Matrices and determinant

Book: Algebra and trigonometry **Chapter: Matrices and determinant**
Time: No matter **Prepare by *M. Fiaz Hussain***

Q1. If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$ then show that, $A^4 = I$.

Q2. Find X if, $X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$.

Q3. If A and B are the squere matrices of same order then show that,

$$(A+B)^2 \neq A^2 + 2AB + B^2$$

Q4. Show that without expansion. $\begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix} = l^2(3a+l)$.

Q5. Show that without expansion. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

Q6. Show that without expansion. $\begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix} = 0$.

Q7. Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$.

Q8. Find the value of λ if A is singular matrix. $A = \begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$.

Q9. Show that $(A^{-1})^t = (A^t)^{-1}$ if $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$.

Q10. if A and B are non-singular matrices. then show that, $(AB)^{-1} = B^{-1}A^{-1}$.

Q11. if $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$ then show that, $A - A^t$ is skew symmetric.

Q12. If $A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$ then show that, $A - (\bar{A})^t$ is skew hermitian.

Q13. If A and B are symmetric matrix and $AB = BA$. Show that AB is symmetric matrix.

Q14. If A is symmetric or skew symmetric, show that A^2 is symmetric.

Q15. Find the rank of the following matrix $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$.

Q16. Solve the following equations.

$$\begin{aligned} x + y &= 2 \\ 2x - z &= 1 \\ 2y - 3z &= -1 \end{aligned}$$

Q17. Solve the following equations.

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 8 \\ x_1 + 2x_2 + 2x_3 &= 6 \\ x_1 - 2x_2 - x_3 &= 1 \end{aligned}$$

Q18. Find the inverse of the following matrix $\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$.

Q19. If A is a square matrix of order 3 then show that, (i) $A + A^t$ is symmetric matrix. (ii) $A - A^t$ is skew symmetric matrix.

Q20. Show that without expansion $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$

Q21. Show that without expansion $\begin{vmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{vmatrix} = a^3 + b^3.$

Q22. Show that without expansion $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0.$

Q23. Show that without expansion $\begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix} = 0.$

Q24. Show that $\begin{vmatrix} mn & l & l^2 \\ nl & m & m^2 \\ lm & n & n^2 \end{vmatrix} = \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$.

Q25. Show that the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$ singular or non-singular.

Q26. If A is a square matrix of order 3 then show that $|kA| = k^3|A|$.

Best of luck