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Federal Board - Annual 2004 Paper II
Mathematics II (Objective), Time Allowed: 20 Min.
Marks: 20 , Available online @ http://www.mathcity.org/fsc

Section – A (20 marks)

Note: Section A is compulsory.

Q. 1(a): Fill in the blanks.

i) The domain of $g(x) = \sqrt{x+1}$ is …….

ii) $\frac{d}{dx}(\text{sech}\, x) =$ …….

iii) $\int \text{cosec}^2 x\, dx =$ …….

iv) $l_1 \perp l_2$ if $1 + m_1 m_2 =$ ……..

v) $A^c \cup B^c =$ …….

Q. 1(b): Encircle the correct answer as true or false.

i) If $0 < e < 1$ then conic is an ellipse. T / F

ii) $\lim_{x\to 0}\left(\frac{e^x - 1}{x}\right) = \log_{10} e$ T / F

iii) $\frac{d}{dx}a^x = xa^{x-1}$ T / F

iv) Gradient of x-axis is zero T / F

v) A non vertical line divides the plane into left and right planes. T / F

Q. 1(c): Choose and tick (✓) the best possible answer.

i) The symbol $\frac{dy}{dx}$ is used for the derivative of y w.r.t x by

a) Newton b) Leibniz
c) Lagrange d) Cauchy

ii) If $b^2 = a^2\left(e^2 - 1\right)$, then the conic is

a) Parabola b) Ellipse
c) Hyperbola d) None

iii) If $\underline{a} \times \underline{b} = 0$ and $\underline{a} \cdot \underline{b} = 0$ then

a) $\underline{a} \parallel \underline{b}$ b) $\underline{a} \perp \underline{b}$
c) At least one is zero d) None

iv) The series $f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + ... \frac{x^n}{n!}f^{(n)}(0) + ...$ is

a) Taylor series b) Maclaurin Series
c) Binomial Series d) None

v) $\lim_{x\to -\infty} e^x$ is equal to

a) ∞ b) 0
c) e d) None

Q. 1(d): Match the column I with column II and write the correct answer in column III.

	Column I	Column II	Column III
i.	$\lim_{x\to 1}\frac{x^2-1}{x^2-x}$	$[0,2]$	
ii	Domain of the function $y=\begin{cases} x & if \quad 0\le x\le 1 \\ x-1 & if \quad 1\le x\le 2 \end{cases}$	$\ln x + c$	
iii	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	2	
iv	$y=mx+c$	Length of latus rectum is $2b^2/a$	
v	$\int \frac{1}{x}dx$	y-intercept is c	

ANSWERS

Q . 1(a):

(i) $[-1,\infty)$ (ii) $-\operatorname{sech} x \tanh x$ (iii) $-\cot x$ (iv) 0 (v) $(A\cap B)^c$

Q . 1(b):

(i) T (ii) F (iii) F (iv) T (v) F

Q . 1(c):

(i) b (ii) c (iii) c (iv) b (v) a

Q . 1(d):

(i) 2 (ii) $[0,2]$ (iii) Length of latus rectum is $2b^2/a$

(iv) y-intercept is c (v) $\ln x + c$

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Section –B (4 ´ 10 =40 marks)	
Q # 2. *Attempt any **TEN** parts. Graph paper will be supplied on demand.*	
(i) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$	Ex 1.3 – 1.5.5 – p23 (Article)
(ii) Determine whether the given function $f(x) = x^{\frac{2}{3}} + 6$ is even or odd.	Ex 1.1 – 9(v) – p11
(iii) If $x = \sin\theta$, $y = \sin m\theta$ show that $\left(1-x^2\right)y_2 - xy_1 + m^2 y = 0$.	Ex 2.7 – 5 – p95
(iv) Show that $y = \frac{\ln x}{x}$ has a maximum value at $x = e$.	Ex 2.9 – 4 – p113
(v) Find the area between the x-axis and the curve $y^2 = 4 - x$ in the first quadrant from $x = 0$ to $x = 3$.	Ex 3.7 – Exp5 – p167
(vi) Find the general solution of $\frac{dy}{dx} - x = xy^2$. Also find the particular solution if $y = 1$ when $x = 0$.	Ex 3.8 – 19 – p178
(vii) Find an equation of horizontal line through $(7,9)$.	Ex 4.3 – 9(i) – p216
(viii) The xy-coordinate axes are rotated about the origin through an angle of $45°$. The new axes are OX and OY. Find the XY-coordinate of the point whose xy-coordinate are $(5,3)$.	Ex 4.2 – 3(i) – p190
(ix) Derive standard equation of parabola.	Ex 6.4 – Art – p273
(x) Minimize $z = 2x + y$ subject to the constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$.	Ex 5.3 – 4 – p248
(xi) Find the centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$.	Ex 6.1 – 2(d) – p255
(xii) Find an equation of the ellipse with given data: foci $(\pm 3, 0)$ and minor axis of length 10.	Ex 6.5 – 1(i) – p290
(xiii) Show that the vector $2\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 3\underline{j} - 5\underline{k}$ and $3\underline{i} - 4\underline{j} - 4\underline{k}$ form the sides of a triangle.	Ex 7.3 – Exp6 – p347
(xiv) Evaluate: $\int \tan^4 x \; dx$	Ex 3.4 – 2(i) – p144

Section C (40 Marks (5+5 each))	
***Note:** Attempt any **FOUR** questions. Graph paper will be supplied on demand.*	
Q # 3 (a) Find the value of m and n so that the given function f is continuous at $x = 3$. $$f(x) = \begin{cases} mx & if \quad x < 3 \\ n & if \quad x = 3 \\ -2x + 9 & if \quad x > 3 \end{cases}$$	Ex 1.4 – 5(i) – p32
(b) If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$. Show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$.	Ex 2.3 – 16 – p61

Q # 4 (a). The perimeter of a triangle is $16\,cm$. If one side is of length $6\,cm$. What are the lengths of the other sides for maximum area of the triangle? **(b)** Evaluate: $\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$.	Ex 2.10 – 4 – p117 Ex 3.6 – 27 – p164
Q # 5 (a) Show that $\cos(x+h) = \cos x - \frac{h}{1!}\sin x - \frac{h^2}{2!}\cos x + \frac{h^3}{3!}\sin x + ...$ and evaluate $\cos 61^\circ$. **(b)** Evaluate: $\int \frac{x+\sin x}{1+\cos x}\,dx$.	Ex 2.8 - 2 – p101 Ex 3.4 – 5(xiv) – p145 (Excluded)
Q # 6 (a) Graph the feasible region of the following system of linear inequalities and find the corner points. $x+y \le 5 \quad , \quad -2x+y \ge 2 \quad , \quad x \ge 0$. **(b)** Find the distance between the parallel lines $l_1 : 2x-5y+13=0$ and $l_2 : -2x+5y-6=0$	Ex 5.2 – 1(iii) - p242 Ex 4.3 – Exp – p213 (Old Book)
Q # 7 (a) The major axis of an ellipse in standard form lies along the *x*-axis and has length $4\sqrt{2}$. The distance between foci equals the length of the minor axis. Write an equation of the ellipse. **(b)** Find an equation of the line through the intersection of the line $x+2y+3=0$ and $3x+4y+7=0$ and making equal intercept on the axis.	Ex 6.5 – 6 – p291 Ex 4.4 – 2(iii) – p223
Q # 8 (a) An arch in the form of half an ellipse is $40\,m$ wide and $15\,m$ high at the centre. Find the height of the arch at a distance of $10\,m$ from its centre. **(b)** Prove that the points whose position vectors are $A\left(-6\underline{i}+3\underline{j}+2\underline{k}\right)$, $B\left(3\underline{i}-2\underline{j}+4\underline{k}\right)$ and $C\left(5\underline{i}+7\underline{j}+3\underline{k}\right)$ are coplanar.	Ex 6.5 – Exp4 – p289 Ex 7.5 – Exp5 -p363 (Need Correction)
Q # 9 (a) Evaluate: $\lim_{\theta\to 0} \frac{\tan\theta - \sin\theta}{\sin^3\theta}$ **(b)** Prove by vector method: $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.	Ex 1.3 – 3(xii) – p27 Ex 7.4 – 8 – p358