1

Determine the left hand limit and the right hand limit and then, find the limit of the following functions when $x \to c$

$$f(x) = 2x^2 + x - 5, \quad c = 1$$

Chapter 1

FUNCTIONS AND LIMITS

Ex 1.4

UCADEMY

$f(x) = 2x^2 + x - 5, \qquad c = 1$

L.H.L

$$\lim_{x \to c^-} f(x) = \lim_{x \to 1^-} f(x)$$
$$= \lim_{x \to 1} (2x^2 + x - 5)$$
$$= 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2$$

R.H.L

$$\lim_{x \to c^+} f(x) = \lim_{x \to 1^+} f(x)$$
$$= \lim_{x \to 1} (2x^2 + x - 5)$$
$$= 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2$$

$$\lim_{x \to 1} f(x) = -2.$$

1 Determine the left hand limit and the right hand limit and then, find the limit of the following functions when $x \to c$

(ii) $f(x) = \dfrac{x^2 - 9}{x - 3}, \quad c = -3$

$$f(x) = \frac{x^2 - 9}{x - 3}, \qquad c = -3$$

L.H.L

$$\lim_{x \to c^-} f(x) = \lim_{x \to -3^-} f(x) = \lim_{x \to -3} \left(\frac{x^2 - 9}{x - 3} \right)$$

$$= \frac{(-3)^2 - 9}{-3 - 3} = \frac{9 - 9}{-6} = 0 \checkmark$$

R.H.L

$$\lim_{x \to c^+} f(x) = \lim_{x \to -3^+} f(x) = \lim_{x \to -3} \left(\frac{x^2 - 9}{x - 3} \right)$$

$$= \frac{(-3)^2 - 9}{-3 - 3} = \frac{9 - 9}{-6} = 0 \checkmark$$

$$\lim_{x \to -3} f(x) = 0.$$

1 Determine the left hand limit and the right hand limit and then, find the limit of the following functions when $x \to c$

(iii) $f(x) = |x-5|, \quad c = 5$

$f(x) = |x-5| \quad , \quad c = 5$

$$f(x) = \begin{cases} x-5 & \text{if } x-5 \geq 0 \\ -(x-5) & \text{if } x-5 < 0 \end{cases}$$

$$f(x) = \begin{cases} x-5 & \text{if } x \geq 5 \ \checkmark \\ -x+5 & \text{if } x < 5 \ \checkmark \end{cases}$$

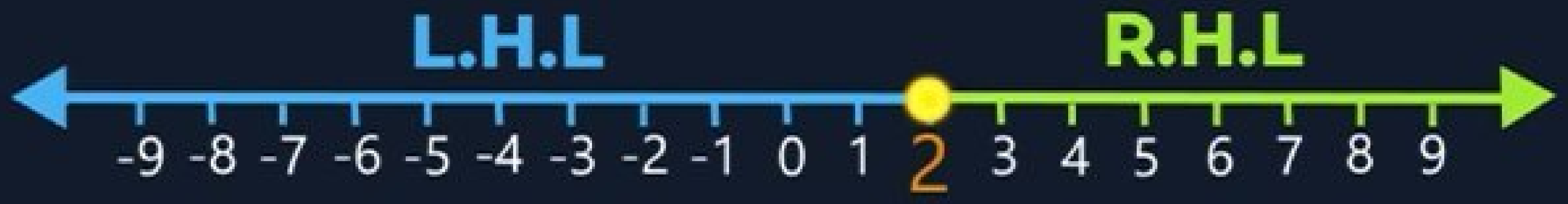
L.H.L $\lim_{x \to 5^-} f(x) = \lim_{x \to 5} (-x+5) = -5+5 = 0. \checkmark$

R.H.L $\lim_{x \to 5^+} f(x) = \lim_{x \to 5} (x-5) = 5-5 = 0 \checkmark$

$$\lim_{x \to 5} f(x) = 0.$$

2 Discuss the continuity of $f(x)$ at $x = c$ (i) $f(x) = \begin{cases} 2x+5 & \text{if} \quad x \le 2 \\ 4x+1 & \text{if} \quad x > 2 \end{cases}$

$$f(x) = \begin{cases} 2x+5 & \text{if} \quad x \le 2 \\ 4x+1 & \text{if} \quad x > 2 \end{cases}, \quad c = 2$$



L.H.L $\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (2x+5) = 2(2)+5 = 9$

R.H.L $\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (4x+1) = 4(2)+1 = 9$

Value $f(2) = 2(2) + 5 = 4+5 = 9$

Since $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

So f is continuous at $c = 2$.

Continuous function

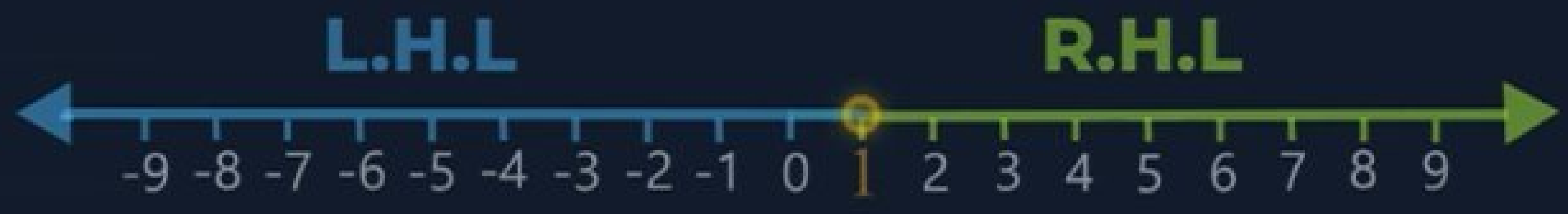
A function $f(x)$ is said to be continuous at c if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

2 Discuss the continuity of $f(x)$ at $x = c$

(ii) $f(x) = \begin{cases} 3x - 1 & \text{if} \quad x < 1 \\ 4 & \text{if} \quad x = 1 \\ 2x & \text{if} \quad x > 1 \end{cases}$

$$f(x) = \begin{cases} 3x - 1 & \text{if} \quad x < 1 \checkmark \\ 4 & \text{if} \quad x = 1 \checkmark \\ 2x & \text{if} \quad x > 1 \checkmark \end{cases}, \qquad c = 1$$



L.H.L. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (3x - 1) = 3(1) - 1 = 3 - 1 = 2$

R.H.L $\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (2x) = 2(1) = 2$

Value $f(1) = 4$

Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq f(1)$

So f is discontinuous at $c = 1$.

Continuous function

A function $f(x)$ is said to be continuous at c if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

3 If $f(x)=\begin{cases} 3x & \text{if} \quad x\le -2 \\ x^2-1 & \text{if} \quad -2<x<2 \\ 3 & \text{if} \quad x\ge 2 \end{cases}$ Discuss continuity at $x=2$ and $x=-2$

$$f(x)=\begin{cases} 3x & \text{if} & x\le -2 \\ x^2-1 & \text{if} & -2<\underline{x<2} \ \checkmark \\ 3 & \text{if} & \underline{x\ge 2} \ \checkmark \end{cases}$$

Continuous function

A function $f(x)$ is said to be continuous at c if

$$\lim_{x\to c^-} f(x)=\lim_{x\to c^+} f(x)=f(c)$$

At $\underline{x=2}$

L.H.L $\quad \lim_{x\to 2^-} f(x)=\lim_{x\to 2}(x^2-1) = 4-1 = 3$

R.H.L $\quad \lim_{x\to 2^+} f(x)=\lim_{x\to 2}(3) = 3$

Value $\quad f(2)=3$

Since $\quad \lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=f(2)$

So f is continuous at $x=2$.

3 If $f(x)=\begin{cases} 3x & \text{if} \quad x\le -2 \\ x^2-1 & \text{if} \quad -2<x<2 \\ 3 & \text{if} \quad x\ge 2 \end{cases}$ **Discuss continuity at** $x=2$ **and** $x=-2$

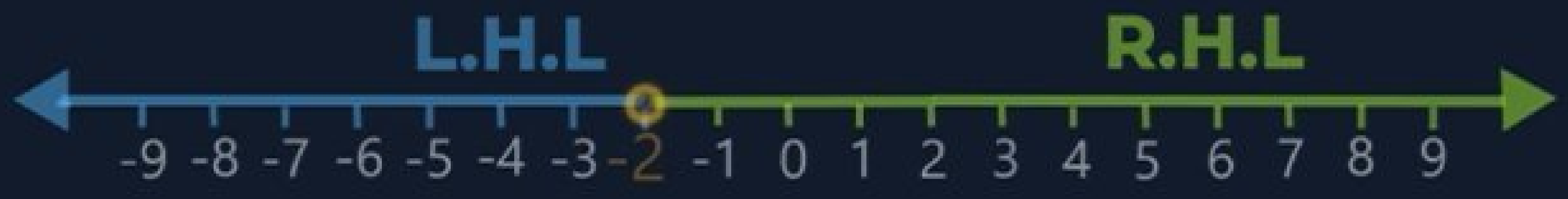
At $\underline{x=-2}$

L.H.L $\quad \lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2} (3x) = 3(-2) = -6$

R.H.L $\quad \lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2} (x^2-1) = (-2)^2-1 = 3$

Since $\quad \lim\limits_{x\to -2^-} f(x) \neq \lim\limits_{x\to -2^+} f(x)$

So f is discontinuous at $x=-2$.



Continuous function

A function $f(x)$ is said to be continuous at c if

$$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

4 If $f(x) = \begin{cases} x+2 & , \quad x \le -1 \\ c+2 & , \quad x > -1 \end{cases}$ find "c" so that $\lim_{x \to 1} f(x)$ exists

$$f(x) = \begin{cases} x+2 & , \quad x \le -1 \ \checkmark \\ c+2 & , \quad x > -1 \ \checkmark \end{cases}$$

Give $\lim_{x \to -1} f(x)$ exists.

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$$

$$\lim_{x \to -1} (x+2) = \lim_{x \to -1} (c+2)$$

$$-1+2 = c+2$$

$$1 = c+2$$

$$1-2 = c$$

$$c = -1.$$

Limit Exist

A function $f(x)$ has limit at c if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

5 Find the values m and n, so that given function f is continuous at $x = 3$

(i) $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x+9 & \text{if } x > 3 \end{cases}$

$$f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x+9 & \text{if } x > 3 \end{cases}$$

Given $f(x)$ is continuous at $x = 3$.

$$\Rightarrow \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

$$\lim_{x \to 3} (mx) = \lim_{x \to 3} (-2x+9) = n$$

$$3m = -2(3) + 9 = n$$

$$3m = 3 = n$$

$3m = 3$, $3 = n$

$m = 1$, $n = 3.$

Continuous function

A function $f(x)$ is said to be continuous at c if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

5 Find the values m and n, so that given function f is continuous at $x = 3$

(ii) $f(x) = \begin{cases} mx & \text{if} \quad x < 3 \\ x^2 & \text{if} \quad x \geq 3 \end{cases}$

$$f(x) = \begin{cases} mx & \text{if} \quad x < 3 \ ✓ \\ x^2 & \text{if} \quad x \geq 3 \ ✓ \end{cases}$$

Given $f(x)$ is continuous at $x = 3$.

$$\Rightarrow \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

$$\lim_{x \to 3} (mx) = \lim_{x \to 3} (x^2) = 3^2$$

$$3m = 3^2 = 9$$

$$3m = 9 = 9$$

$$3m = 9$$

$$m = 3.$$

Continuous function

A function $f(x)$ is said to be continuous at c if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

6 If $f(x)=\begin{cases}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} &, x\neq 2\\ k &, x=2\end{cases}$ Find the values of k so that f is continuous at $x=2$

Continuous function

A function $f(x)$ is said to be continuous at c if

$$\lim_{x\to c^-} f(x)=\lim_{x\to c^+} f(x)=f(c)$$

$$f(x)=\begin{cases}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} &, x\neq 2 \ (x>2,\ x<2)\\ k &, x=2 ✓\end{cases}$$

Given $f(x)$ is continuous at $x=2$

$$\Rightarrow \quad f(2)=\lim_{x\to 2} f(x)$$

$$k=\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$$

$$k=\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\times\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$$

$$k=\lim_{x\to 2}\frac{\left(\sqrt{2x+5}\right)^2-\left(\sqrt{x+7}\right)^2}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$k = \lim_{x \to 2} \frac{2x+5-x-7}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$k = \lim_{x \to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

$$k = \frac{1}{\sqrt{2(2)+5}+\sqrt{2+7}} = \frac{1}{3+3}$$

$$k = \frac{1}{6}.$$

Continuous function

A function $f(x)$ is said to be continuous at c if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

LECTURES BY

AKHTAR ABBAS

UNIVERSITY OF JHANG

03326297570