

IMPORTANT—FORMULAE

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Relation Between l & θ

$$l = r\theta$$

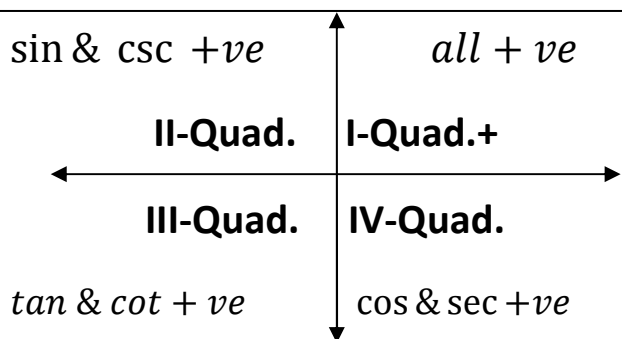
Conversion of Radian

$$1^\circ = \frac{\pi}{180} \text{radian} \quad \& \quad 1 \text{radian} = \frac{180^\circ}{\pi}$$

Fundamental Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

Signs of Trigonometric Function



Values of Trigonometric Functions

θ	0°	$30^\circ = \pi/6$	$45^\circ = \pi/4$	$60^\circ = \pi/3$	$90^\circ = \pi/2$
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

Fundamental Laws of Trigonometry

- $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$
- $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$
- $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
- $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
- $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$
- $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$

Double Angle Identities

- $\sin 2\alpha = 2 \sin\alpha \cos\alpha$
- $\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 2\cos^2\alpha - 1 \\ 1 - 2\sin^2\alpha \end{cases}$
- $\tan 2\alpha = \frac{2 \tan\alpha}{1 - \tan^2\alpha}$

Triple Angle Identities

- $\sin 3\alpha = 3 \sin\alpha - 4 \sin^3\alpha$
- $\cos 3\alpha = 4 \cos^3\alpha - 3 \cos\alpha$
- $\tan 3\alpha = \frac{3 \tan\alpha - \tan^3\alpha}{1 - 3 \tan^2\alpha}$

Sum, Difference & Product

- $2 \sin\alpha \cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
- $2 \cos\alpha \sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$
- $2 \cos\alpha \cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$
- $-2 \sin\alpha \sin\beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$
- $\sin P + \sin Q = 2 \sin \frac{P+Q}{2} \cos \frac{P-Q}{2}$
- $\sin P - \sin Q = 2 \cos \frac{P+Q}{2} \sin \frac{P-Q}{2}$
- $\cos P + \cos Q = 2 \cos \frac{P+Q}{2} \cos \frac{P-Q}{2}$
- $\cos P - \cos Q = -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$

Domain & Range of Trig. Functions

Functions	Domain	Range
$y = \sin x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \cos x$	$-\infty < x < +\infty$	$-1 \leq y \leq 1$
$y = \tan x$	$-\infty < x < +\infty$ $x \neq \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \cot x$	$-\infty < x < +\infty$ $x \neq \frac{(2n+1)\pi}{2}$	$-\infty < y < +\infty$
$y = \sec x$	$-\infty < x < +\infty$ $x \neq n\pi$	$y \geq 1 \text{ or } y \leq -1$
$y = \csc x$	$-\infty < x < +\infty$ $x \neq n\pi$	$y \geq 1 \text{ or } y \leq -1$

where $n \in \mathbb{Z}$.

Period of Trigonometric Functions

Function	Period
$\sin a\theta$	$\frac{2\pi}{a}$
$\cos a\theta$	
$\csc a\theta$	
$\sec a\theta$	
$\tan a\theta$	$\frac{\pi}{a}$
$\cot a\theta$	

The Law of Sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

The Law of Cosine

- $a^2 = b^2 + c^2 - 2bc \cos \alpha$
- $b^2 = c^2 + a^2 - 2ca \cos \beta$
- $c^2 = a^2 + b^2 - 2ab \cos \gamma$

The Law of Tangents

- $\frac{a-b}{a+b} = \frac{\tan(\frac{\alpha-\beta}{2})}{\tan(\frac{\alpha+\beta}{2})}$
- $\frac{b-c}{b+c} = \frac{\tan(\frac{\beta-\gamma}{2})}{\tan(\frac{\beta+\gamma}{2})}$
- $\frac{c-a}{c+a} = \frac{\tan(\frac{\gamma-\alpha}{2})}{\tan(\frac{\gamma+\alpha}{2})}$

Half Angle Formulas

- $\sin \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
- $\sin \frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$
- $\sin \frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$
- $\cos \frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$
- $\cos \frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ca}}$
- $\cos \frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$
- $\tan \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
- $\tan \frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$
- $\tan \frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Area of Triangles (Δ)

- $\Delta = \frac{1}{2}bc \sin \alpha = \frac{1}{2}ca \sin \beta = \frac{1}{2}ab \sin \gamma$
- $\Delta = \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha} = \frac{b^2 \sin \gamma \sin \alpha}{2 \sin \beta} = \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma}$
- $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$
where $s = \frac{a+b+c}{2}$

Circum - Circle (R)

- $R = \frac{a}{2 \sin \alpha} = \frac{b}{2 \sin \beta} = \frac{c}{2 \sin \gamma}$
- $R = \frac{abc}{4\Delta}$

In - Circle (r)

$$r = \frac{\Delta}{S}$$

Escribed - Circle

$$r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$

Inverse Trigonometr Formulas

$$\begin{aligned} \sin^{-1} A + \sin^{-1} B &= \sin^{-1}(A\sqrt{1-B^2} + B\sqrt{1-A^2}) \\ \sin^{-1} A - \sin^{-1} B &= \sin^{-1}(A\sqrt{1-B^2} - B\sqrt{1-A^2}) \\ \cos^{-1} A + \cos^{-1} B &= \cos^{-1}(AB - \sqrt{(1-A^2)(1-B^2)}) \\ \cos^{-1} A - \cos^{-1} B &= \cos^{-1}(AB + \sqrt{(1-A^2)(1-B^2)}) \\ \tan^{-1} A + \tan^{-1} B &= \tan^{-1}\left(\frac{A+B}{1-AB}\right) \\ \tan^{-1} A - \tan^{-1} B &= \tan^{-1}\left(\frac{A-B}{1+AB}\right) \\ 2\tan^{-1} A &= \tan^{-1}\left(\frac{2A}{1-A^2}\right) \end{aligned}$$

Principal Trig. Functions

Function	Domain	Range
$y = \sin x$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	$-1 \leq y \leq 1$
$y = \sin^{-1} x$	$-1 \leq y \leq 1$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
$y = \cos x$	$0 \leq x \leq \pi$	$-1 \leq y \leq 1$
$y = \cos^{-1} x$	$-1 \leq y \leq 1$	$0 \leq x \leq \pi$
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	\mathbb{R}
$y = \tan^{-1} x$	\mathbb{R}	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
$y = \cot x$	$0 < x < \pi$	\mathbb{R}
$y = \cot^{-1} x$	\mathbb{R}	$0 < x < \pi$
$y = \sec x$	$[0, \pi], x \neq \frac{\pi}{2}$	$y \leq -1$ or $y \geq 1$
$y = \sec^{-1} x$	$x \leq -1$ or $x \geq 1$	$[0, \pi], y \neq \frac{\pi}{2}$
$y = \csc x$	$[-\frac{\pi}{2}, \frac{\pi}{2}], x \neq 0$	$y \leq -1$ or $y \geq 1$
$y = \csc^{-1} x$	$x \leq -1$ or $x \geq 1$	$[-\frac{\pi}{2}, \frac{\pi}{2}], y \neq 0$

Best of Luck

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