Exercise 9.4

$\sin\theta$, $\cos\theta$

$$\sin^2\theta + \cos^2\theta = 1. \quad -(1)$$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Divide by $\sin^2\theta$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta \quad -(2)$$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

Divide by $\cos^2\theta$

$$\tan^2\theta + 1 = \sec^2\theta$$

$\sec\theta = \dfrac{1}{\cos\theta}$

$$1 + \tan^2\theta = \sec^2\theta \quad -(3)$$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

(1) $\tan\theta + \cot\theta = \operatorname{cosec}\theta \sec\theta$

L.H.S.

$$\tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta} = \frac{1}{\cos\theta \sin\theta}$$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta \cdot \operatorname{cosec}\theta$$

$$= \operatorname{cosec}\theta \sec\theta = \text{R.H.S.}$$

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(2) $\sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta = 1$

L.H.S $\sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta$

$= \frac{1}{\cancel{\cos\theta}} \cdot \frac{1}{\cancel{\sin\theta}} \cancel{\sin\theta} \cancel{\cos\theta}$

$= 1 = R.H.S$

(3) $\cos\theta + \tan\theta \sin\theta = \sec\theta$

L.H.S $\cos\theta + \tan\theta \sin\theta$

$= \cancel{\cos\theta} + \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cancel{\sin\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta}$

$= \sec\theta = R.H.S$

(4) $\operatorname{cosec}\theta + \tan\theta \sec\theta = \operatorname{cosec}\theta \sec^2\theta$

L.H.S $\operatorname{cosec}\theta + \tan\theta \sec\theta$

$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$= \frac{1}{\cancel{\sin\theta}} + \frac{\sin\theta}{\cos^2\theta \cancel{}}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos^2\theta} = \frac{1}{\sin\theta \cos^2\theta}$

$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos^2\theta} = \operatorname{cosec}\theta \cdot \sec^2\theta = R.H.S$

⑤ $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

L.H.S

$\sec^2\theta - \operatorname{cosec}^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$= (1 + \tan^2\theta) - (1 + \cot^2\theta)$

$= 1 + \tan^2\theta - 1 - \cot^2\theta$

$= \tan^2\theta - \cot^2\theta = \text{R.H.S}$

⑥ $\cot^2\theta - \cos^2\theta = \cot^2\theta \cos^2\theta$

L.H.S

$\cot^2\theta - \cos^2\theta$

$= \dfrac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$

$= \dfrac{\cos^2\theta - \cos^2\theta \sin^2\theta}{\sin^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$= \dfrac{\cos^2\theta\,(1 - \sin^2\theta)}{\sin^2\theta}$

$= \dfrac{\cos^2\theta}{\sin^2\theta} \cdot \cos^2\theta = \cot^2\theta \cdot \cos^2\theta = \text{R.H.S}$

⑦ $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

L.H.S

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)\left(\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)$

$= \left(\dfrac{1 + \sin\theta}{\cos\theta}\right)\left(\dfrac{1 - \sin\theta}{\cos\theta}\right)$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$= \dfrac{1 - \sin^2\theta}{\cos^2\theta} = \dfrac{\cos^2\theta}{\cos^2\theta} = 1$

$= 1 = \text{R.H.S}$

(8) $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

L.H.S $\quad 2\cos^2\theta - 1$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$= 2(1 - \sin^2\theta) - 1$

$= 2 - 2\sin^2\theta - 1$

$= 1 - 2\sin^2\theta = \text{R.H.S}$

(9) $\cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

R.H.S $\quad \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \dfrac{1 - \tan^2\theta}{\sec^2\theta}$

$= \cos^2\theta\left(1 - \tan^2\theta\right)$

$= \cos^2\theta\left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\right) = \cancel{\cos^2\theta}\left(\dfrac{\cos^2\theta - \sin^2\theta}{\cancel{\cos^2\theta}}\right)$

$= \cos^2\theta - \sin^2\theta = \text{L.H.S}$

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(10) $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

R.H.S $\dfrac{\cot\theta - 1}{\cot\theta + 1}$

$= \dfrac{\left(\frac{\cos\theta}{\sin\theta} - 1\right)}{\left(\frac{\cos\theta}{\sin\theta} + 1\right)} = \dfrac{\left(\frac{\cos\theta - \sin\theta}{\sin\theta}\right)}{\left(\frac{\cos\theta + \sin\theta}{\sin\theta}\right)}$

$= \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \text{L.H.S}$

(11) $\dfrac{\sin\theta}{1 + \cos\theta} + \cot\theta = \operatorname{cosec}\theta$

L.H.S $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta$

$= \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos\theta(1+\cos\theta)}{(1+\cos\theta)\sin\theta} = \dfrac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$

$= \dfrac{1 + \cos\theta}{(1+\cos\theta)\sin\theta} = \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta = \text{R.H.S}$