This is a Title with Each Word Capitalized and Support α, β and Break of Line.

A thesis

submitted in partial fulfillment of the

requirement for the degree of

Master of Science in Mathematics

by

Muhammad Kamran

CIIT/FA14-RMT-001/ATK

COMSATS UNIVERSITY ISLAMABAD

Attock Campus

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COMSATS UNIVERSITY ISLAMABAD
ATTOCK CAMPUS

FINAL APPROVAL

This thesis titled

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submitted to the Department of Mathematics

by

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Contents

Abstract

A function is convex if the line segment joining two points on the graph lies above the graph. These functions have important properties and applications in mathematics. Specially, they are very important in optimization and minimization problems. Also these functions are used in statistic and functional analysis. A positive function f is logarithmic convex if log f is convex. It would seem that log convex functions unremarkable because they are so simply related to convex functions. But they have some surprising properties.

In the first chapter we generalize results for logarithmic convexity of Giaccardi's difference for classes of functions with the help of divided difference.

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Student Name

Chapter 1

Introduction

In this chapter we will give the notions and definitions of monotone function, convex function, some important inequalities and properties which will frequently use in the rest of chapters to proof our results.

1.1 Monotone function

The monotone function are such function which maintain the order of inequalities. It is often defined on the interval I, where I is also an order set (see [**?**, **?**]).

Definition 1.1.1. A function $f : I \to \mathbb{R}$ is said to be nondecreasing (respectively nonincreasing) if $x_1 < x_2$ implies $f(x_1) \leq f(x_2)$ (respectively $f(x_1) \geq f(x_2)$) for $x_1, x_2 \in I$. We say that f is increasing (respectively decreasing) if $x_1 < x_2$ implies $f(x_1) < f(x_2)$ (respectively $f(x_1) > f(x_2)$) for all $x_1, x_2 \in I$.

Example 1.1.2. The function $f : [0, \infty) \to \mathbb{R}$ given by $f(x) = x^2$ is increasing on $[0, \infty)$, while $f : (-\infty, 0] \to \mathbb{R}$ given by $f(x) = x^2$ is decreasing on $(-\infty, 0]$

Definition 1.1.3. A function is said to be monotone on an interval I, if it is either increasing or decreasing.

The following criteria is often used to investigate the monotonicity of function (see [**?**, **?**]).

Proposition 1.1.4. *Suppose that a function f is continuous on the closed interval $[a, b]$ and has a derivative at each point of the open interval (a, b).*

1. *If $f'(x)$ is positive for all x in (a, b), then f is increasing function on $[a, b]$.*

2. *If $f'(x)$ is negative for all x in (a, b), then f is decreasing function on $[a, b]$.*

1.2 Convex function

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} \leq \frac{f(x_2) - f(x_3)}{x_2 - x_3} \tag{1.1}$$

Convex geometry as a new field of mathematics takes it origin from the publication of the book by Minkowski [**?**]. This book influenced the formation of a new field in mathematics, viz., functional analysis [**?**]

Definition 1.2.1. A function $f : I \to \mathbb{R}$ is said to be convex if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) \tag{1.2}$$

for all $x, y \in I$ and $t \in [0, 1]$.

A function f is said to strictly convex on I if (1.2) is strict for $x \neq y$.

If f is convex function on I, then for $x_1 < x_2 < x_3$ the inequality

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2} \leq \frac{f(x_2) - f(x_3)}{x_2 - x_3} \tag{1.3}$$

holds.

Lemma 1.2.2. *This is an example of lemma $x + 2y$ and this is good.*

Proposition 1.2.3. *This is an example of proposition $x + 2y$ and this is good.*

Theorem 1.2.4. *This is an example of thorem $x + 2y$ and $2x - y$ and this is good.*

Corollary 1.2.5. *This is an example of thorem $x + 2y$ and $2x - y$ and this is good.*

(1) This is first

(2) This is second

(3) This is third

Here is one more.

A- This is first

B- This is second

C- This is third

Chapter 2

Title of 2nd chap

good

- good
- very good

2.1 Title 1st Section

$$x^2 \tag{2.1}$$

$$y^2 \tag{2.2}$$

2.2 Title of 2nd Section

$$z^2 \tag{2.3}$$

[1] [2] [3] [4]

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