ$

$\Rightarrow \alpha + \beta = 180^\circ - 108^\circ$

$\Rightarrow \alpha + \beta = 72^\circ$

$\Rightarrow \alpha = \beta = 36^\circ$



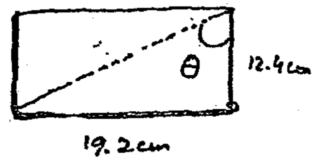
Now $\tan\alpha = \frac{h}{10}$

$h = 10\tan\alpha \Rightarrow h = 10\tan 36^\circ$

$\boxed{h = 7.265\text{cm}}$ Ans

Q:8 The length and width of a rectangle are 19.2 cm and 12.4 cm respectively. Find the angle between a diagonal and the shorter side of the rectangle.

Figure



Sol Angle b/w diagonal and shorter side $= \theta$

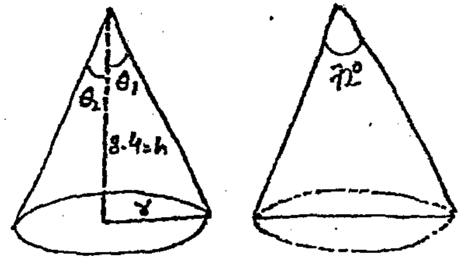
$\tan\theta = \frac{19.2}{12.4}$

$\Rightarrow \tan\theta = 1.548$

$\Rightarrow \theta = \tan^{-1}(1.548)$

$\Rightarrow \theta = 57.14^\circ$ Ans

Q:9 If a cone is 8.4cm high and has a vertical angle of 72°, calculate the diameter of its base.



Sol $\theta_1 = \theta_2 = 36^\circ$

$\tan\theta_1 = \frac{r}{h}$

$r = h\tan\theta_1$

$r = 8.4\tan 36^\circ$

$\boxed{r = 6.1}$

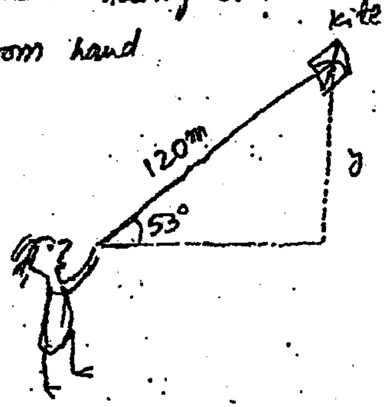
Then diameter $= d = 2r$

$= 2(6.1)$

$= 12.2$ cm Ans

Q:10 A kite has 120m of string attached to it when it flies at an elevation of 53°. How far is it above the hand holding it?

Sol Let height of kite from hand holding it $= h$



Then $\sin 53^\circ = \dfrac{y}{120}$

$y = 120 \sin 53^\circ$

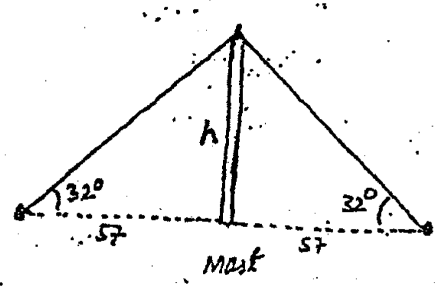
$y = 95.8\,m$ Ans

Exercise # 11.2

Q:1 An aerial mast is supported by two wires attached to points on the ground each 57m away from the foot of the mast. If each wire makes an angle of 32° with the horizontal, find the height of the mast.

Sol From the figure

Let height $= h$



Then $\tan 32^\circ = \dfrac{h}{57}$

$\Rightarrow h = 57 \times \tan 32^\circ$

$\Rightarrow h = 57 \times (0.6248)$

$\Rightarrow h = 35.61\,m$

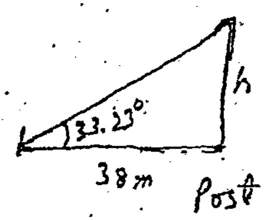
Q:2 The angle of elevation of the top of a post from a point on a level ground 38m away is 33.23°. Find the height of the post.

Sol h = height

Then $\tan 33.23^\circ = \dfrac{h}{38}$

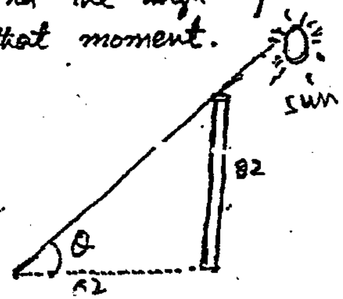
$\Rightarrow h = 38 \times \tan 33.23^\circ$

$\Rightarrow h = 24.89\,m$



Q:3 A mosque minar is 82 m high and casts a shadow 62 m long. Find the angle of elevation of sun at that moment.

Sol From the figure.



$$\tan\theta = \frac{82}{62}$$

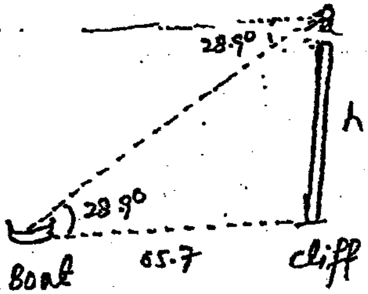
$$\Rightarrow \tan\theta = 1.322$$

$$\Rightarrow \theta = \tan^{-1}(1.322)$$

$$\Rightarrow \theta = 52.9^\circ$$

Q:4 The angle of depression of a boat 65.7 m from the base of a cliff is 28.9°. How high is the cliff?

Sol



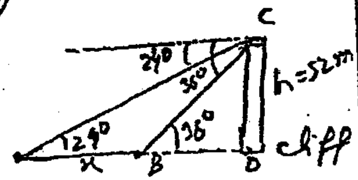
$$\tan 28.9^\circ = \frac{h}{65.7}$$

$$\Rightarrow h = 65.7 \times \tan 28.9^\circ$$

$$h = 36.26\,m$$

Q:5 : From the top of a cliff 52 m high, the angle of depression of two ships are 36° and 24° respectively. Find the distance between the two ships.

Sol Let A and B are the two ships and x is the distance between them



Now $\tan 36^\circ = \dfrac{h}{BD}$ and $\tan 24^\circ = \dfrac{h}{AD}$

$$\Rightarrow BD = \frac{h}{\tan 36^\circ} \quad \& \quad AD = \frac{h}{\tan 24^\circ}$$

$$\Rightarrow BD = \frac{52}{0.7265} \qquad AD = \frac{52}{0.44}$$

$$\Rightarrow BD = 71.57\,m \qquad \Rightarrow AD = 116.8\,m$$

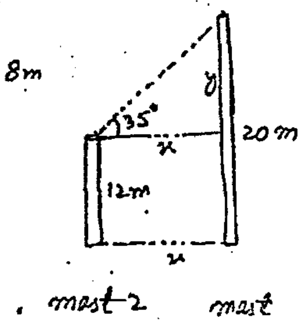
Now $x = AD - BD$

$$x = 116.8 - 71.57$$

$$x = 45.22\,m \quad \text{Ans}$$

Q:6: Two masts are 20m and 12 m high. If the line joining their tops makes an angle of 35° with horizontal find the distance b/w them.

Sol From the figure $y = 20 - 12 = 8\,m$



Now $\tan 35^\circ = \dfrac{y}{x}$

$$\Rightarrow x = \frac{y}{\tan 35^\circ} = \frac{8}{0.7}$$

$$\Rightarrow \boxed{x = 11.4\,m} \quad \text{Ans}$$

where x is the distance between the two masts.

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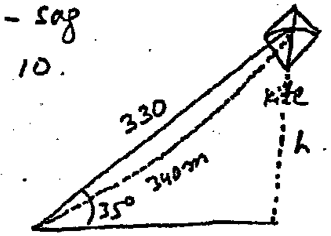
Q.7. The measure of the angle of elevation of a kite is 35°. The string of the kite is 340 m long. If the sag in the string is 10 m. Find the height of the kite.

Sol Net distance = total − sag

$= 340 - 10$

$= 330\,m$

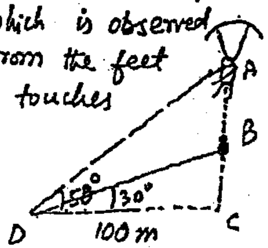


Then $\sin 35^\circ = \frac{h}{330}$

$\Rightarrow h = 330 \times \sin 35^\circ$

$\Rightarrow h = 189.28\,m$ Ans

Q.8 A parachutist is descending vertically. How far does the parachutist fall as the angle of elevation changes from 50° to 30° which is observed from a point 100 m away from the feet of the parachutist where he touches the ground?



Sol $AB = ?$

$\tan 50^\circ = \frac{AC}{100}$ and $\tan 30^\circ = \frac{BC}{100}$

$\Rightarrow AC = 100 \times \tan 50^\circ$ & $BC = 100 \times \tan 30^\circ$

$\Rightarrow AC = 119.17\,m$ & $BC = 57.7\,m$

Now $AB = AC - BC$

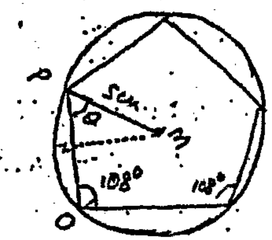
$= 119.17 - 57.7$ $\Rightarrow \boxed{AB = 61.43\,m}$ A

Q.9 A regular pentagon (five sided figure of equal sides) is inscribed in a circle of radius 5 cm. Find the length of a side of the pentagon.

Sol Sum of all angles $= (5-2) \times 180^\circ$

$= 3 \times 180^\circ$

$= 540^\circ$



Each angle $= \frac{540^\circ}{5} = 108^\circ$

and $\angle MPN = \frac{108^\circ}{2} = 54^\circ$

Now

$\cos\theta = \frac{PN}{PM}$

$\Rightarrow \overline{PN} = PM \cos\theta$

$\overline{PN} = 5 \times \cos 54^\circ$

$\overline{PN} = 2.939$

Now $\overline{PO} = 2 \times \overline{PN}$

$= 2 \times 2.939$

$\boxed{\overline{PO} = 5.877\,m}$

Formula for the sum of all the angles of a polygon $= (n-2) \times 180^\circ$ where n is # of sides

Therefore each side of the pentagon is $5.877\,m$

Exercise # 11.3

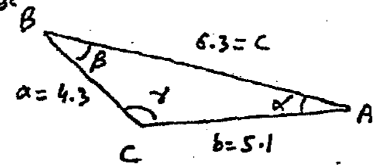
Q:1: Find the measure of the smallest angle of a triangle whose sides have lengths.

(a) 4.3, 5.1 and 6.3

Sol let $a = 4.3$, $b = 5.1$, $c = 6.3$

By theorem the angle opposite to the smallest side is the smallest

Hence α is the smallest angle.



Now by law of cosine

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\Rightarrow 2bc\cos\alpha = b^2 + c^2 - a^2$

$\Rightarrow \cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow \cos\alpha = \dfrac{5.1^2 + 6.3^2 - 4.3^2}{2 \times 5.1 \times 6.3}$

$\Rightarrow \cos\alpha = 0.73 \Rightarrow \alpha = \cos^{-1}(0.73) \Rightarrow \boxed{\alpha = 42.7^\circ}$ Ans

(b) $a = 3$, $b = 4.2$ & $c = 3.8$

Sol α will be the smallest

$\Rightarrow \cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{4.2^2 + 3.8^2 - 3^2}{2 \times 4.2 \times 3.8}$

$\Rightarrow \cos\alpha = 0.72 \Rightarrow \alpha = \cos^{-1} 0.72 \Rightarrow \boxed{\alpha = 43.7^\circ}$ Ans

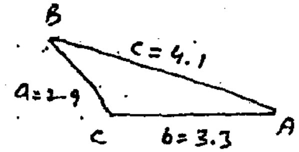
Q:2 Find the measure of largest angle of a triangle whose sides are... CH-11 P-04

(a) 2.9, 3.3 and 4.1

Sol let $a = 2.9$ $b = 3.3$ and $c = 4.1$

By theorem the angle opposite to the largest side is the greatest

Hence γ is the largest angle



Now $c^2 = b^2 + a^2 - 2ba\cos\gamma$

$\Rightarrow 2ba\cos\gamma = b^2 + a^2 - c^2$

$\Rightarrow \cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$

$= \dfrac{2.9^2 + 3.3^2 - 4.1^2}{2 \times 2.9 \times 3.3}$

$\Rightarrow \cos\gamma = 0.13 \Rightarrow \gamma = \cos^{-1}(0.13) \Rightarrow \boxed{\gamma = 82.5^\circ}$ A

(b) $a = 6$, $b = 8$ and $c = 9.4$

$\Rightarrow \gamma$ is largest angle

Now $\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \cos\gamma = \dfrac{6^2 + 8^2 - 9.4^2}{2 \times 6 \times 8}$

$\Rightarrow \cos\gamma = 0.12125$

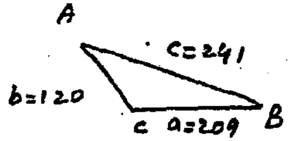
$\Rightarrow \gamma = \cos^{-1}(0.12125)$

$\Rightarrow \boxed{\gamma = 83.03^\circ}$ A

Q: Solve the triangle (find the missing parts).

③ $a = 209$, $b = 120$, $c = 241$

Required: $\alpha, \beta, \gamma = ?$



Now $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\Rightarrow 2bc\cos\alpha = b^2 + c^2 - a^2$

$\Rightarrow \cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow \cos\alpha = \dfrac{120^2 + 241^2 - 209^2}{2 \times 120 \times 241}$

$\Rightarrow \cos\alpha = 0.498 \Rightarrow \alpha = \cos^{-1}(0.498) \Rightarrow \boxed{\alpha = 60.13^\circ}$

Now to find β

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$\Rightarrow 2ac\cos\beta = a^2 + c^2 - b^2$

$\Rightarrow \cos\beta = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \cos\beta = \dfrac{209^2 + 241^2 - 120^2}{2 \times 209 \times 241} \Rightarrow \cos\beta = 0.867$

$\Rightarrow \beta = \cos^{-1}(0.867) \Rightarrow \boxed{\beta = 29.8^\circ}$

Last γ

$\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \gamma = 180^\circ - \alpha - \beta$

$\Rightarrow \gamma = 180^\circ - 60.13^\circ - 29.8^\circ$

$\Rightarrow \boxed{\gamma = 90.07^\circ}$ A

Q:4 $a = 120$, $b = 240$, $\gamma = 32^\circ$.

Required: $\alpha, \beta, c = ?$

Sol By law of cosine

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 120^2 + 240^2 - 2(120)(240)\cos 32^\circ$

$\Rightarrow c^2 = 23152.43$

$\Rightarrow c = \sqrt{23152.43} \Rightarrow \boxed{c = 152.16}$ Ans

Now $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{240^2 + 152.16^2 - 120^2}{2 \times 240 \times 152.16}$

$\cos\alpha = 0.908 \Rightarrow \alpha = \cos^{-1}(0.908) \Rightarrow \boxed{\alpha = 24.7^\circ}$

Finally $\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \beta = 180^\circ - \alpha - \gamma$

$\beta = 180^\circ - 24.7^\circ - 32^\circ$

$\boxed{\beta = 123.3^\circ}$ Ans

Q:5 $a = 24.5$, $c = 43.8$ and $\beta = 112^\circ$

Sol $b^2 = a^2 + c^2 - 2ac\cos\beta$

$\Rightarrow b^2 = 24.5^2 + 43.8^2 - (2 \times 24.5 \times 43.8 \times \cos 112^\circ)$

$\Rightarrow b^2 = 3322.6$

$\Rightarrow b = \sqrt{3322.6} \Rightarrow \boxed{b = 57.64}$ Ans

Now $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos\alpha = \dfrac{57.64^2 + 43.8^2 - 24.5^2}{2\times 57.64 \times 43.8}$

$\Rightarrow \cos\alpha = 0.919 \Rightarrow \alpha = \cos^{-1}(0.919) \Rightarrow \boxed{\alpha = 23.2^\circ}$

Finally $\alpha+\beta+\gamma = 180^\circ$

$\gamma = 180^\circ - \alpha - \beta$

$\Rightarrow \gamma = 180^\circ - 23.2^\circ - 112^\circ$

$\Rightarrow \boxed{\gamma = 44.78^\circ}$ Ans

Q:6 $a = 0.7$, $c = 0.8$, $\beta = 141^\circ 30'$

Sol $b^2 = a^2 + c^2 - 2ac\cos\beta$

$\Rightarrow b^2 = 0.7^2 + 0.8^2 - 2\times 0.7 \times 0.8 \times \cos 141^\circ 30'$

$\Rightarrow b^2 = 2.006 \Rightarrow b = \sqrt{2.006} \Rightarrow \boxed{b = 1.41}$ Ans

Now $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos\alpha = \dfrac{1.41^2 + 0.8^2 - 0.7^2}{2\times 1.41 \times 0.8}$

$\Rightarrow \cos\alpha = 0.956 \Rightarrow \alpha = \cos^{-1}(0.956) \Rightarrow \boxed{\alpha = 17.08^\circ}$

or $\alpha = 17^\circ 4' 48'$

Finally $\alpha+\beta+\gamma = 180^\circ$

$\Rightarrow \gamma = 180^\circ - \alpha - \beta$

$\Rightarrow \gamma = 180^\circ - 17.08^\circ - 141^\circ 30'$

$\Rightarrow \boxed{\gamma = 21.42^\circ}$ or $\gamma = 21^\circ 25'$

Q:7 $a = 34$, $b = 23$, $c = 58$

Required $\alpha, \beta, \gamma = ?$

Sol By law of cosines

$\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos\alpha = \dfrac{23^2 + 58^2 - 34^2}{2\times 23 \times 58}$

$\Rightarrow \cos\alpha = 1.025$

$\Rightarrow \alpha = \cos^{-1}(1.025)$

$\Rightarrow \alpha =$ Undefined $\Rightarrow$ Hence such a triangle is not possible.

Q:8 $a = 15.6$, $b = 18$, $\gamma = 35^\circ 10'$

Sol By law of cosines

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$\Rightarrow c^2 = 15.6^2 + 18^2 - 2(15.6)(18)\cos 35^\circ 10'$

$\Rightarrow c^2 = 108.26 \Rightarrow c = \sqrt{108.26} \Rightarrow \boxed{c = 10.4}$ Ans

Then $\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos\alpha = \dfrac{18^2 + 10.4^2 - 15.6^2}{2\times 18 \times 10.4}$

$\Rightarrow \cos\alpha = 0.5045$

$\Rightarrow \alpha = \cos^{-1}(0.5045)$

$\Rightarrow \boxed{\alpha = 59.7^\circ}$

Finally $\alpha+\beta+\gamma = 180^\circ$

$\Rightarrow \beta = 180^\circ - \alpha - \gamma$

$\beta = 180^\circ - 59.7^\circ - 35^\circ 10'$

$\boxed{\beta = 85.13^\circ}$ or $\beta = 85^\circ 8'$

Q:9 $b = 1.6 \quad c = 3.2 \quad \text{and} \quad \alpha = 100^\circ 24'$

Sol $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\Rightarrow a^2 = 1.6^2 + 3.2^2 - (2 \times 1.6 \times 3.2 \times \cos 100^\circ 24')$

$\Rightarrow a^2 = 14.64 \Rightarrow a = \sqrt{14.64} \Rightarrow \boxed{a = 3.82}$ Ans

Now $b^2 = a^2 + c^2 - 2ac\cos\beta$

$\Rightarrow \cos\beta = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \cos\beta = \dfrac{3.82^2 + 3.2^2 - 1.6^2}{2 \times 3.82 \times 3.2}$

$\Rightarrow \cos\beta = 0.91 \Rightarrow \beta = \cos^{-1}(0.91) \Rightarrow \boxed{\beta = 24.03^\circ}$ or $\beta = 24^\circ 1' 48''$

Finally $\alpha + \beta + \gamma = 180^\circ$

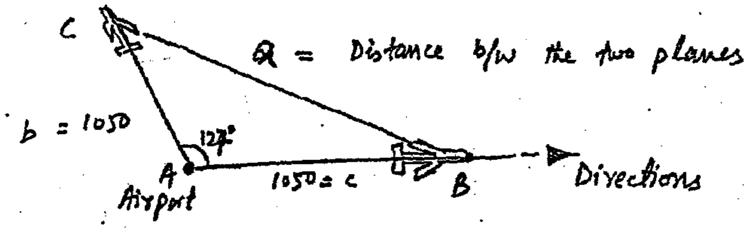
$\Rightarrow \gamma = 180^\circ - \alpha - \beta$

$\Rightarrow \gamma = 180^\circ - 100^\circ 24' - 24.03^\circ$

$\Rightarrow \gamma = 55.57^\circ$ or $\gamma = 55^\circ 34' 12''$

Q:10 Two planes start from Karachi International Airport at the same time and fly in the directions that make 127° with each other. Their speeds are 525 km/h. How far apart they are at the end of 2 hours of flying time.

Sol Figure is shown



Speeds = 525 km/hour.

After two hours distance travelled is

$= 525 \times 2 = 1050$ km

By law of cosine

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$\Rightarrow a^2 = 1050^2 + 1050^2 - 2(1050)(1050)\cos 127^\circ$

$\Rightarrow a^2 = 3532002$

$\Rightarrow a = \sqrt{3532002} \Rightarrow \boxed{a = 1879.3 \text{ km}}$

Q:11 Two sides of a parallelogram are 25 cm and 35 cm long, and one of its angle is 36°. Find the lengths of its diagnols.

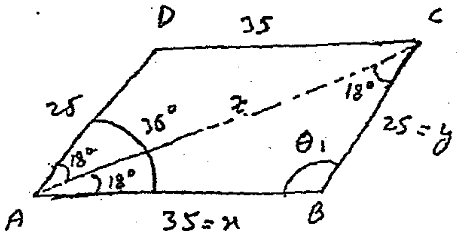
Sol To find θ_1

$\theta_1 + 18^\circ + 18^\circ = 180^\circ$

$\theta_1 + 36^\circ = 180^\circ$

$\theta_1 = 180^\circ - 36^\circ$

$\boxed{\theta_1 = 144^\circ}$



Now diagnal. $AC = z$

By law of cosine

$z^2 = x^2 + y^2 - 2xy\cos\theta,$

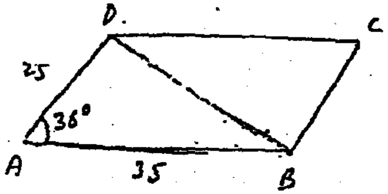
$z^2 = 35^2 + 25^2 - 2\times35\times25\times\cos144^\circ$

$\Rightarrow z^2 = 2665.8 \Rightarrow z = \sqrt{2665.8} \Rightarrow \boxed{z = 51.6\text{ cm}}$

Hence diagnal AC = 51.6cm Ans

Now diagnal BD

By law of cosines



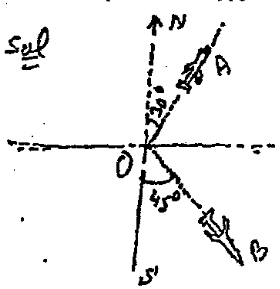
$\overline{BD}^2 = 25^2 + 35^2 - 2(25)(35)\times\cos36^\circ$

$\Rightarrow \overline{BD}^2 = 434.22$

$\Rightarrow \overline{BD} = \sqrt{434.22} \Rightarrow \boxed{\overline{BD} = 20.84\text{ cm}}$ Ans

Q.12 Two aeroplanes leave a field at the same time. One flies 30° east of north at 250 km/hr and the other flies 45° east of south at 300 km/hr. How far apart are they at the end of 2 hours?

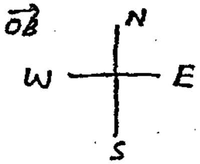
Sol



Angle b/w $\overrightarrow{OA}$ & $\overrightarrow{OB}$

$= 180^\circ - 30^\circ - 45^\circ$

$= 105^\circ$



1st Airoplane $= 2\times250$

$= 500$ km

2nd airoplane

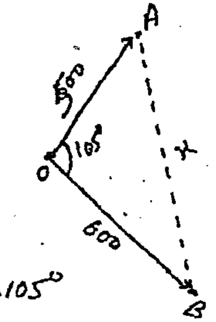
velocity = 300 km/hr

Distance travelled in 2 hours

$= 300\times2$

$= 600$ km

Then the simplified figure is



Distance b/w the two airoplanes = x

By law of cosines

$x^2 = 500^2 + 600^2 - 2\times500\times600\cos105^\circ$

$x^2 = 720.62 \Rightarrow x^2 = 76529.42$

$x = \sqrt{720.62} \Rightarrow x = \sqrt{76529.42} \Rightarrow \boxed{x = 874.8\text{ km}}$ Ans

Q.13 By law of cosines prove that

(a) $1+\cos\alpha = \dfrac{(b+c+a)(b+c-a)}{2bc}$

Sol

L.H.S

$1+\cos\alpha$

$= 1 + \dfrac{b^2+c^2-a^2}{2bc}$

$= \dfrac{2bc + b^2+c^2-a^2}{2bc}$

$= \dfrac{(b+c)^2 - a^2}{2bc} = \dfrac{(b+c+a)(b+c-a)}{2bc}$

$=$ R.H.S

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$2bc\cos\alpha = b^2+c^2-a^2$

$\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$

(b) $1 - \cos\alpha = \dfrac{(a-b+c)(a+b-c)}{2bc}$

L.H.S

$$1 - \cos\alpha$$

$$= 1 - \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{2bc - (b^2+c^2-a^2)}{2bc}$$

$$= \frac{2bc - b^2 - c^2 + a^2}{2bc}$$

$$= \frac{a^2 - b^2 - c^2 + 2bc}{2bc}$$

$$= \frac{a^2 - (b^2+c^2-2bc)}{2bc}$$

$$= \frac{a^2 - (b-c)^2}{2bc} \quad A^2-B^2 \text{ formula}$$

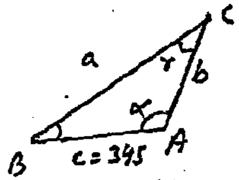
$$= \frac{\{a+(b-c)\}\{a-(b-c)\}}{2bc}$$

$$= \frac{(a+b-c)(a-b+c)}{2bc} = R.H.S$$

Exercise # 11.4

Solve the triangle

Q:1 $\alpha = 100^\circ$, $c = 345$, $\gamma = 56.4^\circ$



Sol As $\alpha + \beta + \gamma = 180^\circ$

$$\beta = 180^\circ - \alpha - \gamma$$

$$\beta = 180^\circ - 100^\circ - 56.4^\circ$$

$$\boxed{\beta = 23.6^\circ}$$

Now by law of sine

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \quad \& \quad \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\Rightarrow a = \frac{c\sin\alpha}{\sin\gamma} \qquad \Rightarrow b = \frac{c\sin\beta}{\sin\gamma}$$

$$\Rightarrow a = \frac{345 \times \sin 100^\circ}{\sin 56.4^\circ} \qquad b = \frac{345 \times \sin 23.6^\circ}{\sin 56.4^\circ}$$

$$\Rightarrow \boxed{a = 407.9} \text{ Ans} \quad \& \quad \boxed{b = 165.82} \text{ Ans}$$

Q2 $\alpha = 35^\circ$, $\beta = 70^\circ$, $c = 115$

Sol $\alpha + \beta + \gamma = 180^\circ$

$$\Rightarrow \gamma = 180^\circ - \alpha - \beta$$

$$\Rightarrow \gamma = 180^\circ - 35^\circ - 70^\circ \Rightarrow \boxed{\gamma = 75^\circ} \text{ Ans}$$

Now by law of sines

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \quad \& \quad \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\Rightarrow a = \frac{c\sin\alpha}{\sin\gamma} \quad \& \quad b = \frac{c\sin\beta}{\sin\gamma}$$

$$\Rightarrow a = \frac{115\sin 35^\circ}{\sin 75^\circ} \quad \& \quad b = \frac{115\sin 70^\circ}{\sin 75^\circ}$$

$$\Rightarrow \boxed{a = 68.28} \text{ Ans} \qquad \boxed{b = 118.87} \text{ Ans}$$

Q:3 $\beta = 39^\circ 30'$, $\gamma = 34^\circ 10'$, $a = 240$.

Sol

$$\alpha + \beta + \gamma = 180^\circ$$

$$\Rightarrow \alpha = 180^\circ - \beta - \gamma$$

$$\Rightarrow \alpha = 180^\circ - 39^\circ 30' - 34^\circ 10'$$

$$\Rightarrow \boxed{\alpha = 106^\circ 20'} \text{ Ans}$$

By law of sines

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \quad , \quad \frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$$

$$\Rightarrow b = \frac{a\sin\beta}{\sin\alpha} \qquad \Rightarrow c = \frac{a\sin\gamma}{\sin\alpha}$$

$$\Rightarrow b = \frac{240 \times \sin 39^\circ 30'}{\sin 106^\circ 20'} \qquad \Rightarrow c = \frac{240\sin 34^\circ 10'}{\sin 106^\circ 20'}$$

$$\Rightarrow \boxed{b = 159.1} \text{ Ans} \qquad \Rightarrow \boxed{c = 140.45} \text{ Ans}$$

Q:4 $a = 37.5$, $b = 12.4$, $\beta = 72^\circ$

Sol

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \quad \text{(By law of sines)}$$

$$\Rightarrow \sin\alpha = \frac{a\sin\beta}{b}$$

$$\Rightarrow \sin\alpha = \frac{37.5 \times \sin 72^\circ}{12.4}$$

$$\Rightarrow \sin\alpha = 2.87$$

$$\Rightarrow \alpha = \sin^{-1}(2.87)$$

$\Rightarrow \alpha =$ Undefined $\Rightarrow$ such a triangle is not possible.

Q:5 $a = 58.4$, $\beta = 37.2^\circ$, $\gamma = 100^\circ$

Sol

$$\alpha + \beta + \gamma = 180^\circ$$

$$\Rightarrow \alpha = 180^\circ - \beta - \gamma$$

$$\Rightarrow \alpha = 180^\circ - 37.2^\circ - 100^\circ$$

$$\Rightarrow \boxed{\alpha = 42.8^\circ} \text{ Ans}$$

Now by law of sines

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \quad , \quad \frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$$

$$\Rightarrow b = \frac{a\sin\beta}{\sin\alpha} \qquad \Rightarrow c = \frac{a\sin\gamma}{\sin\alpha}$$

$$\Rightarrow b = \frac{58.4 \times \sin 37.2^\circ}{\sin 42.8^\circ} \qquad \Rightarrow c = \frac{58.4 \times \sin 100^\circ}{\sin 42.8^\circ}$$

$$\Rightarrow \boxed{b = 51.96} \text{ Ans} \qquad \Rightarrow \boxed{c = 84.64} \text{ Ans}$$

Q:6 $c = 13.6 \quad \alpha = 30^\circ 24' \quad \beta = 72^\circ 6'$

Sol

$\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \gamma = 180^\circ - \alpha - \beta$

$\Rightarrow \gamma = 180^\circ - 30^\circ 24' - 72^\circ 6'$

$\Rightarrow \boxed{\gamma = 77.5^\circ}$ Ans

Now $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$ & $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

$\Rightarrow a = \dfrac{c \sin\alpha}{\sin\gamma}$ $\quad\Rightarrow b = \dfrac{c \sin\beta}{\sin\gamma}$

$\Rightarrow a = \dfrac{13.6 \sin 30^\circ 24'}{\sin 77.5^\circ}$ $\quad\Rightarrow b = \dfrac{13.6 \times \sin 72^\circ 6'}{\sin 77.5^\circ}$

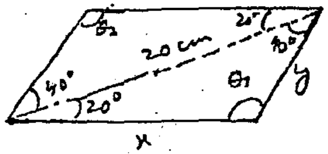
$\Rightarrow \boxed{a = 7.05}$ Ans $\quad \boxed{b = 13.25}$ Ans

Q:7 One diagnol of a parallelogram is 20 cm long and at one end forms angles 20° and 40° with the sides of the parallelogram. Find the lengths of sides.

Sol From the figure

$\theta_1 = 180^\circ - 20^\circ - 40^\circ$

$\boxed{\theta_1 = 120^\circ}$



Now by law of sines

$\dfrac{x}{\sin 40^\circ} = \dfrac{20}{\sin 120^\circ}$ & $\dfrac{y}{\sin 20^\circ} = \dfrac{20}{\sin 120^\circ}$

$\Rightarrow x = \dfrac{20 \times \sin 40^\circ}{\sin 120^\circ}$ $\quad\Rightarrow y = \dfrac{20 \times \sin 20^\circ}{\sin 120^\circ}$

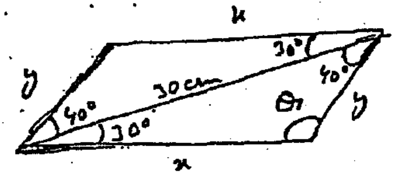
$\Rightarrow \boxed{x = 14.84\text{ cm}}$ $\quad\Rightarrow \boxed{y = 7.9\text{ cm}}$ Ans

Q:8 The diagnals of a parallelogram meets the sides at angle of 30° and 40°. If the length of the diagnol is 30 cm, find the perimeter of the parallelogram.

Sol From the figure

$\theta_1 = 180^\circ - 30^\circ - 40^\circ$

$\boxed{\theta_1 = 110^\circ}$



Now $\dfrac{x}{\sin 40^\circ} = \dfrac{30}{\sin 110^\circ}$ and $\dfrac{y}{\sin 30^\circ} = \dfrac{30}{\sin 110^\circ}$

$\Rightarrow x = \dfrac{30 \times \sin 40^\circ}{\sin 110^\circ}$ $\quad\Rightarrow y = \dfrac{30 \times \sin 30^\circ}{\sin 110^\circ}$

$\Rightarrow \boxed{x = 20.52}$ $\quad\Rightarrow \boxed{y = 15.96}$

Now perimeter $= x + y + x + y$

$\Rightarrow$ Perimeter $= 2x + 2y$

$= 2(20.52) + 2(15.96)$

Perimeter $= 72.96$ cm Ans

Q:9 A robbin on a branch 40 ft up in a tree spots a worm at an angle of depression of 14°. From a branch 15 ft above the robin, a crow spots the same worm at an angle of depression of 19°. How far is each bird from the worm?

Sol: For Robin

$\sin 14^\circ = \frac{40}{\overline{BC}}$

$\overline{BC} = \frac{40}{\sin 14^\circ}$

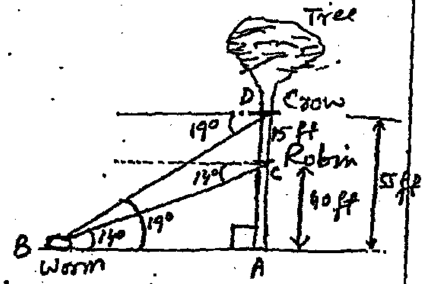
$\boxed{\overline{BC} = 165.3 ft}$

For Crow

$\sin 19^\circ = \frac{55}{\overline{BD}}$

$\overline{BD} = \frac{55}{\sin 19^\circ}$

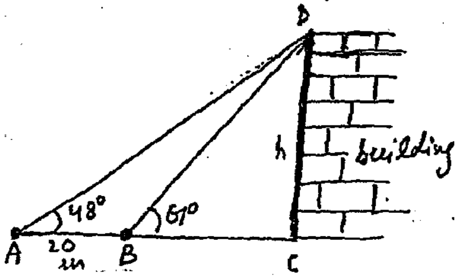
$\boxed{\overline{BD} = 168.93 ft}$



Q:10: The angle of elevation of a building is 48° from A and 61° from B. If $\overline{AB}$ is 20 m find the heigth of the building?

Sol From the figure

$\overline{AB} + \overline{BC} = \overline{AC}$



In ΔACD

$\tan 48^\circ = \frac{h}{\overline{AC}}$

$\Rightarrow h = \overline{AC} \tan 48^\circ$ ①

In ΔBCD

$\tan 61^\circ = \frac{h}{\overline{BC}}$

$h = \overline{BC} \tan 61^\circ$ ②

Compare eqns ① and ②, we get.

$\overline{AC} \tan 48^\circ = \overline{BC} \tan 61^\circ$

$\Rightarrow (\overline{AB} + \overline{BC}) \tan 48^\circ = \overline{BC} \tan 61^\circ$.

$\Rightarrow (20 + \overline{BC}) \tan 48^\circ = \overline{BC} \tan 61^\circ$

$\Rightarrow 20 \tan 48^\circ + \overline{BC} \tan 48^\circ = \overline{BC} \tan 61^\circ$

$22.21 + \overline{BC}(1.11) = \overline{BC}(1.804)$

$\Rightarrow 22.21 + 1.11 \overline{BC} = 1.804 \overline{BC}$

$\Rightarrow 22.21 = 1.804 \overline{BC} - 1.11 \overline{BC}$

$\Rightarrow 22.21 = 0.694 \overline{BC} \Rightarrow \boxed{\overline{BC} = 32 m}$

Eqn② $\Rightarrow h = \overline{BC} \tan 61^\circ$

$\Rightarrow h = 32 \times \tan 61^\circ$

$\Rightarrow \boxed{h = 57.7 m}$ Ans

hence height = 57.7 m

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Exercise # 11.5

Solve the triangle ABC using the law of tangents.

(i) $a = 48$ $\quad b = 32$ $\quad \gamma = 57^\circ$

Sol As, $\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \alpha + \beta = 180^\circ - \gamma$

$\Rightarrow \alpha + \beta = 180^\circ - 57^\circ$

$\Rightarrow \alpha + \beta = 123^\circ \longrightarrow (i)$

Now by law of tangents

$$\frac{a+b}{a-b} = \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

$$\Rightarrow \frac{48+32}{48-32} = \frac{\tan\left(\frac{123^\circ}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} \Rightarrow \frac{80}{16} = \frac{1.84177}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

$$\Rightarrow 5 = \frac{1.84177}{\tan\left(\frac{\alpha-\beta}{2}\right)} \Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right) = \frac{1.84177}{5}$$

$\Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right) = 0.368$

$\Rightarrow \frac{\alpha-\beta}{2} = \tan^{-1}(0.368)$

$\Rightarrow \frac{\alpha-\beta}{2} = 20.22$

$\Rightarrow \alpha - \beta = 40.44 \longrightarrow (ii)$

Eq (i) + Eq (ii)

$$\begin{aligned} \alpha + \beta &= 123^\circ \\ \alpha - \beta &= 40.44^\circ \\ \hline 2\alpha &= 163.44^\circ \end{aligned}$$

$\Rightarrow 2\alpha = 163.44^\circ \Rightarrow \boxed{\alpha = 81.72^\circ}$ Ans

Since $\alpha + \beta = 123^\circ$

$\Rightarrow \beta = 123^\circ - \alpha$

$\Rightarrow \beta = 123^\circ - 81.72^\circ$

$\Rightarrow \boxed{\beta = 41.28^\circ}$ Ans

Now again by tangent rule

$$\frac{a+c}{a-c} = \frac{\tan\left(\frac{\alpha+\gamma}{2}\right)}{\tan\left(\frac{\alpha-\gamma}{2}\right)}$$

$$\Rightarrow \frac{48+c}{48-c} = \frac{\tan\left(\frac{81.72^\circ+57^\circ}{2}\right)}{\tan\left(\frac{81.72^\circ-57^\circ}{2}\right)} \Rightarrow \frac{48+c}{48-c} = \frac{2.6548}{0.219132}$$

$$\Rightarrow \frac{48+c}{48-c} = 12.115$$

$\Rightarrow (48+c) = (48-c)(12.115)$

$\Rightarrow 48 + c = 581.522 - 12.115c$

$\Rightarrow 12.115c + c = 581.522 - 48$

$\Rightarrow 13.115c = 533.52$

$\Rightarrow c = \frac{533.52}{13.115}$

$\Rightarrow \boxed{c = 40.68}$ units Ans

verification by law of cosines

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 48^2 + 32^2 - 2(48)(32)\cos 57^\circ$

$c^2 = 1654.86$

$\Rightarrow c = \sqrt{1654.86}$

$\Rightarrow c = 40.68$ Ans

(2) $b = 12.5$ $\quad c = 23$ $\quad$ & $\quad \alpha = 38^\circ 20' = 38.33^\circ$

Sol $\quad \dfrac{b+c}{b-c} = \dfrac{\tan\frac{1}{2}(\beta+\gamma)}{\tan\frac{1}{2}(\beta-\gamma)}$

$\beta + \gamma = 180^\circ - \alpha$

$\beta + \gamma = 180^\circ - 38^\circ 20'$

$\beta + \gamma = 141.66 \longrightarrow$ (i)

$\Rightarrow \dfrac{12.5+23}{12.5-23} = \dfrac{\tan\frac{1}{2}(141.66)}{\tan\frac{1}{2}(\beta-\gamma)}$

$\Rightarrow -3.3809 = \dfrac{2.87645}{\tan\frac{1}{2}(\beta-\gamma)} \Rightarrow \tan\frac{1}{2}(\beta-\gamma) = -0.8508$

$\Rightarrow \frac{1}{2}(\beta-\gamma) = \tan^{-1}(-0.8508)$

$\Rightarrow \frac{1}{2}(\beta-\gamma) = -40.39$

$\Rightarrow \beta - \gamma = -80.78 \longrightarrow$ (ii)

Now Eq(i) + Eq(ii)

$\beta + \gamma = 141.66$

$\beta - \gamma = -80.78$

$2\beta = 60.87$

$\Rightarrow \boxed{\beta = 30.43^\circ}$ Ans

P.T.V of β

$30.43 - \gamma = -80.78$

$\Rightarrow 30.43 + 80.78 = \gamma$

$\Rightarrow \boxed{111.21^\circ = \gamma}$ Ans

Now $\quad \dfrac{a+b}{a-b} = \dfrac{\tan\frac{1}{2}(\alpha+\beta)}{\tan\frac{1}{2}(\alpha-\beta)}$

$\Rightarrow \dfrac{a+12.5}{a-12.5} = \dfrac{\tan\frac{1}{2}(38.33^\circ + 30.43^\circ)}{\tan\frac{1}{2}(38.33^\circ - 30.43^\circ)}$

$\Rightarrow \dfrac{a+12.5}{a-12.5} = \dfrac{0.6842}{0.06904}$

$\Rightarrow \dfrac{a+12.5}{a-12.5} = 9.91 \Rightarrow a + 12.5 = 9.91(a - 12.5)$

$\Rightarrow a + 12.5 = 9.91a - 123.85$

$\Rightarrow 123.85 + 12.5 = 9.91a - a$

$\Rightarrow 136.35 = 8.91a \Rightarrow \boxed{a = 15.47}$ Ans

Q3 $\quad b = 35$, $\quad c = 37$ $\quad$ and $\quad \alpha = 23^\circ 25' \Rightarrow \alpha = 23.41^\circ$

Sol $\quad$ By law of tangent

As $\beta + \gamma + \alpha = 180^\circ$

$\beta + \gamma = 180^\circ - \alpha$

$\beta + \gamma = 180^\circ - 23.41^\circ$

$\Rightarrow \beta + \gamma = 156.58^\circ \longrightarrow$ (i)

$\dfrac{b+c}{b-c} = \dfrac{\tan\frac{1}{2}(\beta+\gamma)}{\tan\frac{1}{2}(\beta-\gamma)}$

$\Rightarrow \dfrac{35+37}{35-37} = \dfrac{\tan\frac{1}{2}(156.58^\circ)}{\tan\frac{1}{2}(\beta-\gamma)}$

$\Rightarrow -36 = \dfrac{4.824}{\tan\frac{1}{2}(\beta-\gamma)} \Rightarrow \tan\frac{1}{2}(\beta-\gamma) = -0.134$

$\Rightarrow \frac{1}{2}(\beta-\gamma) = \tan^{-1}(-0.134)$

$\Rightarrow \frac{1}{2}(\beta-\gamma) = -7.633$

$\Rightarrow \beta - \gamma = -15.266 \longrightarrow$ (ii)

Now Eq(i) + Eq(ii)

$\beta + \gamma = 156.58^\circ$

$\beta - \gamma = -15.26$

$2\beta = 141.31$

$\Rightarrow \boxed{\beta = 70.65^\circ}$

P.T.V of β

$70.65^\circ - \gamma = -15.266$

$\Rightarrow \boxed{85.9^\circ = \gamma}$

Now to find a

$\dfrac{b+a}{b-a} = \dfrac{\tan\frac{1}{2}(\beta+\alpha)}{\tan\frac{1}{2}(\beta-\alpha)}$

$\Rightarrow \dfrac{35+a}{35-a} = \dfrac{\tan\frac{1}{2}(70.65^\circ + 23.41^\circ)}{\tan\frac{1}{2}(\beta-\alpha)} = \dfrac{\tan\frac{1}{2}(70.65^\circ + 23.41^\circ)}{\tan\frac{1}{2}(70.65^\circ - 23.41^\circ)}$

$$\Rightarrow \frac{35+a}{35-a} = \frac{1.0735}{0.4373}$$

$$\Rightarrow \frac{35+a}{35-a} = 2.4549 \Rightarrow 35+a = 2.4549(35-a)$$

$$\Rightarrow 35+a = 85.91 - 2.4549a$$

$$\Rightarrow 2.4549a + a = 85.91 - 35$$

$$\Rightarrow 3.4549a = 50.918$$

$$\Rightarrow \boxed{a = 14.73} \text{ Ans}$$

Q:4 $a = 88$, $b = 48$, $\gamma = 75^\circ 51' \Rightarrow \gamma = 75.85^\circ$

Sol By law of tangent

$$\frac{a+b}{a-b} = \frac{\tan\frac{1}{2}(\alpha+\beta)}{\tan\frac{1}{2}(\alpha-\beta)}$$

$$\alpha+\beta+\gamma = 180^\circ$$

$$\Rightarrow \beta+\alpha = 180^\circ - \gamma$$

$$\Rightarrow \alpha+\beta = 180^\circ - 75.85^\circ$$

$$\Rightarrow \alpha+\beta = 104.15^\circ \quad (i)$$

$$\Rightarrow \frac{88+48}{88-48} = \frac{\tan\frac{1}{2}(104.15^\circ)}{\tan\frac{1}{2}(\alpha-\beta)}$$

$$\Rightarrow 3.4 = \frac{1.2834}{\tan\frac{1}{2}(\alpha-\beta)} \Rightarrow \tan\frac{1}{2}(\alpha-\beta) = \frac{1.2834}{3.4}$$

$$\Rightarrow \tan\frac{1}{2}(\alpha-\beta) = 0.37747$$

$$\Rightarrow \frac{1}{2}(\alpha-\beta) = \tan^{-1}(0.37747)$$

$$\Rightarrow \frac{1}{2}(\alpha-\beta) = 20.68^\circ$$

$$\Rightarrow \alpha-\beta = 41.36^\circ \longrightarrow (ii)$$

Now Equ (i) + Equ (ii)

$$\alpha+\beta = 104.15^\circ$$

$$\alpha-\beta = 41.36^\circ$$

$$2\alpha = 145.51$$

$$\Rightarrow \boxed{\alpha = 72.75^\circ}$$

P.T.V of α

$$72.75^\circ - \beta = 41.36^\circ$$

$$\boxed{31.4^\circ = \beta} \text{ Ans}$$

Now $$\frac{c+b}{c-b} = \frac{\tan\frac{1}{2}(\gamma+\beta)}{\tan\frac{1}{2}(\gamma-\beta)}$$

$$\Rightarrow \frac{c+48}{c-48} = \frac{\tan\frac{1}{2}(75.85^\circ+31.4^\circ)}{\tan\frac{1}{2}(75.85^\circ-31.4^\circ)}$$

$$\Rightarrow \frac{c+48}{c-48} = \frac{1.3576}{0.408}$$

$$\Rightarrow \frac{c+48}{c-48} = 3.3225 \Rightarrow (c+48) = (3.3225)(c-48)$$

$$\Rightarrow c+48 = 3.3225c - 159.48$$

$$\Rightarrow 159.48 + 48 = 3.3225c - c$$

$$\Rightarrow 207.48 = 2.3225c$$

$$\Rightarrow \boxed{89.33 = c} \text{ Ans}$$

Q:5 $a = 168$, $c = 319$, $\beta = 110^\circ 22'$

$\beta = 110.36^\circ$

Sol By law of tangents

or $$\frac{c+a}{c-a} = \frac{\tan\frac{1}{2}(\gamma+\alpha)}{\tan\frac{1}{2}(\gamma-\alpha)}$$

As $\alpha+\beta+\gamma = 180^\circ$

$$\Rightarrow \alpha+\gamma = 180^\circ - \beta$$

$$\Rightarrow \alpha+\gamma = 180^\circ - 110.36^\circ$$

$$\Rightarrow \alpha+\gamma = 69.63^\circ \longrightarrow (i)$$

$$\Rightarrow \frac{319+168}{319-168} = \frac{\tan\frac{1}{2}(69.63^\circ)}{\tan\frac{1}{2}(\gamma-\alpha)}$$

$$\Rightarrow 3.225 = \frac{0.6954}{\tan\frac{1}{2}(\gamma-\alpha)}$$

$\Rightarrow \tan\frac{1}{2}(\gamma-\alpha) = \frac{0.6954}{3.225}$

$\Rightarrow \tan\frac{1}{2}(\gamma-\alpha) = 0.2156$

$\Rightarrow \frac{1}{2}(\gamma-\alpha) = \tan^{-1}(0.2156) \Rightarrow \frac{1}{2}(\gamma-\alpha) = 12.168$

$\Rightarrow \gamma - \alpha = 24.33^\circ \rightarrow (ii)$

Now eqn (i) + eqn (ii)

$\gamma + \alpha = 69.63^\circ$
$\gamma - \alpha = 24.33^\circ$

$2\gamma = 93.966 \Rightarrow \boxed{\gamma = 46.98^\circ}$ Ans

Eqn (ii) $\Rightarrow \gamma - \alpha = 24.33^\circ$

$\Rightarrow \alpha = \gamma - 24.33^\circ$

$\Rightarrow \alpha = 46.98^\circ - 24.33^\circ$

$\Rightarrow \boxed{\alpha = 22.65^\circ}$ Ans

Now $\frac{b+c}{b-c} = \frac{\tan\frac{1}{2}(\beta+\gamma)}{\tan\frac{1}{2}(\beta-\gamma)}$

$\Rightarrow \frac{b+319}{b-319} = \frac{\tan\frac{1}{2}(110.36^\circ + 46.98^\circ)}{\tan\frac{1}{2}(110.36^\circ - 46.98^\circ)}$

$\Rightarrow \frac{b+319}{b-319} = \frac{4.991}{0.6173} \Rightarrow \frac{b+319}{b-319} = 8.0842$

$\Rightarrow b+319 = 8.0842(b-319)$

$\Rightarrow b+319 = 8.0842b - 2578.88$

$\Rightarrow 2578.88 + 319 = 8.0842b - b$

$2897.88 = 7.0842b$

$\Rightarrow \boxed{b = 409.06}$ Ans

Q:- Find the measure of the largest angle CH-11 P-10

(6) $a = 74, \quad b = 52, \quad c = 47$

Sol $s = \frac{a+b+c}{2}$

$\Rightarrow s = \frac{74+52+47}{2} \Rightarrow s = 86.5$

Since a is the largest side $\Rightarrow \alpha$ will be the largest angle

By half angle ~~iden~~ formula

$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$

$\Rightarrow \cos\frac{\alpha}{2} = \sqrt{\frac{86.5(86.5-74)}{52\times 47}}$

$\Rightarrow \cos\frac{\alpha}{2} = 0.665 \Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.665) \Rightarrow \frac{\alpha}{2} = 48.307$

$\Rightarrow \boxed{\alpha = 96.61^\circ}$ Ans

Q:7 $a = 7, \quad b = 9$ and $c = 7$

Sol b is largest side $\Rightarrow \beta$ is largest angle

$s = \frac{a+b+c}{2} \Rightarrow s = \frac{7+9+7}{2} \Rightarrow \boxed{s = 11.5}$

Now $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$

$\Rightarrow \cos\frac{\beta}{2} = \sqrt{\frac{11.5(11.5-9)}{7\times 7}} \Rightarrow \cos\frac{\beta}{2} = 0.7659$

$\Rightarrow \frac{\beta}{2} = \cos^{-1}(0.7659)$

$\Rightarrow \frac{\beta}{2} = 40.0052$

$\Rightarrow \boxed{\beta = 80.01^\circ}$ Ans

Q:8 $a = 2.3$, $b = 1.5$ and $c = 2.7$

Sol c is largest side $\Rightarrow$ γ is largest angle

$$s = \frac{a+b+c}{2} \Rightarrow s = \frac{2.3+1.5+2.7}{2} \Rightarrow s = 3.25$$

Now $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$

$$\Rightarrow \cos\frac{\gamma}{2} = \sqrt{\frac{3.25(3.25-2.7)}{2.3\times1.5}} \Rightarrow \cos\frac{\gamma}{2} = 0.719$$

$$\Rightarrow \frac{\gamma}{2} = \cos^{-1}(0.719)$$

$$\Rightarrow \frac{\gamma}{2} = 43.96^\circ$$

$$\Rightarrow \boxed{\gamma = 87.92^\circ} \text{ Ans}$$

Solve the triangle

(9) $a = 9$, $b = 7$, $c = 5$

Sol $s = \frac{a+b+c}{2} \Rightarrow s = \frac{9+7+5}{2} \Rightarrow \boxed{s = 10.5}$

Now $\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$, $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$

$\Rightarrow \cos\frac{\alpha}{2} = \sqrt{\frac{10.5(10.5-9)}{7\times5}}$, $\cos\frac{\beta}{2} = \sqrt{\frac{10.5(10.5-7)}{9\times5}}$, $\cos\frac{\gamma}{2} = \sqrt{\frac{10.5(10.5-5)}{9\times7}}$

$\Rightarrow \cos\frac{\alpha}{2} = 0.67$, $\cos\frac{\beta}{2} = 0.9036$, $\cos\frac{\gamma}{2} = 0.957$

$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.67)$, $\frac{\beta}{2} = \cos^{-1}(0.9036)$, $\frac{\gamma}{2} = \cos^{-1}(0.957)$

$\Rightarrow \frac{\alpha}{2} = 47.86$, $\frac{\beta}{2} = 25.35$, $\frac{\gamma}{2} = 16.77^\circ$

$\Rightarrow \boxed{\alpha = 95.74^\circ}$ $\Rightarrow \boxed{\beta = 50.7^\circ}$ $\Rightarrow \boxed{\gamma = 33.5^\circ}$ Ans

(10) $a = 1.2$ $b = 9$ $c = 10$

Sol. $s = \frac{a+b+c}{2} = \frac{1.2+9+10}{2} = 10.1$

Now $\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$, $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$

$\cos\frac{\alpha}{2} = \sqrt{\frac{10.1(10.1-1.2)}{9\times10}}$ $\cos\frac{\beta}{2} = \sqrt{\frac{10.1(10.1-9)}{1.2\times10}}$ $\cos\frac{\gamma}{2} = \sqrt{\frac{10.1(10.1-10)}{1.2\times9}}$

$\Rightarrow \cos\frac{\alpha}{2} = 0.999$ $\cos\frac{\beta}{2} = 0.962$ $\cos\frac{\gamma}{2} = 0.305$

$\Rightarrow \frac{\alpha}{2} = 2.003$ $\Rightarrow \boxed{\beta = 31.6^\circ}$ Ans $\Rightarrow \boxed{\gamma = 144.38^\circ}$ Ans

$\Rightarrow \boxed{\alpha = 4.006^\circ}$

(11) $a = 6$, $b = 8$, $c = 12$

Sol $s = \frac{a+b+c}{2} = \frac{6+8+12}{2} = 13$

$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$ Now $\alpha+\beta+\gamma = 180^\circ$

$\Rightarrow \cos\frac{\alpha}{2} = \sqrt{\frac{13(13-6)}{8\times12}}$ $\cos\frac{\beta}{2} = \sqrt{\frac{13(13-8)}{6\times12}}$ $\Rightarrow \gamma = 180^\circ - \alpha - \beta$

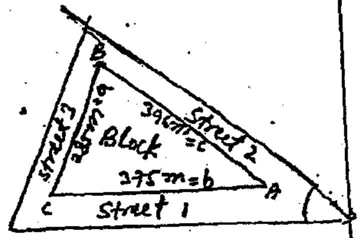
$\Rightarrow \cos\frac{\alpha}{2} = 0.973$ $\cos\frac{\beta}{2} = 0.950146$ $\Rightarrow \gamma = 180^\circ - 26.38^\circ - 36.33^\circ$

$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.973)$ $\frac{\beta}{2} = 18.168$ $\Rightarrow \boxed{\gamma = 117.28^\circ}$ Ans

$\Rightarrow \frac{\alpha}{2} = 13.19$ $\Rightarrow \boxed{\beta = 36.33^\circ}$ Ans

$\Rightarrow \boxed{\alpha = 26.38^\circ}$ Ans

Q.12 A city block is bounded by three streets. If the measure of the sides of the block are 285, 375 and 396 meters, find the measure of the angles of the streets making with each other?



Sol Let $a = 285$, $b = 375$, $c = 396$ m

$s = \dfrac{285+375+396}{2} \Rightarrow \boxed{s = 528}$

Now $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$

$\Rightarrow \cos\dfrac{\alpha}{2} = \sqrt{\dfrac{528(528-285)}{375\times 396}}$

$\Rightarrow \cos\dfrac{\alpha}{2} = 0.9295$

$\Rightarrow \dfrac{\alpha}{2} = \cos^{-1}(0.9295)$

$\Rightarrow \dfrac{\alpha}{2} = 21.64$

$\Rightarrow \alpha = 43.28^\circ$ Ans

and $\cos\left(\dfrac{\beta}{2}\right) = \sqrt{\dfrac{s(s-b)}{ac}}$, $\cos\left(\dfrac{\gamma}{2}\right) = \sqrt{\dfrac{s(s-c)}{ab}}$

$\Rightarrow \cos\dfrac{\beta}{2} = \sqrt{\dfrac{528(528-375)}{285\times 396}}$, $\cos\dfrac{\gamma}{2} = \sqrt{\dfrac{528(528-396)}{285\times 375}}$

$\Rightarrow \cos\left(\dfrac{\beta}{2}\right) = 0.846$, $\cos\dfrac{\gamma}{2} = 0.8075$

$\Rightarrow \dfrac{\beta}{2} = \cos^{-1}(0.846)$, $\Rightarrow \dfrac{\gamma}{2} = 36.143$

$\Rightarrow \dfrac{\beta}{2} = 32.21^\circ$, $\Rightarrow \boxed{\gamma = 72.28^\circ}$ Ans

$\Rightarrow \boxed{\beta = 64.43^\circ}$ Ans

Exercise # 11.6

Find the areas of the triangles

(1) $a = 15$ $b = 80$ $\gamma = 38^\circ$ (S.A.S case)

Sol Area $= \frac{1}{2} ab\sin\gamma$

$\Delta = \frac{1}{2}(15)(80)\sin 38^\circ$

$\Rightarrow \boxed{\Delta = 369.4 \text{ unit}^2}$ Ans

(2) $b = 14$, $c = 12$, $\alpha = 82^\circ$ (S.A.S case)

Sol Area $= \frac{1}{2} bc\sin\alpha$

$= \frac{1}{2}(14)(12)\sin 82^\circ$

$\boxed{\Delta = 83.18 \text{ unit}^2}$ Ans

(3) $a = 30$ $\beta = 50^\circ$ $\gamma = 100^\circ$ (A.S.A case)

Sol $\alpha + \beta + \gamma = 180^\circ \Rightarrow \alpha = 180^\circ - \beta - \gamma \Rightarrow \alpha = 180^\circ - 50^\circ - 100^\circ$

$\Rightarrow \boxed{\alpha = 30^\circ}$

Area $= \dfrac{1}{2}\,\dfrac{a^2\sin\beta\sin\gamma}{\sin\alpha}$

$\Rightarrow \Delta = \dfrac{1}{2}\,\dfrac{30^2\sin 50^\circ\sin 100^\circ}{\sin 30^\circ}$

$\Rightarrow \boxed{\Delta = 678.96 \text{ unit}^2}$ Ans

(4) $b = 40$ $\alpha = 50^\circ$ $\gamma = 60^\circ$

Sol $\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \beta = 180^\circ - \alpha - \gamma$

$\beta = 180^\circ - 50^\circ - 60^\circ \Rightarrow \boxed{\beta = 70^\circ}$

Now Area $= \Delta = \frac{1}{2} \cdot \frac{b^2 \sin\alpha \sin\gamma}{\sin\beta}$

$\Delta = \frac{1}{2} \frac{40^2 \sin 50^\circ \sin 60^\circ}{\sin 70^\circ}$

$\boxed{\Delta = 564.8 \text{ unit}^2}$ Ans

Q:5 $a = 7$, $b = 8$, $c = 2$ (S.S.S case)

Sol $S = \frac{a+b+c}{2} \Rightarrow S = \frac{7+8+2}{2} \Rightarrow \boxed{S = 8.5}$

Area $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{8.5(8.5-7)\cdot(8.5-8)(8.5-2)}$

$= \sqrt{8.5(1.5)(0.5)(6.5)} = \sqrt{41.4375}$

$\Rightarrow \boxed{\Delta = 6.437 \text{ unit}^2}$ Ans.

Q:6 $a = 11$ $b = 9$ & $c = 8$ (S.S.S case)

Sol $S = \frac{a+b+c}{2} \Rightarrow S = \frac{11+9+8}{2} \Rightarrow \boxed{S = 14}$

Area $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{14(14-11)(14-9)(14-8)}$

$= \sqrt{14(3)(5)(6)}$

Area $= \Delta = \sqrt{1260}$

$\Rightarrow \boxed{\Delta = 35.49 \text{ unit}^2}$

Q:7

$b = 414$, $c = 485$ and $\alpha = 49^\circ 47'$

$\alpha = 49.78^\circ$

Sol Area $= \frac{1}{2} bc \sin\alpha$

$= \frac{1}{2}(414)(485) \sin 49.78^\circ$

$\Delta = 76658.68 \text{ unit}^2$ Ans

Q:8 $a = 32$ $\beta = 47^\circ 24'$ & $\gamma = 70^\circ 16'$

Sol $\alpha + \beta + \gamma = 180^\circ \Rightarrow \alpha = 180^\circ - \beta - \gamma$

$\alpha = 180^\circ - 47^\circ 24' - 70^\circ 16'$

$\Rightarrow \alpha = 62^\circ 20'$

Area $= \Delta = \frac{1}{2} \frac{a^2 \sin\beta \sin\gamma}{\sin\alpha}$

$\Rightarrow \Delta = \frac{1}{2} \frac{32^2 \sin 47^\circ 24' \sin 70^\circ 16'}{\sin 62^\circ 20'}$

$\Rightarrow \boxed{\Delta = 400.5 \text{ unit}^2}$

Q:9 $b = 47$, $\alpha = 60^\circ 25'$, $\gamma = 41^\circ 35'$

Sol Area $= \Delta = \frac{1}{2} \frac{b^2 \sin\alpha \sin\gamma}{\sin\beta}$

$\Delta = \frac{1}{2} \frac{47^2 \sin 60^\circ 25' \sin 41^\circ 35'}{\sin 78^\circ 10'}$

$\boxed{\Delta = 83.7 \text{ unit}^2}$ Ans

$\alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \beta = 180^\circ - \alpha - \gamma$

$\Rightarrow \beta = 180^\circ - 60^\circ 25' - 41^\circ 25'$

$\Rightarrow \beta = 78^\circ 10'$

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Q:10 $c=57$, $\alpha=23^\circ 24'$ & $\beta=71^\circ 36'$

Sol $\alpha+\beta+\gamma=180^\circ \Rightarrow \gamma=180^\circ-\alpha-\beta$

$\gamma=180^\circ-23^\circ 24'-71^\circ 36'$

$\gamma=85^\circ$

Now Area $=\Delta=\frac{1}{2}\frac{c^2\sin\alpha\sin\beta}{\sin\gamma}$

$\Rightarrow \Delta=\frac{1}{2}\frac{57^2\sin 23^\circ 24'\sin 71^\circ 36'}{\sin 85^\circ}$

$\Rightarrow \Delta=614.52$ unit^2 Ans

Q:11 $a=925$ $c=433$ and $\beta=42^\circ 17'$

Sol Area $=\frac{1}{2}ac\sin\beta$

$=\frac{1}{2}(925)(433)\sin 42^\circ 17'$

$\boxed{\text{Area}=134.73}$

Q:12 $a=98$ $b=71$ $\gamma=56^\circ 14'$

Sol Area $=\Delta=\frac{1}{2}ab\sin\gamma$

$\Delta=\frac{1}{2}(98)(71)\sin 56^\circ 14'$

$\boxed{\Delta=2892.12 \text{ unit}^2}$ Ans

Exercise # 11.7

Problem 1-4 find r and R.

Q:1 $a=3$, $b=5$ and $c=6$

Sol $s=\frac{a+b+c}{2}$

$=\frac{3+5+6}{2}$

$\boxed{s=7}$

Then $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$\Delta=\sqrt{7(7-3)(7-5)(7-6)}$

$\Delta=\sqrt{7(4)(2)(1)}=\sqrt{56}$

$\boxed{\Delta=7.48 \text{ unit}^2}$

Now Inradius

$r=\frac{\Delta}{s}=\frac{7.48}{7}=1.069$

$\Rightarrow \boxed{r=1.069}$ Ans

Circum radius

$R=\frac{abc}{4\Delta}=\frac{3\times5\times6}{4(7.48)}$

$\boxed{R=3}$ Ans

Q:2 $a=21$, $b=20$, $c=29$

Sol $s=\frac{a+b+c}{2}$

$s=\frac{21+20+29}{2}$

$\boxed{s=35}$

Then $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{35(35-21)(35-20)(35-29)}$

$\Delta=\sqrt{35(14)(15)(6)}$

$\Delta=342.92$

Now

$r=\frac{\Delta}{s}=\frac{342.92}{35}$

$\boxed{r=9.8}$ Ans

and $R=\frac{abc}{4\Delta}=\frac{21\times20\times29}{4(342.92)}$

$\Rightarrow \boxed{R=8.88}$ Ans

Q:3 $a=117$, $b=44$, $c=125$

Sol

$$S = \frac{a+b+c}{2} = \frac{117+44+125}{2} \Rightarrow S = 143$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)} \Rightarrow \Delta = \sqrt{143(143-117)(143-44)(143-125)}$$

$$\Delta = \sqrt{143(26)(99)(18)}$$

$$\Delta = 2574$$

Now

$$r = \frac{\Delta}{S}$$

$$= \frac{2574}{143} = 18 \text{ Ans}$$

and

$$R = \frac{abc}{4\Delta} = \frac{117 \times 44 \times 125}{4(2574)} = 62.5 \text{ Ans}$$

Q:4 $a=20$, $b=99$, $c=101$

Sol

$$S = \frac{a+b+c}{2} = \frac{20+99+101}{2} = 110$$

$$\Delta = \sqrt{110(110-20)(110-99)(110-101)} \Rightarrow \Delta = \sqrt{110(90)(11)(9)}$$

$$\boxed{\Delta = 990}$$

Now

$$r = \frac{\Delta}{S}$$

$$r = \frac{990}{110} = 9 \text{ Ans}$$

&

$$R = \frac{abc}{4\Delta}$$

$$= \frac{20 \times 99 \times 101}{4(990)}$$

$$R = 50.5 \text{ Ans}$$

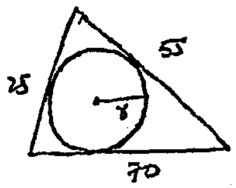
Q:5 Find the area of the inscribed circle of the triangle with measures of the sides 55 m, 25 m and 70 m.

Sol

$a = 55m$

$b = 25m$

$c = 70m$



Then

$$S = \frac{a+b+c}{2} = \frac{55+25+70}{2} = 75$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{75(75-55)(75-25)(75-70)}$$

$$\Delta = \sqrt{75(20)(50)(5)} = \sqrt{375000}$$

$$\Delta = 612.37$$

Now

$$r = \frac{\Delta}{S} = \frac{612.37}{75} \Rightarrow \boxed{r = 8.165}$$

Then Area of the inscribed circle is

$$\text{Area} = \pi r^2$$

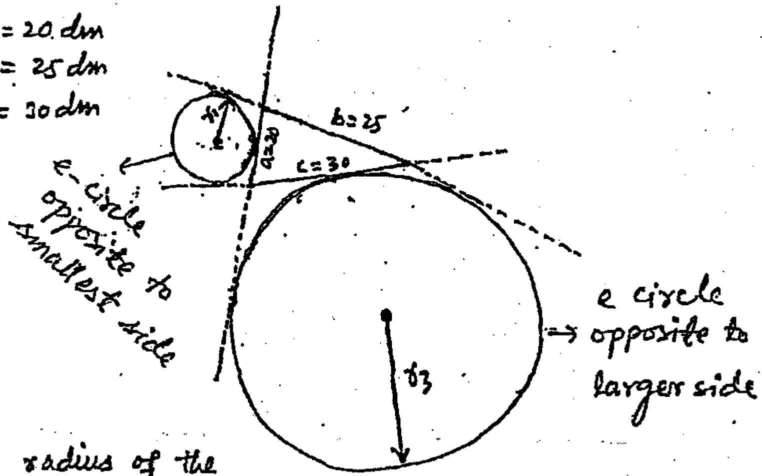
$$= 3.14(8.165)^2$$

$$\boxed{\text{Area} = 209.33} \; m^2$$

CH-11
P-13

Q:6 The measures of sides of a triangle are 20, 25 and 30 dm. Find the radius of the escribed circle
(a) opposite to larger side

Sol Let $a = 20$ dm
$b = 25$ dm
$c = 30$ dm



r_3 is the radius of the e-circle opposite to the larger side

$$r_3 = \frac{\Delta}{s-c}$$

$$= \frac{248.04}{37.5-30}$$

$$\boxed{r_3 = 33.07 \text{ dm}}$$

$$s = \frac{20+25+30}{2}$$

$$s = 37.5 \text{ dm}$$

Then $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$\Delta = \sqrt{37.5(37.5-20)(37.5-25)(37.5-30)}$$

$$\Delta = \sqrt{37.5(17.5)(12.5)(7.5)}$$

$$\Delta = \sqrt{61523.43}$$

$$\Delta = 248.04 \text{ dm}^2$$

(b) opposite to smaller side

$$r_1 = \frac{\Delta}{s-a} = \frac{248.04}{37.5-20}$$

$$\Rightarrow \boxed{r_1 = 14.17 \text{ dm}}$$

Q:7 Show that $\sqrt{r r_1 r_2 r_3} = \Delta$

L.H.S $\sqrt{r\, r_1\, r_2\, r_3}$

$$= \sqrt{\frac{\Delta}{s} \cdot \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}}$$

$$= \sqrt{\frac{\Delta^4}{s(s-a)(s-b)(s-c)}} = \frac{\Delta^2}{\sqrt{s(s-a)(s-b)(s-c)}}$$

$$= \frac{\Delta^2}{\Delta} = \Delta = R.H.S$$

Q:8 $\frac{abc}{4s}(\sin\alpha + \sin\beta + \sin\gamma) = \Delta$

L.H.S $\frac{abc}{4s}(\sin\alpha + \sin\beta + \sin\gamma)$

$$= \frac{abc}{4s}\sin\alpha + \frac{abc}{4s}\sin\beta + \frac{abc}{4s}\sin\gamma$$

$$= \frac{a}{2s}\,\frac{bc\sin\alpha}{2} + \frac{b}{2s}\,\frac{ac\sin\beta}{2} + \frac{c}{2s}\,\frac{ab\sin\gamma}{2}$$

$$= \frac{a}{2s}\Delta + \frac{b}{2s}\Delta + \frac{c}{2s}\Delta$$

$$= \frac{\Delta}{2s}\{a + b + c\}$$

$$s = \frac{a+b+c}{2}$$

$$= \frac{\Delta}{2s}(2s)$$

$$= \Delta = R.H.S$$

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Q:9 Prove that

$$r_1 + r_2 + r_3 - r = 4R$$

L.H.S

$$r_1 + r_2 + r_3 - r$$

$$= \frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s}$$

$$= \Delta\left\{\frac{1}{s-a} + \frac{1}{s-b}\right\} + \Delta\left\{\frac{1}{s-c} - \frac{1}{s}\right\}$$

$$= \Delta\left\{\frac{(s-b)+(s-a)}{(s-a)(s-b)}\right\} + \Delta\left\{\frac{s-(s-c)}{s(s-c)}\right\}$$

$$= \Delta\left\{\frac{2s-a-b}{(s-a)(s-b)}\right\} + \Delta\frac{c}{s(s-c)}$$

$$= \Delta\left\{\frac{a+b+c-a-b}{(s-a)(s-b)}\right\} + \frac{\Delta c}{s(s-c)}$$

$$= \frac{\Delta c}{(s-a)(s-b)} + \frac{\Delta c}{s(s-c)}$$

take Δc as common

$$= \Delta c\left\{\frac{1}{(s-a)(s-b)} + \frac{1}{s(s-c)}\right\}$$

$$= \Delta c\left\{\frac{s(s-c)+(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right\}$$

$$= \frac{\Delta c\{s^2 - sc + s^2 - bs - as + ab\}}{\Delta^2}$$

$$= \frac{c\{2s^2 - as - bs - cs + ab\}}{\Delta}$$

$$= \frac{c\{2s^2 - s(a+b+c) + ab\}}{\Delta}$$

$$= \frac{c\{2s^2 - s(2s) + ab\}}{\Delta}$$

$$= \frac{c\{2s^2 - 2s^2 + ab\}}{\Delta}$$

$$= \frac{abc}{\Delta} = 4\left(\frac{abc}{4\Delta}\right) = 4R = R.H.S$$

(10) Show that

$$r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2$$

L.H.S

$$r_1 r_2 + r_2 r_3 + r_3 r_1$$

$$= \frac{\Delta}{s-a}\frac{\Delta}{s-b} + \frac{\Delta}{s-b}\frac{\Delta}{s-c} + \frac{\Delta}{s-c}\cdot\frac{\Delta}{s-a}$$

$$= \frac{\Delta^2}{(s-a)(s-b)} + \frac{\Delta^2}{(s-b)(s-c)} + \frac{\Delta^2}{(s-a)(s-c)}$$

$$= \Delta^2\left\{\frac{1}{(s-a)(s-b)} + \frac{1}{(s-b)(s-c)} + \frac{1}{(s-a)(s-c)}\right\}$$

$$= \Delta^2\left\{\frac{(s-c)+(s-a)+(s-b)}{(s-a)(s-b)(s-c)}\right\}$$

$$= \Delta^2\left\{\frac{3s-a-b-c}{(s-a)(s-b)(s-c)}\right\} = \Delta^2\left\{\frac{3s-(a+b+c)}{(s-a)(s-b)(s-c)}\right\}$$

$$= \frac{\Delta^2\{3s-2s\}}{(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta^2 \, S}{(s-a)(s-b)(s-b)}$$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta^2 = s(s-a)(s-b)(s-c)$

P.T.V of Δ^2

$$= \frac{s(s-a)(s-b)(s-c) \quad s}{(s-a)(s-b)(s-c)}$$

$$= s^2 = R.H.S$$

Q:11 Prove that

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$$

L.H.S $\quad \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$$= \frac{1}{\frac{\Delta}{s-a}} + \frac{1}{\frac{\Delta}{s-b}} + \frac{1}{\frac{\Delta}{s-c}}$$

$$= \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$$

$$= \frac{1}{\Delta}\left\{(s-a) + (s-b) + (s-c)\right\}$$

$$= \frac{1}{\Delta}\left\{3s - a - b - c\right\}$$

$$= \frac{1}{\Delta}\left\{3s - (a+b+c)\right\}$$

$$= \frac{1}{\Delta}\left\{3s - 2s\right\}$$

$$= \frac{1}{\Delta}\left\{s\right\}$$

$$= \frac{1}{\Delta/s}$$

$$= \frac{1}{r} = R.H.S$$

Q:12: The sides of a triangle are in the ratio 3:7:8. The radius of the inscribed circle is 2m. Find the sides of the triangle.

Sol Let the sides of the triangle are

$3x$, $7x$ & $8x$

Then $\quad s = \frac{3x + 7x + 8x}{2} \Rightarrow \boxed{s = 9x}$

and $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$\Rightarrow \Delta = \sqrt{9x(9x-3x)(9x-7x)(9x-8x)}$$

$$\Rightarrow \quad \Delta = \sqrt{9x\,(6x)\,(2x)\,(x)}$$

$$\Rightarrow \quad \Delta = \sqrt{9 \times 12 \times x^4} \quad \Rightarrow \quad \Delta = 3\sqrt{12}\; x^2$$

$$\Rightarrow \quad \Delta = 3\sqrt{4 \times 3}\; x^2$$

$$\Rightarrow \quad \Delta = 3\sqrt{4}\sqrt{3}\; x^2$$

$$\Rightarrow \quad \boxed{\Delta = 6\sqrt{3}\cdot x^2}$$

Now we know that

$$r = \frac{\Delta}{s}$$

$$\Rightarrow \quad r = \frac{6\sqrt{3}\,x^2}{9x}$$

$$\Rightarrow \quad 2 = \frac{2\sqrt{3}\,x}{3}$$

$$\Rightarrow \quad 1 = \frac{\sqrt{3}}{3}\,x \quad \Rightarrow \quad 1 = \frac{x}{\sqrt{3}} \quad \Rightarrow \quad x = \sqrt{3}$$

Hence the sides are

$$\left.\begin{aligned} 3x &= 3\sqrt{3} \\ 7x &= 7\sqrt{3} \\ 8x &= 8\sqrt{3} \end{aligned}\right\} \text{Ans}$$

$\boxed{Q{:}13}$ Show that $r_1 r_2 r_3 = r s^2$

L.H.S $\quad r_1\; r_2\; r_3$

$$= \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}$$

$$= \frac{\Delta^3}{(s-a)(s-b)(s-c)}$$

$\times$ and $\div$ by s

$$= \frac{s\,\Delta^3}{s\,(s-a)\,(s-b)\,(s-c)}$$

$$= \frac{s\,\Delta^3}{\Delta^2}$$

$$= s\Delta$$

$$= s\,(r s)$$

$$= r s^2 = R.H.S$$

$$r = \frac{\Delta}{s}$$

$$\Rightarrow \quad \Delta = r s$$

Hurrah! That the end of chapter #11.

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