1107 (Inter. Part-I) Session 2006-2008

Warning:- Please, do not write any thing on this Question Paper except your Roll No.

Mathematics (Subjective) Group -- I Paper (I)

Time Allowed:- 2.30 hours Maximum Marks:- 80

Section ---------------- I

2. Write short answers of any 25 questions; every question is of 2 marks. 50

(i) Simplify $(2, 6) \div (3, 7)$ (ii) $\forall Z \in C$, show that $Z^2 + \bar{Z}^{2}$ is real number (iii) If a, b are elements of a group G, then show that $(ab)^{-1} = b^{-1}a^{-1}$ (iv) Determine whether the statement is a tautology $q \vee (\sim q \vee p)$. (v) Write two proper subsets of $\{x / x \in Q \wedge 0 < x \leq 2\}$ (vi) Define a skew-symmetric matrix. (vii) If a square matrix A has two identical rows or two identical columns, then $|A| = 0$ (viii) Find value of α if A is singular $A = \begin{bmatrix} 4 & \alpha & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$

(ix) Solve the equation by completing the square $x^2 + 6x - 567 = 0$

(x) Show that $x^3 - y^3 = (x - y)(x - \omega y)(x - \omega^2 y)$

(xi) Discuss nature of roots of equation $25x^2 - 30x + 9 = 0$ (xii) The product of one less than a certain positive number and 2 less than three times the number is 14. Find the number.

(xiii) Resolve $\frac{7x + 25}{(x+3)(x+4)}$ into partial fractions. (xiv) Write the first four terms of the following A.Sequence when $a_5 = 17$ and $a_9 = 37$ (xv) Resolve $\frac{1}{x^2 - 1}$ into partial fractions.

(xvi) Sum the series $3 + 7 + 11 + \text{----}$ upto 16 terms. (xvii) Find 10^{th} term of the G. Sequence 3, 6, 12 ---- (xviii) Sum the series $2 + (1 - i) + \frac{1}{i} + \text{-----}$ to 8 terms

(xix) Find a_{12} of the H.Sequence $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}$ -------- (xx) How many words can be formed from the letters of "PLANE". (xxi) If ${}^nC_{12} = {}^nC_6$ find n

(xxii) Let $N(S) = 6$ $N(E) = 2$ Find the probability of the event E (xxiii) Find the Number of arrangements of the letters of the word "MATHEMATICS" (xxiv) Calculate by means of binomial theorem $(9.98)^4$ (xxv) Expand $\frac{1}{1+2x}$ to three terms, such that the expansion is valid.

(xxvi) Write $\sqrt[5]{31}$ in the form, so that the Binomial series may be applied.

(xxvii) Verify that $\sin^2 \pi/6 : \sin^2 \frac{\pi}{4} : \sin^2 \pi/3 = 1 : 2 : 3$ (xxviii) What is the circular measure of the angle between the hands of a clock at 4 O' Clock. (xxix) Without using table, find the value of $\cos 315^\circ$.

(xxx) Show that $\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$ (xxxi) Find the period of $\tan \frac{x}{3}$

(xxxii) In a right triangle, $\gamma = 90^\circ$, $b = 30.8$, $c = 37.2$, find a.

(xxxiii) Write the formulae of $\sin \gamma/2$ & $\cos \gamma/2$ in terms of the measures of the sides of the ΔABC. (xxxiv) Find the Area of the triangle ABC in which $a = 21.6$, $c = 30.2$ and $\alpha = 52^\circ, 40'$

(xxxv) Using the relation $R = \frac{b}{2\sin\beta}$, Prove that $R = \frac{abc}{4\Delta}$ (xxxvi) Show that $\cos(2\sin^{-1}x) = 1 - 2x^2$ (xxxvii) Solve the trigonometric equation $\sin 2x + \sin x = 0$ in the interval $[0, 2\pi]$

Section ----------------- II

Note:- Attempt any THREE questions. All questions carry equal marks. $(10 \times 3 = 30)$

3. (a) If $(G, *)$ is a group with e its identity. Then Prove that e is unique.

(b) If the roots of $px^2 + qx + q = 0$ are α and β then prove that $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{q}{p}} = 0$

4. (a) Without expansion prove that $\begin{vmatrix} mn & l & l^2 \\ nl & m & m^2 \\ lm & n & n^2 \end{vmatrix} = \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$

(b) With usual notations, Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

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5. (a) Find n so that $\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be H.M. between 'a' and 'b'

(b) Show that $\dfrac{n^3 + 2n}{3}$ represents an integer $\forall\, n \in N$ by mathematical induction.

6. (a) If $\tan\theta = \dfrac{1}{\sqrt{7}}$ and terminal arm of the angle is not the III Quadrant. Find the values of $\dfrac{\operatorname{Cosec}^2\theta - \operatorname{Sec}^2\theta}{\operatorname{Cosec}^2\theta + \operatorname{Sec}^2\theta}$

(b) Reduce $\operatorname{Sin}^4\theta$ to Expansion involving only function of multiple of θ raised to first power.

7. (a) Prove that $a\,b\,c\,(\operatorname{Sin}\alpha + \operatorname{Sin}\beta + \operatorname{Sin}\gamma) = 4\,\Delta\,S$

(b) Prove $\operatorname{Sin}^{-1}\dfrac{1}{\sqrt{5}} + \operatorname{Cot}^{-1} 3 = \pi/4$

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$\tan\frac{x}{3}$

$\left(\frac{x}{3} + \pi\right)$

$\frac{1}{3}(x + 3\pi$