DEFINITIONS

Textbook of Algebra and Trigonometry Class XI

Punjab Textbook Board

Mr. Aurang Zaib
Qualification: M.Phil in Mathematics
Job Status: Lecturer
Contact: itxaurangzaib@gmail.com
https://www.mathcity.org/people/aurang-zaib

1 Chapter 01: Number System

1.1 Rational Number:

A number which can be expressed in the form $\frac{p}{q}$, where $p, q \in \mathbb{Z}$ and $q \neq 0$, is termed as a rational number.

Example:

$\frac{3}{4}$, $\frac{7}{2}$

1.2 Irrational Number:

A real number that cannot be represented as a fraction of two integers is called an irrational number.

Example:

$\sqrt{2}$, π

1.3 Real Number:

The set comprising all rational and irrational numbers is referred to as the real numbers, denoted as $\mathbb{R}$.

1.4 Terminating Decimal:

A decimal number that has a finite number of digits in its decimal part.

Example:

0.25, 3.75

1.5 Recurring Decimal:

A decimal in which one or more digits repeat indefinitely.

Example:

$0.3333...$, $1.234234...$

1.6 Non-terminating Decimal or Non-recurring Decimal

A non-terminating decimal, also known as a non-recurring decimal, is a decimal representation of a number that neither terminates nor repeats. These decimals cannot be expressed as a fraction with integer numerator and denominator. They often represent irrational numbers.

Example: π (pi) is a well-known non-terminating, non-recurring decimal. Its decimal representation is $3.1415....$ Similarly, $\sqrt{2}$ is another example, with its decimal representation being $1.41421356....$

1.7 Binary Operations

A binary operation on a set A is a rule, typically denoted by $\circ$ or $\star$, that assigns to any pair of elements in A another element of A.

Example: In the set of real numbers $\mathbb{R}$, two important binary operations are addition (+) and multiplication ($\times$). For example, for any real numbers a and b, $a + b$ and $a \times b$ are also real numbers.

1.8 Complex Number

A complex number is a number of the form $z = x+iy$, where x and y are real numbers, and $i = \sqrt{-1}$ is the imaginary unit. Here, x is called the real part, and y is called the imaginary part of z.

Example: Some examples of complex numbers include 2, $3 + \sqrt{3}i$, and $\frac{1}{2} + i$.

1.9 Real Plane or Coordinate Plane

The real plane, also known as the coordinate plane, is the geometric plane where a coordinate system, typically consisting of horizontal and vertical axes, has been specified.

Example: In the Cartesian coordinate system, the real plane consists of a horizontal x-axis and a vertical y-axis. Any point in this plane can be represented by an ordered pair (x, y), where x represents the horizontal position (abscissa) and y represents the vertical position (ordinate).

1.10 Argand Diagram

An Argand diagram is a graphical representation of complex numbers on the complex plane. It is similar to the Cartesian coordinate system, where the horizontal axis represents the real part, and the vertical axis represents the imaginary part of complex numbers.

Example: Consider the complex number $z = 3 + 2i$. In the Argand diagram, the real part $x = 3$ corresponds to the horizontal axis, and the imaginary part $y = 2$ corresponds to the vertical axis. Thus, z would be plotted as a point at coordinates $(3, 2)$ on the Argand diagram.

1.11 Modulus of Complex Number

The modulus of a complex number $z = x + iy$ is the distance from the origin to the point representing the number on the complex plane. It is denoted by $|z|$ or $|(x, y)|$.

Example: For the complex number $z = 3 + 4i$, the modulus $|z|$ is calculated as the square root of the sum of squares of its real and imaginary parts: $|z| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$. Therefore, $|z| = 5$.

2 Chapter 02: Set and Operations

2.1 Set

A set is a well-defined collection of distinct objects or elements.

Example:

- Consider the set of natural numbers: $N = \{1, 2, 3, 4, \ldots\}$, where each element is distinct and well-defined.

2.2 Methods to describe a set

There are 3 method to describe a set

Descriptive Method

Sets can be described using words.

Example: N can be described descriptively as the set of all natural numbers.

Tabular Method

Sets can be listed by enumerating their elements within braces.

Example: N can be represented tabularly as $N = \{1, 2, 3, 4, \ldots\}$.

Set-builder Method

In this method, a property common to all elements is described using set-builder notation.

Example: Let $A = \{x \mid x \text{ is any natural number}\}$. This implies that A consists of all natural numbers.

2.3 Order of a Set

The number of elements in a set.

Example: If $A = \{3, 4\}$, then the order of A is 2.

2.4 Equal Set

Two sets are equal if they contain the same elements, regardless of the order.

Example: $A = \{2, 4, 6, 8\}$ and $B = \{2, 8, 6, 4\}$ are equal sets since they have the same elements.

2.5 Equivalent Set

Two sets are equivalent if there exists a one-to-one correspondence between their elements.

Example: $A = \{2, 4, 6, 8\}$ and $B = \{a, b, c, d\}$ are equivalent sets if each element in A corresponds to an element in B.

2.6 Singleton Set

A singleton set is a set that contains only one element.

Example:

- $A = \{2\}$
- $B = \{5\}$

2.7 Null Set

A null set, also known as an empty set, is a set that contains no elements.

Example:

- $\emptyset$
- $C = \{\}$

2.8 Finite Set

A finite set is a set that contains a limited, countable number of elements.
Example:

- $D = \{1, 2, 3, 4\}$
- $E = \{a, b, c\}$

2.9 Infinite Set

An infinite set is a set that contains an unlimited, uncountable number of elements.
Example:

- $F = \{1, 2, 3, ...\}$ (Set of natural numbers)
- $G = \{a, b, c, ...\}$ (Set of letters in the alphabet)

2.10 Subset

A subset is a set that contains elements from another set, where every element of the subset is also an element of the larger set.
Example:

- If $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then $A \subseteq B$.

2.11 Proper Subset

A proper subset is a subset that contains some, but not all, elements of another set.
Example:

- If $C = \{1\}$ and $D = \{1, 2\}$, then $C \subset D$.

2.12 Improper Subset

An improper subset is a subset that contains all elements of another set, including itself.
Example:

- If $E = \{1, 2\}$, then $E \subseteq E$.

2.13 Power Set

The power set of a set is the set of all its subsets, including the empty set and itself.
Example:

- If $A = \{1, 2\}$, then $P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

2.14 Universal Set

The universal set is the set containing all objects or elements under consideration.

Example:

- If we are considering the set of all integers, then the universal set could be denoted as $\mathbb{Z}$.

2.15 Complement of a Set

The complement of a set relative to the universal set is the set of all elements not contained in the given set.

Example:

- If $U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2\}$, then $A' = \{3, 4, 5\}$.

2.16 Deduction

Deduction is the process of drawing logical conclusions from known facts or premises.

Example:

- If it is known that all men are mortal, and Socrates is a man, then it can be deduced that Socrates is mortal.

2.17 Induction

Induction is the process of drawing general conclusions based on specific instances or observations.

Example:

- If we observe that a coin is heads-up five times in a row, we might induce that it will always land heads-up, which may or may not be true.

2.18 Aristotelian Logic

Aristotelian logic is a deductive system in which every statement is regarded as either true or false.

Example:

- "All humans are mortal" would be regarded as either true or false in Aristotelian logic.

2.19 Non-Aristotelian Logic

Non-Aristotelian logic is a deductive system that allows for more than two truth values, such as true, false, and unknown.

Example:

- In some systems of non-Aristotelian logic, a statement may be true, false, or neither true nor false.

2.20 Truth Table

A truth table is a table that shows all possible truth values of a given compound statement based on the truth values of its component parts.

Example: Consider the truth table for the logical statement $p \wedge q$:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2.21 Tautology

A tautology is a statement that is true for all possible truth values of its variables.

Example: $p \to q \leftrightarrow (\neg q \to \neg p)$ is a tautology because its truth table shows that it is always true, regardless of the truth values of p and q.

2.22 Contradiction

A contradiction is a statement that is always false, regardless of the truth values of its variables.

Example: $p \wedge \neg p$ is a contradiction because its truth table shows that it is always false.

2.23 Contingency

A contingency is a statement that can be either true or false depending on the truth values of its variables.

Example: $(p \to q) \wedge (p \vee q)$ is a contingency because its truth table shows that it is true for some combinations of truth values of p and q and false for others.

2.24 Function

A function is a relation between two non-empty sets A and B, where each element of set A is related to exactly one element of set B.

Example: Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. A function $f : A \to B$ could be defined as $f(1) = a, f(2) = b, f(3) = c$.

2.25 Bijective Function

A bijective function is a function that is both injective (one-to-one) and surjective (onto).

Example: Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = 2x$. It is both injective and surjective, hence bijective.

2.26 Injective Function

An injective function is a function that assigns distinct elements of the domain to distinct elements of the codomain.

Example: The function $f : \{1, 2, 3\} \to \{a, b, c\}$ defined as $f(1) = a, f(2) = b, f(3) = c$ is injective.

2.27 Groupoid

A groupoid is a non-empty set closed under a given binary operation.

Example: Let $G = \{a, b, c\}$ be a set with a binary operation $*$. If for any elements x, y in G, $x * y$ is also in G, then G is a groupoid under $*$.

2.28 Binary Operation

A binary operation on a set G is a mapping from the Cartesian product $G \times G$ into G. It associates every ordered pair of elements from G with a unique element of G.

Example: Addition $+$ on the set of integers $\mathbb{Z}$ is a binary operation because for any two integers a and b, their sum $a + b$ is also an integer.

2.29 Semi group

A semi group is a non-empty set equipped with a binary operation that is associative.

Example: The set of positive integers $\mathbb{Z}^+$ with the operation of multiplication $\times$ forms a semi group because multiplication is associative for positive integers.

2.30 Monoid

A monoid is a semi group with an identity element, i.e., a unique element that acts as an identity under the binary operation.

Example: The set of non-negative integers $\mathbb{Z}_0^+$ with the operation of addition $+$ forms a monoid because 0 serves as the identity element for addition.

2.31 Group

A group is a non-empty set equipped with a binary operation that is associative, has an identity element, and every element has an inverse.

Example: The set of integers $\mathbb{Z}$ with the operation of addition $+$ forms a group because for every integer a, there exists its additive inverse $-a$, and addition is associative.

2.32 Abelian Group

An Abelian group (or commutative group) is a group where the binary operation is commutative.

Example: The set of real numbers $\mathbb{R}$ with the operation of addition $+$ forms an Abelian group because addition of real numbers is commutative.

2.33 Linear Function

A linear function is a function of the form $f(x) = mx+c$, where m and c are constants, and the graph of the function is a straight line.

Example: The function $f(x) = 2x + 3$ is a linear function because its graph is a straight line with slope 2 and y-intercept 3.

2.34 Quadratic Function

A quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants, and the highest power of the variable is 2.

Example: The function $f(x) = x^2+3x+2$ is a quadratic function because its highest power of the variable is 2.

2.35 Unary Operation

A unary operation is an operation that takes a single input and produces a single output.

Example: The square root function $\sqrt{x}$ is a unary operation because it takes a single number x as input and produces its square root as output. For example, $\sqrt{9} = 3$.

3 Chapter 03:Matrices and determinants

3.1 Matrix

Definition: An arrangement of different elements in rows and columns, within square brackets is called a matrix.

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 4 \\ 2 & 1 & 1 \end{bmatrix}$$

3.2 Order of Matrix

Definition: The order of a matrix tells us the number of rows and columns it contains. It is denoted as $m \times n$, where m is the number of rows and n is the number of columns.

Example: For matrix $A = \begin{bmatrix} 3 & 1 & 7 \\ 0 & 5 & 4 \end{bmatrix}$, the order of A is 2×3.

3.3 Row Matrix

Definition: A matrix with only one row is called a row matrix.

Example:

$$B = \begin{bmatrix} 1 & 4 & 6 \end{bmatrix}$$

3.4 Column Matrix

Definition: A matrix with only one column is called a column matrix.

Example:

$$B = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

3.5 Square Matrix

Definition: A matrix with an equal number of rows and columns is called a square matrix.

Example:

$$C = \begin{bmatrix} 1 & 4 & 2 \\ 5 & 3 & 0 \end{bmatrix}$$

3.6 Rectangular Matrix

Definition: A matrix with a different number of rows and columns is called a rectangular matrix.

Example:

$$B = \begin{bmatrix} 1 & 4 & 6 \\ 1 & 3 & 0 \end{bmatrix}$$

3.7 Diagonal Matrix

Definition: A square matrix where all elements except those on the main diagonal are zero.

Example:

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3.8 Scalar Matrix

Definition: A diagonal matrix where all diagonal elements are the same.

Example:

$$B = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

3.9 Unit Matrix or Identity Matrix

Definition: A square matrix where all diagonal elements are 1, and all other elements are 0.

Example:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.10 Null Matrix or Zero Matrix

Definition: A matrix where all elements are zero.

Example:

$$O_{2\times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

3.11 Equal Matrix

Definition: Two matrices are said to be equal if they are of the same order and corresponding elements are equal.

Example:

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}$$

are equal matrices.

3.12 Upper Triangular Matrix

Definition: A square matrix where all elements below the main diagonal are zero.

Example:

$$B = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

3.13 Lower Triangular Matrix

Definition: A square matrix where all elements above the main diagonal are zero.

Example:

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 6 \end{bmatrix}$$

3.14 Singular Matrix

Definition: A square matrix whose determinant is zero.

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

3.15 Non-Singular Matrix

Definition: A square matrix whose determinant is nonzero.

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

3.16 Symmetric Matrix

Definition: A square matrix that is equal to its transpose.
Example:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

3.17 Skew Symmetric Matrix

Definition: A square matrix whose transpose is equal to its negative.
Example:

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

3.18 Hermitian Matrix

Definition: A square matrix that is equal to the conjugate transpose of itself.
Example:

$$A = \begin{bmatrix} 1 & 2i \\ -2i & 3 \end{bmatrix}$$

3.19 Skew Hermitian Matrix

Definition: A square matrix whose conjugate transpose is equal to its negative.
Example:

$$A = \begin{bmatrix} 0 & -2i \\ 2i & 0 \end{bmatrix}$$

4 Chapter 04: Quadratic Equations

4.1 Quadratic Equation:

An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants and $a \neq 0$.

Example:

$x^2 - 4 = 0$, $5x^2 - 7x = 0$

4.2 Exponential Equation:

Equations where the variable appears in the exponent.

Example:

$2^x = 16, 5^x = 125$

4.3 Reciprocal Equation:

An equation that remains unchanged when the variable is replaced by its reciprocal.

Example:

$x + \frac{1}{x} = 5$

4.4 Radical Equation:

Equations involving radical expressions of the variable.

Example:

$\sqrt{x+2} + \sqrt{x-3} = 7$

4.5 Remainder Theorem:

If a polynomial $f(x)$ of degree $n \geq 1$ is divided by $(x - a)$ until no x term remains in the remainder, then $f(x)$ is the remainder.

4.6 Polynomial Function:

An expression of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_n \neq 0$ and n is a non-negative integer.

4.7 Factor Theorem:

If $(x - a)$ is a factor of the polynomial $f(x)$, then $f(x) = 0$.

Example:

If $(x - 3)$ is a factor of $f(x) = x^2 - 9$, then $f(x) = 0$.

5 Chapter 05: Partial fractions

5.1 Partial fraction:

A partial fraction is a method used to decompose a complex rational function into simpler fractions. It's particularly useful in integration and solving differential equations.

$$\text{Example:} \quad \frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

5.2 Identity:

An identity is an equation that holds true for all values of the variables involved. It's a fundamental concept in mathematics, often used in proofs and problem-solving.

$$\text{Example:} \quad (x+1)^2 = x^2 + 2x + 1$$

5.3 Rational fraction:

A rational fraction is a fraction where both the numerator and denominator are polynomials, with the denominator not being zero and having no common factors.

$$\text{Example:} \quad \frac{x^2+3x+2}{2x^2-x-3}$$

5.4 Proper rational fraction:

In a rational fraction, if the degree of the numerator is less than the degree of the denominator, it's termed as a proper rational fraction.

$$\text{Example:} \quad \frac{x+2}{x^2+1}$$

5.5 Improper rational fraction:

If the degree of the numerator is greater than or equal to the degree of the denominator in a rational fraction, it's called an improper rational fraction.

$$\text{Example:} \quad \frac{x^2+3x+2}{2x+1}$$

5.6 Conditional equation:

A conditional equation is an equation that holds true only for specific values of the variable.

$$\text{Example:} \quad \frac{x}{x-1} = 1 \quad \text{holds true only for } x = 2$$

6 Chapter 06: Sequence and series

6.1 Sequence:

A sequence is an ordered list of numbers arranged according to a rule or pattern.

$$\text{Example:} \quad 1, 4, 7, 10, 13, ...$$

6.2 Real sequence:

A real sequence is a sequence where all elements are real numbers.

$$\text{Example:} \quad 0.5, 1.5, 2.5, 3.5, ...$$

6.3 Finite sequence:

A finite sequence is a sequence with a limited number of terms.

$$\text{Example:} \quad 2, 4, 6, 8, 10$$

6.4 Infinite sequence:

An infinite sequence is a sequence with an unlimited number of terms.

$$\text{Example:} \quad 1, 2, 3, 4, ...$$

6.5 Series:

A series is the sum of the terms of a sequence.

$$\text{Example:} \quad 1 + 2 + 3 + 4 + ... + n$$

6.6 Arithmetic sequence:

An arithmetic sequence is a sequence in which each term is obtained by adding a common difference to the previous term.

$$\text{Example:} \quad 2, 5, 8, 11, 14, ...$$

6.7 Arithmetic mean:

The arithmetic mean of two numbers is the average of those numbers.

$$\text{Example:}\quad \text{The arithmetic mean of } 2 \text{ and } 5 \text{ is } \frac{2+5}{2} = 3.5$$

6.8 Geometric progression:

A geometric progression is a sequence in which each term is obtained by multiplying the previous term by a common ratio.

$$\text{Example:}\quad 2, 6, 18, 54, ...$$

6.9 Geometric mean:

The geometric mean of two numbers is the positive square root of their product.

$$\text{Example:}\quad \text{The geometric mean of } 2 \text{ and } 8 \text{ is } \sqrt{2\times 8} = \sqrt{16} = 4$$

6.10 Harmonic progression:

A harmonic progression is a sequence in which each term is the reciprocal of an arithmetic progression.

$$\text{Example:}\quad \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...$$

6.11 Harmonic mean:

The harmonic mean of two numbers is the reciprocal of the arithmetic mean of the reciprocals of those numbers.

$$\text{Example:}\quad \text{The harmonic mean of } 2 \text{ and } 6 \text{ is } \frac{2}{\frac{1}{2}+\frac{1}{6}} = \frac{2}{\frac{4+1}{6}} = \frac{12}{5}$$

7 Chapter 07: Permutation, combination, probability

7.1 Permutation:

A permutation is an arrangement of objects in a specific order.
For example, if we have the letters A, B, C,
the permutations of these letters would include ABC, ACB, BAC, BCA, CAB, and CBA.

7.2 Circular permutation:

A circular permutation is a permutation of objects arranged in a circular manner. For instance, if we have three people sitting around a table, the number of circular permutations of these people would be 2.

7.3 Probability:

Probability is the likelihood of an event occurring, expressed as a numerical value between 0 and 1. For example, the probability of rolling a 6 on a fair six-sided die is $\frac{1}{6}$.

7.4 Sample space:

The sample space is the set of all possible outcomes of a given experiment. For example, if we toss a coin, the sample space would be $\{H, T\}$, where H represents heads and T represents tails.

7.5 Combination:

A combination is a selection of objects where the order is not considered. For example, if we have a group of 5 people and we want to select a committee of 3 people, the number of combinations would be $\binom{5}{3} = 10$.

7.6 Event:

An event is a subset of the sample space, representing a particular outcome or set of outcomes. For instance, if we roll a six-sided die, the event of rolling an odd number would include the outcomes $1, 3$, and 5.

7.7 Equally likely:

Two events are equally likely if each has the same probability of occurring. For example, when rolling a fair six-sided die, each outcome (1 through 6) is equally likely.

7.8 Mutually exclusive:

Two events are mutually exclusive if they cannot both occur simultaneously. For instance, when flipping a coin, the events of getting heads and getting tails are mutually exclusive.

8 Chapter 08: Mathematical induction and binomial theorem

8.1 Mathematical Induction

Mathematical Induction is a technique of proving a statement, theorem or formula which is thought to be true, for each and every natural number n. By generalizing this in form of a principle which we would use to prove any mathematical statement is 'Principle of Mathematical Induction.

8.2 Binomial Theorem

The Binomial Theorem is a fundamental concept in mathematics that provides a systematic way to expand expressions of the form $(a+b)^n$, where n is a positive integer. It states that:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

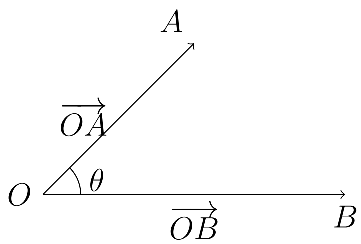
9 Chapter 09: Trigonometry

9.1 Trigonometry

Trigonometry, derived from the Greek words "Trei" (three), "Goni" (angles), and "Metron" (measurement), deals with the measurement of triangles and the relationships between their angles and sides.

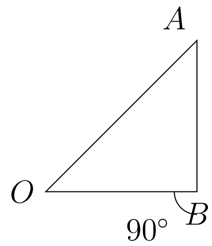
9.2 Angle

An angle is formed when two rays share a common endpoint, called the vertex. For example, consider the angle formed by the rays OA and OB in the figure below:



9.3 Degree

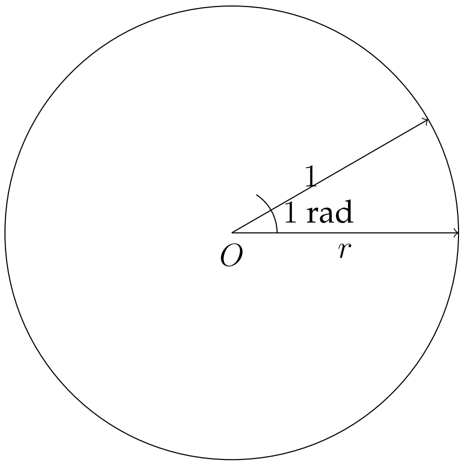
In the sexagesimal system, if a circle is divided into 360° equal parts, each called a degree (°). For instance, a right angle measures 90°, as shown:



9.4 Circular System (Radians)

In the circular system, angles are measured in radians. it is also SI unit of angle.
One radian is the angle subtended at the center of a circle by an arc whose arc length is equal to the radius of the circle.

here's a diagram representing 1 radian:



In this diagram, O represents the center of the circle, r represents the radius of the circle, and the arc subtends an angle of 1 radian at the center.

9.5 Sexagesimal System

This system measures angles in degrees, minutes, and seconds. For example, $16^\circ 13' 9''$ represents an angle in the sexagesimal system.

9.6 Trigonometric Functions:

These functions relate angles to the ratios of side lengths in a right-angled triangle. For instance:

$$\sin(\theta) = \frac{opposite}{hypotenuse}$$
$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$
$$\tan(\theta) = \frac{opposite}{adjacent}$$

These functions play crucial roles in various fields like physics, engineering, and astronomy.

10 Chapter 10: Period of Trigonometric Functions

10.1 Distance formula

Distance formula in two dimensions between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

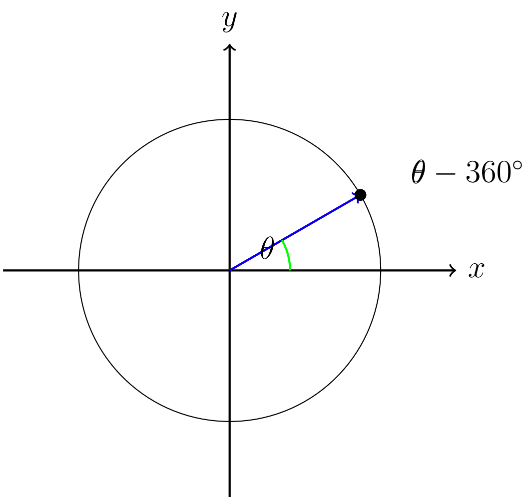
This formula calculates the Euclidean distance or straight-line distance between the two points.

10.2 General Angle (Coterminal Angle)

Definition

Angles having the same initial and terminal sides are called general angles or coterminal angles.

Diagram

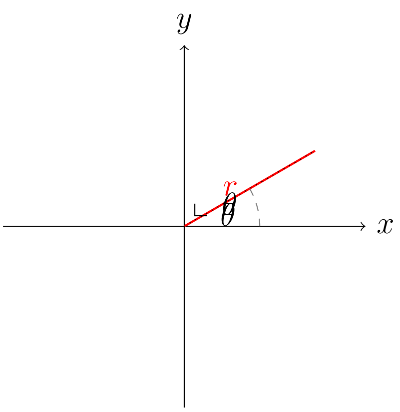


10.3 Angle in Standard Position

Definition

In a rectangular coordinate system, an angle is said to be in standard position if its vertex lies at the origin and its initial ray is along the positive x-axis.

Diagram

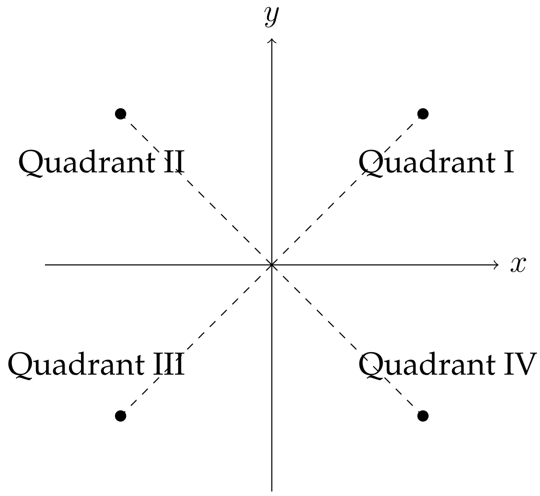


10.4 Quadrantal Angles

Definition

If the terminal side of an angle falls on the X-axis or Y-axis, then it is called a quadrantal angle.

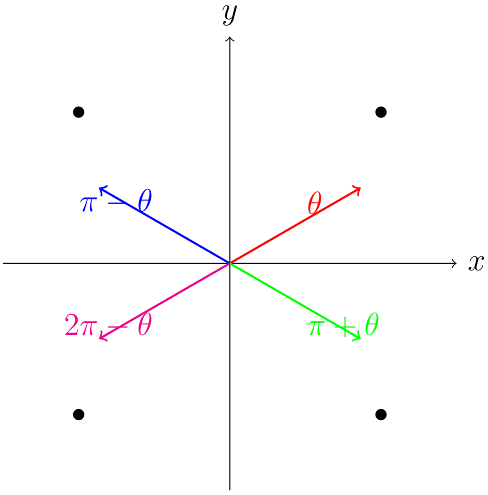
Diagram



10.5 Allied Angles

These are angles associated with a given angle θ, including angles like $90^\circ \pm \theta$, $180^\circ \pm \theta$, etc.

Diagram



11 Chapter 11: Trigonometry Functions and their Graph

11.1 Period

The period of a trigonometric function is the smallest positive number that, when added to the original circular measure of the angle, gives the same function value. For example, consider $\sin(\alpha + 2\pi) = \sin(\alpha)$.

11.2 Domains and ranges of trigonometric functions

Here's a table showing the domains and ranges of the six basic trigonometric functions:

Trigonometric Function	Domain	Range
$\sin(x)$	$(-\infty, \infty)$	$[-1, 1]$
$\cos(x)$	$(-\infty, \infty)$	$[-1, 1]$
$\tan(x)$	$\left(-\frac{\pi}{2} + n\pi, \frac{\pi}{2} + n\pi\right), n \in \mathbb{Z}$	$(-\infty, \infty)$
$\csc(x)$	$(-\infty, \infty)\backslash\{k\pi \mid k \in \mathbb{Z}\}$	$(-\infty, -1] \cup [1, \infty)$
$\sec(x)$	$(-\infty, \infty)\backslash\left\{\frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\right\}$	$(-\infty, -1] \cup [1, \infty)$
$\cot(x)$	$(0 + n\pi, \pi + n\pi), n \in \mathbb{Z}$	$(-\infty, \infty)$

12 Chapter 12: Applications of trigonometry

12.1 Angle of Elevation

Definition

When an object is at a higher level from the observer's eye, the angle made by the observer's eye to the object is called the angle of elevation.

Example

When looking up at the top of a tower from the ground, the angle formed between the ground level and the line of sight to the top of the tower is the angle of elevation.

12.2 Angle of Depression

Definition

When an object is at a lower level from the observer's eye, the angle made by the observer's eye to the object is called the angle of depression.

Example

When looking down from a cliff to a boat in the water below, the angle formed between the horizontal line of sight and the line of sight to the boat is the angle of depression.

12.3 Oblique Triangle

Definition

A triangle that is not right-angled, meaning it does not have a 90-degree angle.

Example

Any triangle that does not have a right angle, such as an equilateral triangle or a scalene triangle, is considered an oblique triangle.

12.4 Circumcircle

Definition

The circle passing through all three vertices of a triangle is called the circumcircle.

Properties

The center of the circumcircle is called the circumcenter, and its radius is called the circumradius (denoted by R).

12.5 Incircle

Definition

A circle drawn inside a triangle, tangent to all three sides of the triangle.

Properties

The center of the incircle is called the incenter, and its radius is called the inradius (denoted by r).

12.6 Escribed Circle

Definition

A circle that touches one side of the triangle externally and the extensions of the other two sides internally.

Properties

The center of the escribed circle is called the escribed center, and its radius is called the escribed radius.

13 Chapter 13: Inverse Trigonometric Functions

13.1 Principal Valued Functions in Trigonometry:

In trigonometry, the principal valued functions refer to the primary or principal values of trigonometric functions within a specific range that makes the functions single-valued. The principal values are chosen such that the functions remain well-defined and continuous over a defined interval.

14 Chapter 14: Solutions of trigonometric equations

14.1 Trigonometry Equation:

These equations involve trigonometric functions like sine, cosine, tangent, etc. For example:

$$\begin{aligned} \sin(x) &= \frac{1}{2} \\ \cos(x) - \tan(x) &= 0 \end{aligned}$$