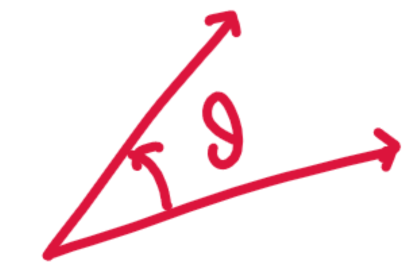
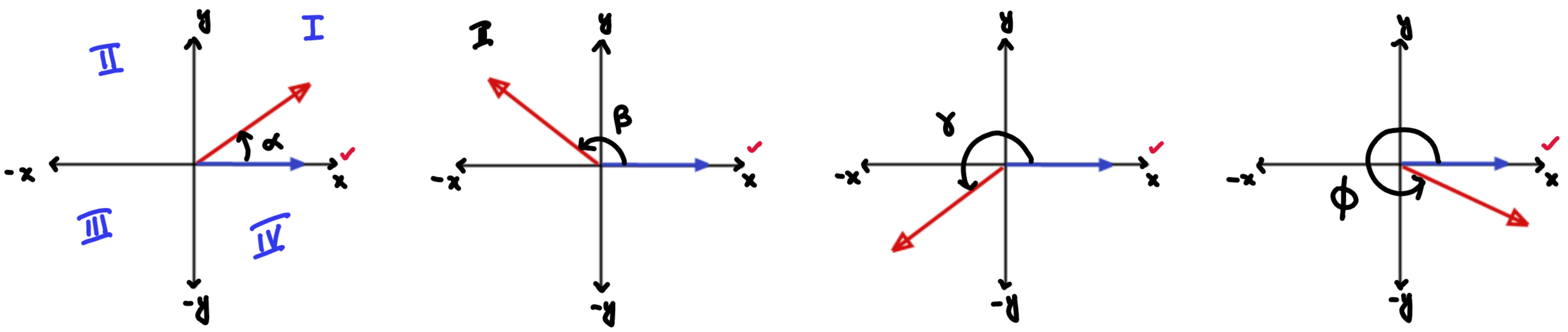
Exercise 9.2



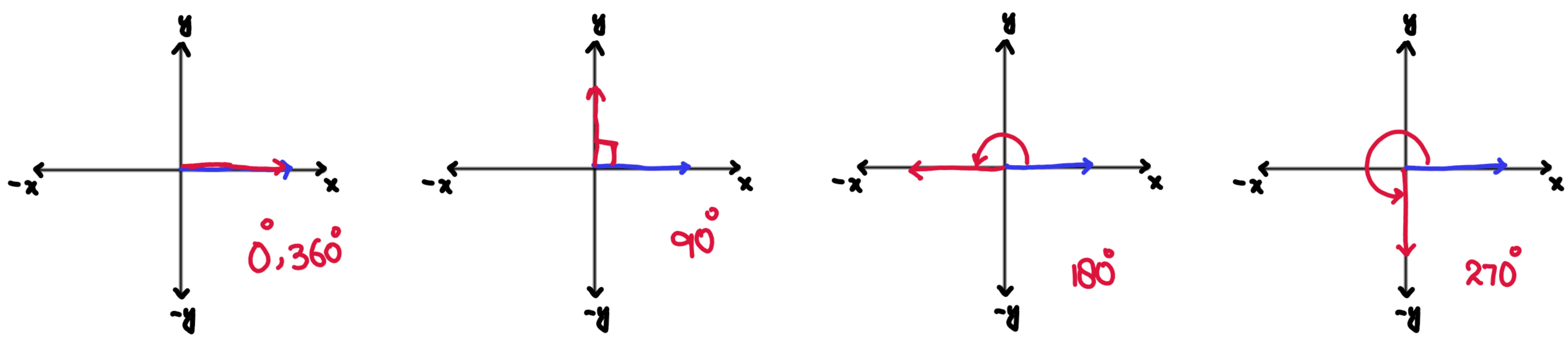
Angle in the standard position

An angle is in standard position if its vertex lies at the origin and its initial side along +ive x-axis.



Quadrantal Angles

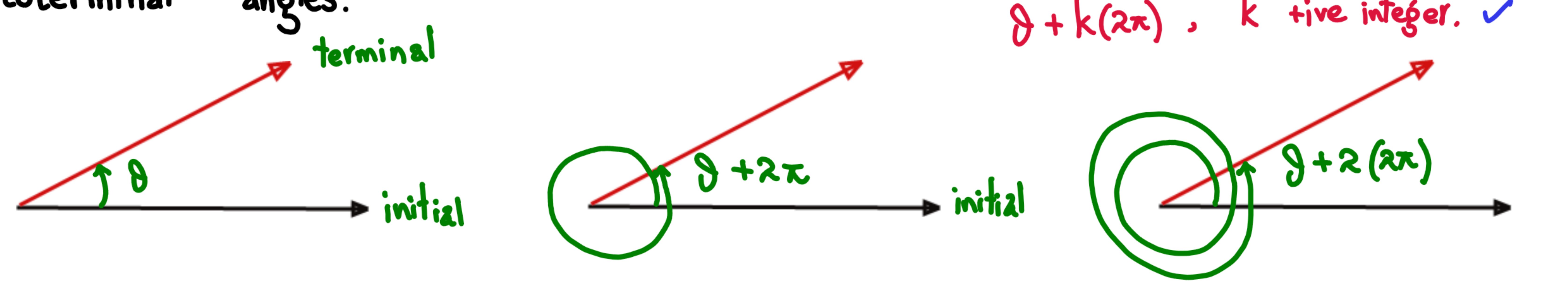
$0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$



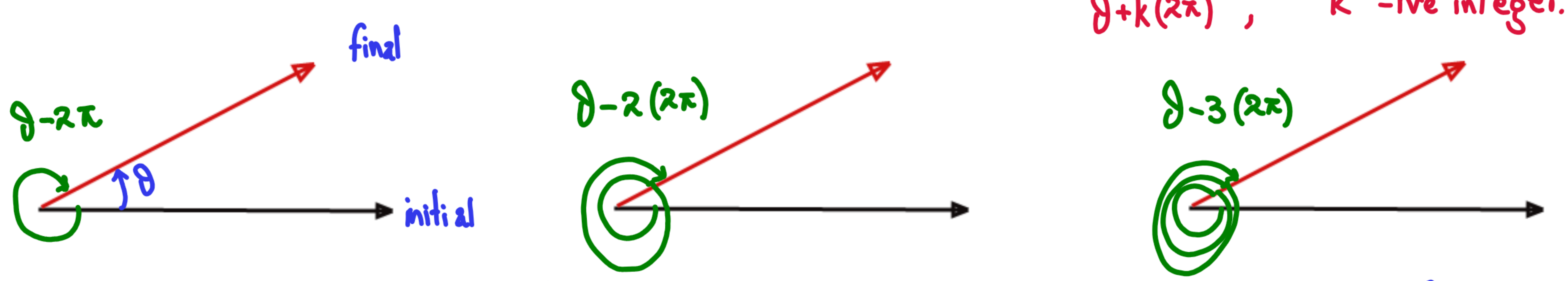
Coterminal Angles

Angles with same initial and terminal sides are called coterminal angles.

$\theta + k(2\pi)$, k +ive integer.



$\theta + k(2\pi)$, k -ive integer.



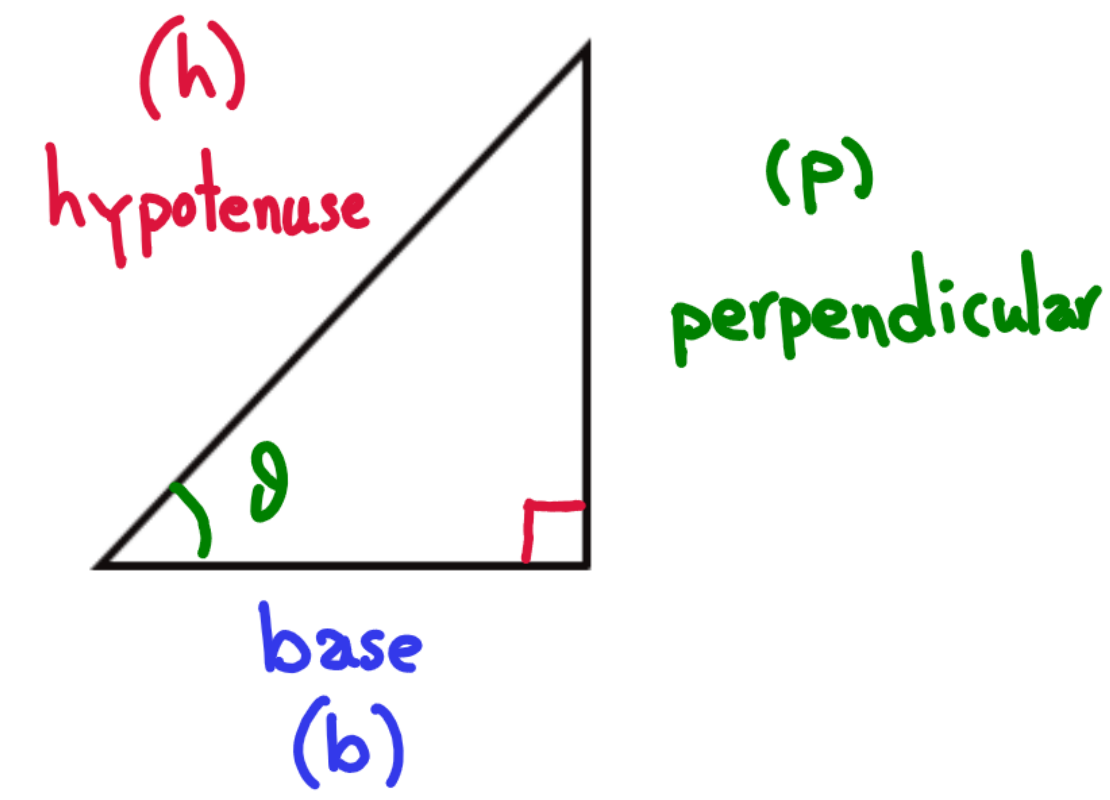
$\theta + k(2\pi)$, $k \in \mathbb{Z}$.

α, β are coterminal if $\alpha - \beta$ is multiple of 2π.

Trigonometric Functions

$\sin\theta = \frac{p}{h}$, $\operatorname{cosec}\theta = \frac{h}{p}$

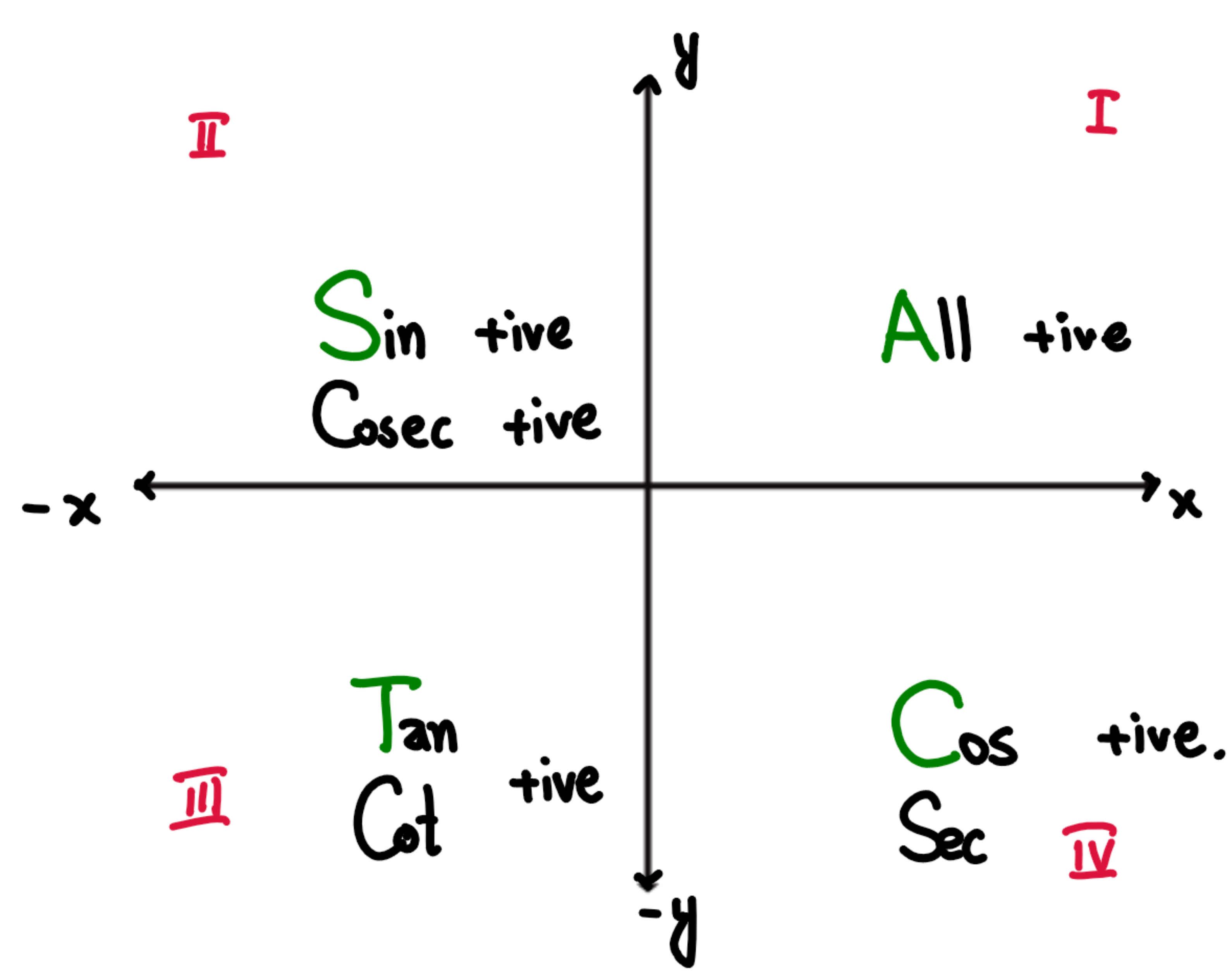
$\cos\theta = \frac{b}{h}$, $\sec\theta = \frac{h}{b}$



$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{p}{b}$, $\cot\theta = \frac{b}{p}$.

Signs of the Trigonometric Functions

After School To College.



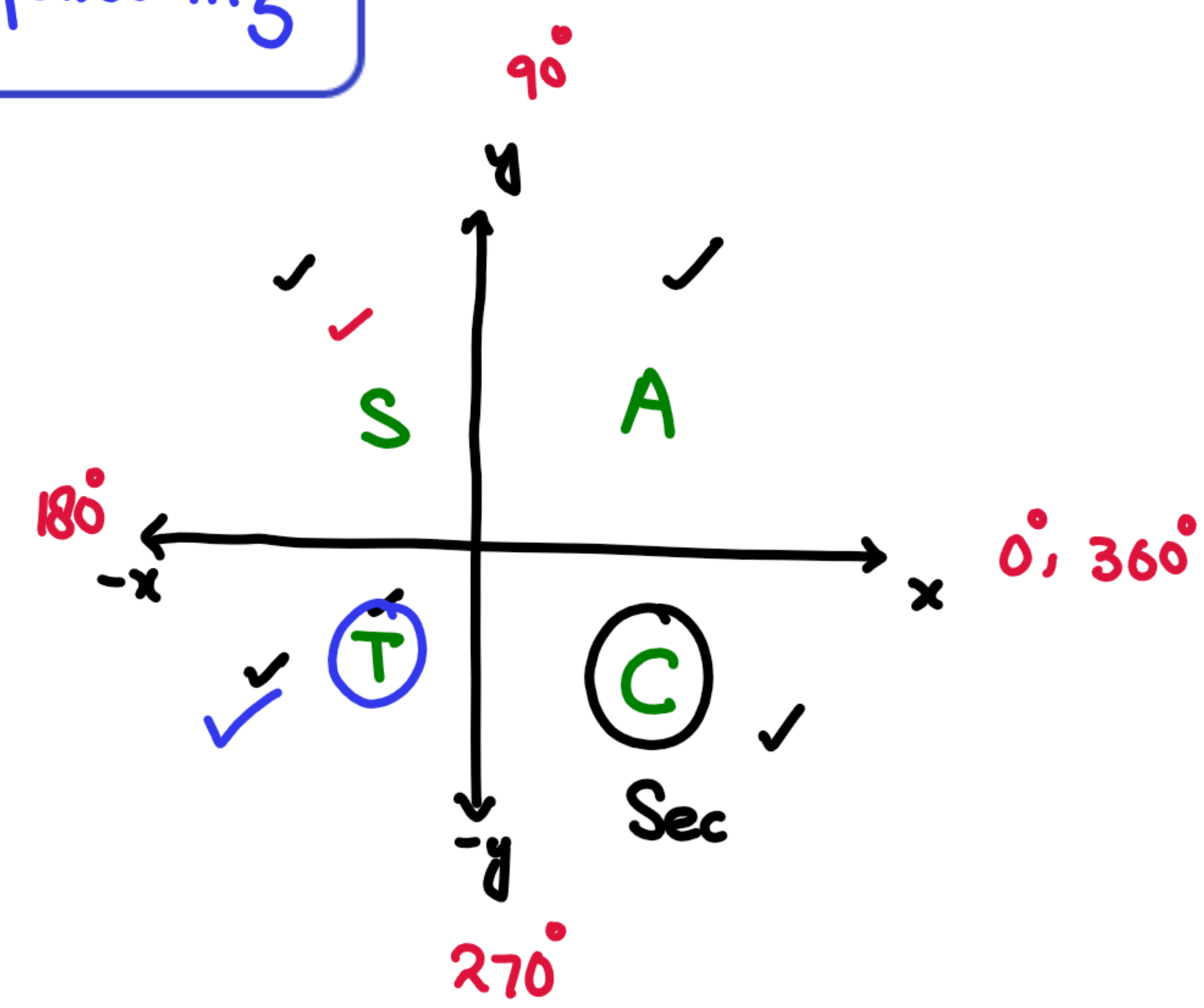
$\sin(-\theta) = -\sin\theta$, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

✓ $\cos(-\theta) = \cos\theta$, $\sec(-\theta) = \sec\theta$ ✓

✓ $\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$. ✓

Exercise 9.2

Q1 Find the signs of the following:



(i) $\sin 160^\circ$

Positive.

(ii) $\cos 190^\circ$

Negative

(iii) $\tan 115^\circ$

Negative.

(iv) $\sec 245^\circ$

Negative

(v) $\cot 80^\circ$

Positive.

(vi) $\operatorname{cosec} 297^\circ$

Negative.

Q2 Fill in the blanks:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

(i) $\sin(-310^\circ) = -\sin 310^\circ$

(ii) $\cos(-75^\circ) = +\cos 75^\circ$

(iii) $\tan(-182^\circ) = -\tan 182^\circ$

(iv) $\cot(-137^\circ) = -\cot 137^\circ$

(v) $\sec(-216^\circ) = +\sec 216^\circ$

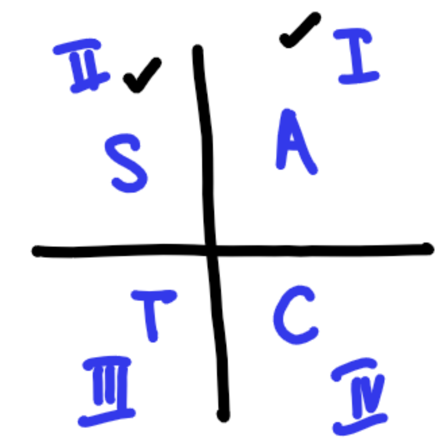
(vi) $\operatorname{cosec}(-15^\circ) = -\operatorname{cosec} 15^\circ$

③ In which quadrant are the terminal arms of the angle lie when

(i) $\sin\theta < 0$ and $\cos\theta > 0$

III, IV $\qquad$ I, IV

Answer : quad. IV



(ii) $\cot\theta > 0$ and $\csc\theta > 0$

I, III $\qquad$ I, II

Answer : quad. I.

(iii) $\tan\theta < 0$ and $\cos\theta > 0$

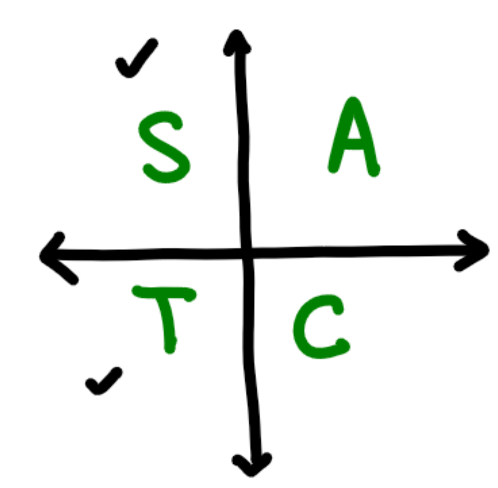
II, IV $\qquad$ I, IV

Answer : quad. IV

(iv) $\sec\theta < 0$ and $\sin\theta < 0$

II, III $\qquad$ III, IV

Answer: quad. III



(v) $\cot\theta > 0$ and $\sin\theta < 0$

I, III $\qquad$ III, IV

Answer: quad. III

(vi) $\cos\theta < 0$ and $\tan\theta < 0.$

II, III $\qquad$ II, IV

Answer: quad. II.

4. Find the values of the remaining trigonometric functions:

(i) $\sin\theta = \frac{12}{13}$ and the terminal arm of angle is in quad I.

Soln Given $p = 12$, $h = 13$ $\sin\theta = \frac{p}{h}$

Terminal arm in quad. I.

Since $h^2 = p^2 + b^2$

$$13^2 = 12^2 + b^2$$

$$169 - 144 = b^2$$

$$b^2 = 25$$

$$b = 5$$

$\cos\theta = +\frac{5}{13}$, $\sec\theta = +\frac{13}{5}$, $\csc\theta = +\frac{13}{12}$.

$\tan\theta = +\frac{12}{5}$, $\cot\theta = +\frac{5}{12}$

(ii) $\cos\theta = \frac{9}{41}$ and the terminal arm of angle is in quad IV.

Soln Given $b = 9$, $h = 41$, terminal arm in quad. IV.

only $\cos\theta$ and $\sec\theta$ are +ive.

Since $h^2 = p^2 + b^2$

$$41^2 = p^2 + 9^2$$

$$1681 - 81 = p^2$$

$$p^2 = 1600$$

$$p = 40$$

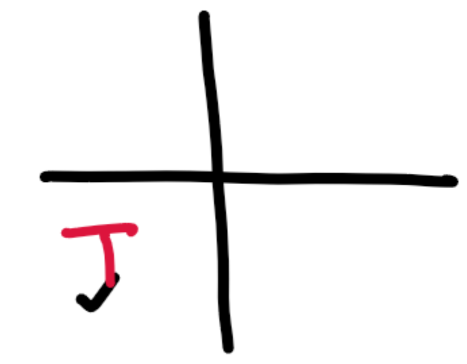
$\sin\theta = -\frac{40}{41}$, $\csc\theta = -\frac{41}{40}$, $\sec\theta = +\frac{41}{9}$.

$\tan\theta = -\frac{40}{9}$, $\cot\theta = -\frac{9}{40}$

(iii) $\cos\theta = -\frac{\sqrt{3}}{2}$ and the terminal arm of angle is in quad III.

Soln

Given $b = \sqrt{3}$, $h = 2$, quad - III.



Since $h^2 = p^2 + b^2$

$$2^2 = p^2 + (\sqrt{3})^2$$

$$4 - 3 = p^2$$

$$p^2 = 1$$

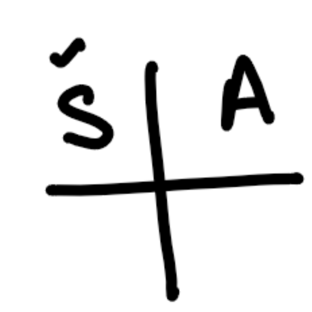
$$p = 1$$

$\sin\theta = -\frac{1}{2}$, $\operatorname{cosec}\theta = -2$, $\sec\theta = -\frac{2}{\sqrt{3}}$

$\tan\theta = +\frac{1}{\sqrt{3}}$, $\cot\theta = +\sqrt{3}$

(iv) $\tan\theta = -\frac{1}{3}$ and the terminal arm of angle is in quad. II.

Given $p = 1$, $b = 3$, quad - II.



Since $h^2 = p^2 + b^2$

$$h^2 = 1^2 + 3^2 = 1 + 9 = 10$$

$$h = \sqrt{10}$$

$\sin\theta = +\frac{1}{\sqrt{10}}$, $\operatorname{cosec}\theta = +\sqrt{10}$, $\cot\theta = -3$

$\cos\theta = -\frac{3}{\sqrt{10}}$, $\sec\theta = -\frac{\sqrt{10}}{3}$

(v) $\sin\theta = -\frac{1}{\sqrt{2}}$ and the terminal arm of angle is not in quad. III.

Given $p = 1, \quad h = \sqrt{2}$, $\sin\theta$ is negative in quad III, or quad IV.

Since $h^2 = p^2 + b^2$ $\Rightarrow$ terminal arm of θ is in quad. IV.

$$(\sqrt{2})^2 = 1^2 + b^2$$

$$2 - 1 = b^2$$

$$1 = b^2$$

$$b = 1$$

C

$\cos\theta = +\frac{1}{\sqrt{2}}$, $\sec\theta = +\sqrt{2}$, $\operatorname{cosec}\theta = -\sqrt{2}$

$\tan\theta = -1$, $\cot\theta = -1$

(5) If $\cot\theta = \frac{15}{8}$ and the terminal arm of angle is not in quad. I, find the values of $\cos\theta$ and $\operatorname{cosec}\theta$.

Given $b = 15, \quad p = 8$, terminal arm of θ is not in quad-I. $\cot\theta = \frac{b}{p}$

Since $h^2 = p^2 + b^2$ terminal arm of θ is in quad-III.

$$h^2 = 8^2 + 15^2$$

$$h^2 = 64 + 225 = 289$$

$$h = 17$$

S A
T C

$\sin\theta = \frac{p}{h}$

So

$\cos\theta = -\frac{15}{17}$ and $\operatorname{cosec}\theta = -\frac{17}{8}$

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Suppose Math.

6. If $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ and $m>1$ ✓ $(0<\theta<\frac{\pi}{2})$, find the values of the remaining trigonometric functions.

Given $h = m^2+1$, $p = 2m$, θ in quad - I. $\qquad \operatorname{cosec}\theta = \frac{h}{p}$

Since $h^2 = p^2+b^2$

$$(m^2+1)^2 = (2m)^2 + b^2$$

$$(m^2)^2 + 1^2 + 2m^2 = 4m^2 + b^2$$

$$(m^2)^2 + 1^2 + 2m^2 - 4m^2 = b^2$$

$$b^2 = (m^2)^2 + 1^2 - 2m^2 = (m^2-1)^2$$

$$\sqrt{b^2} = \sqrt{(m^2-1)^2}$$

$$b = \pm (m^2-1)$$

$m>1$ ✓

$m^2>1$

$m^2-1>0$. ✓

$$b = m^2-1 \checkmark$$

$\sin\theta = \frac{2m}{m^2+1}$, $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$, $\tan\theta = \frac{2m}{m^2-1}$

$\cos\theta = \frac{m^2-1}{m^2+1}$, $\sec\theta = \frac{m^2+1}{m^2-1}$, $\cot\theta = \frac{m^2-1}{2m}$

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(7) If $\tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in quad III, find the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$.

Given $p = 1$, $b = \sqrt{7}$, θ is not in quad-III

$\Rightarrow \theta$ is in quad-I.

$\sin\theta = \frac{p}{h}$

Since $h^2 = p^2 + b^2$

$h^2 = 1^2 + (\sqrt{7})^2 = 1 + 7 = 8$

$h = \sqrt{8}$

$\operatorname{cosec}\theta = \sqrt{8}$, $\sec\theta = \frac{\sqrt{8}}{\sqrt{7}}$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(\sqrt{8})^2 - \left(\frac{\sqrt{8}}{\sqrt{7}}\right)^2}{(\sqrt{8})^2 + \left(\frac{\sqrt{8}}{\sqrt{7}}\right)^2}$$

$$= \frac{\left(8 - \frac{8}{7}\right)}{\left(8 + \frac{8}{7}\right)} = \frac{\left(\frac{56-8}{\cancel{7}}\right)}{\left(\frac{56+8}{\cancel{7}}\right)} = \frac{\cancel{48}^{\,3}}{\cancel{64}_{\,4}}$$

$$= \frac{3}{4}$$ Ans.

Notes by:

Akhtar Abbas.

8. If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in quad. I, find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$.

Given $b = 5$, $p = 2$, θ in quad-I

Since $h^2 = p^2 + b^2$

$$h^2 = 2^2 + 5^2 = 4 + 25 = 29$$

$$h = \sqrt{29}$$

$$\sin\theta = \frac{2}{\sqrt{29}}, \qquad \cos\theta = \frac{5}{\sqrt{29}}$$

$$\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \frac{3\left(\frac{2}{\sqrt{29}}\right) + 4\left(\frac{5}{\sqrt{29}}\right)}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}} = \frac{\left(\frac{6}{\sqrt{29}} + \frac{20}{\sqrt{29}}\right)}{\left(\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}\right)}$$

$$= \frac{\left(\frac{26}{\cancel{\sqrt{29}}}\right)}{\left(\frac{3}{\cancel{\sqrt{29}}}\right)} = \frac{26}{3} \quad \text{Ans.}$$

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Mukhtar Abbas